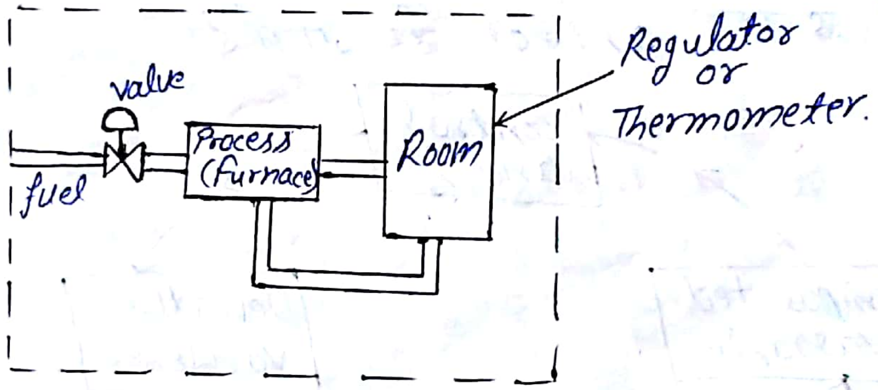
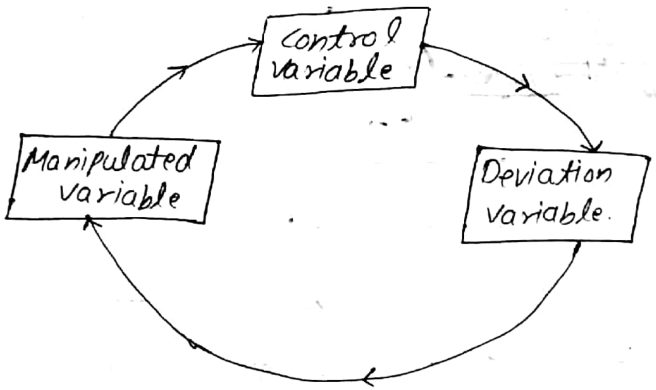
Automatic Control :→ किसी system में मापी गयी राशि की desired value से तुलना करने पर जो difference आता है, उसे दूर करने के लिए जो action प्रारम्भ किया जाता है, Automatic control कहलाता है।

अतः Automatic control द्वारा measured value तथा desired value के मान के difference को दूर किया जाता है।



Example :→ माना किसी कमरे का temp. 72°F पर maintain किया जाता है। इस temperature को desired value या set point कहते हैं। इस system में एक Thermometer कमरे की दीवार के एक side में लगा देते हैं जो कि कमरे का temp. measure करता है, इस temp. को control variable कहते हैं। कोई व्यक्ति thermometer की सहायता से room का temp. 69°F measure करता है इस प्रकार हम देखते हैं कि measured value का मान desired value से कम है।

इन दोनों values के difference को actuating signal कहते हैं। इसलिए वहाँ पर actuating signal का मान $(72-69)^{\circ}F = 3^{\circ}F$ होता है। इस difference को maintain करने के लिए hot gaseous को furnace में भेजा जाता है। Hot gases को manipulated variable कहते हैं। इस प्रकार Control variable, Deviation variable और manipulated variable में एक close loop बन जाता है। इस प्रकार कमरे का temperature automatic control द्वारा control हो जाता है।



Advantages of Automatic control :→

(1.) Product की एकरूपता में सुधार होता है।
(2.) Product की quantity में वृद्धि करना।
(3.) Processing raw material की बचत होती है।
(4.) Plant Equipment की संख्या में कमी।
(5.) Mannual Power की कम जरूरत पड़ती है।

(6.) प्रयोग किये जाने वाले Apparatus को damage से बचाता है।
(7.) आवश्यक energy saving.।
(8.) Processing time को कम करता है।
(9.) Increased (Power) Production, व optimum Cost.।

Industrial Application of Automatic Process control :→

Automatic Process control का use Industrial operation की लगभग सभी phases में होता है।

(1.) Chemical, Petroleum, food, Power, steel etc की Processing Industries में temp. flow Rate, Press. आदि variables को control करने में।
(2.) Goods Manufacture Industries में (Automatic Parts Refrigrator etc.) assembly operations, work, flow, heat treating जैसे अनेक operations को control करने में।
(3.) Railways, Aeroplanes, free missile and Ships जैसे transporations system में।
(4.) Machine, tools, Compressor, Pumps जैसी Power machine और Power supply unit में Position, speed Power को control करने में।

Difference between Automatic Control & Mannual Control

Automatic Control:→

Automatic Process Control का use वहाँ किया जाता है जहाँ पर किसी राशि का मान बहुत तेजी से बदलता है। Automatic Control में error की सम्भावना काफी कम होती है। इसमें error की calculation machine द्वारा की जाती है। इसके द्वारा प्राप्त Product की quality काफी अच्छी होती है।

Example:→ माना हमें किसी Thermometer द्वारा 1 sec. में 10 reading लेनी हो तो हम mannual Method द्वारा time note नहीं कर सकते क्योंकि हमारी आँख इतनी sensible नहीं होती है। अतः हम Automatic Control का use करते हैं।

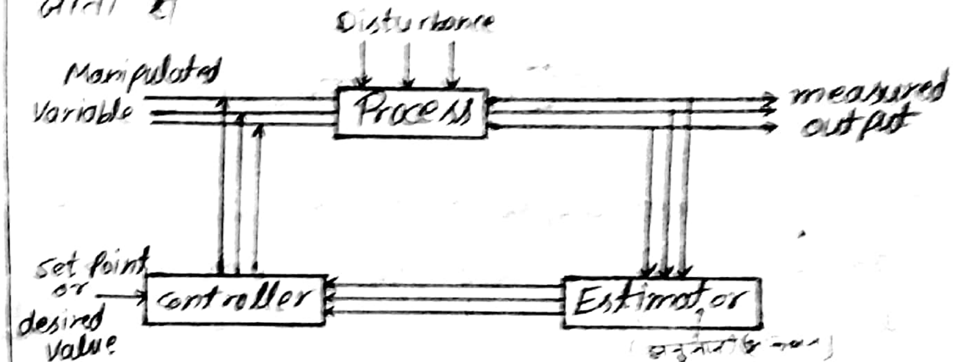
Mannual Control:→

Mannual process Control में मनुष्य स्वयं Process को control करता है। इसमें error की सम्भावनायें काफी अधिक होती हैं। इसके द्वारा प्राप्त Product की quality अच्छी नहीं होती है। इसमें desired value को maintain मनुष्य द्वारा किया जाता है। इस प्रकार के control को mannual Control कहते हैं।

Classification of Automatic Control Systems→

Automatic Control System को निम्न दो भागों में बाँटा जा सकता है:-

(1) Open Loop or feed forward Control:→ यदि main variable का use input को adjust करने में नहीं किया जाता है, तो इस प्रकार के System का use किया जाता है।

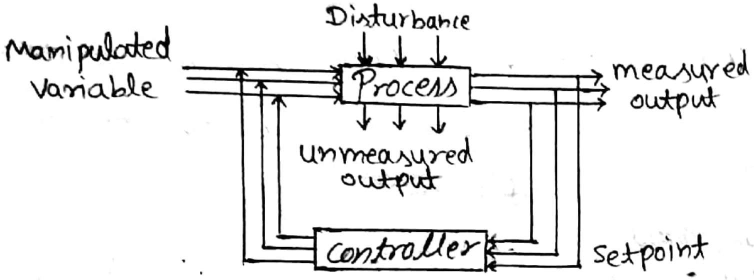


इस प्रकार के Process में manipulated Variable को disturbance द्वारा disturb किया जाता है जिससे measured output प्राप्त होता है। अतः measured output के द्वारा Estimator से होते हुए Controller द्वारा manipulated variable को control किया जाता है। अतः यह Process चलता रहता है इसलिए इसे open loop या feed forward variable कहते हैं। इसे physical और Block diagram द्वारा देख सकते हैं।

(2) Close Loop or feed backward Control:→

माना इस प्रकार के process में manipulated variable disturbance द्वारा disturb हो रहे हैं, जिससे दो प्रकार के output प्राप्त हो रहे हैं।

(a) measured output (b) unmeasured output.



अतः measured output के मान को set point के controller के द्वारा manipulated variable के मान को control किया जाता है। इसे physical या block diagram के द्वारा देख सकते हैं।

Advantages of open Loop:→

(1) Open loop का use वहाँ किया जाता है जहाँ पर input variable के मान में परिवर्तन का कोई महत्व नहीं होता है।

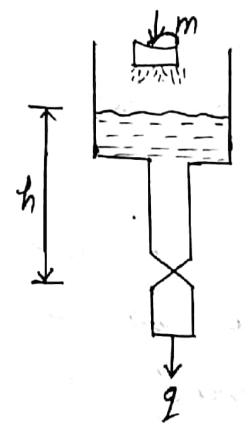
(2) इसका use वहाँ किया जाता है जहाँ पर Closed or feed backward का use नहीं किया जाता है।

Arrows:→ Arrows के द्वारा fluid flow की direction तथा process variable एवं controlled variable ज्ञात किये जाते हैं।

Block:→ Block का use transfer function को गणितीय रूप में show करने में किया जाता है।

Physical Diagram:→

Physical diagram main रूप से किसी system के main part को show करता है। यह material एवं energy के flow को show करता है। यह Automatic तथा mannual control दोनों के लिए use होता है। यह variable के मध्य relation को show नहीं करता है। चित्र में एक 'm' inflow तथा 'q' out flow fluid है जिसको vessel में store किया जा रहा है।



where,

m = inflow of liquid

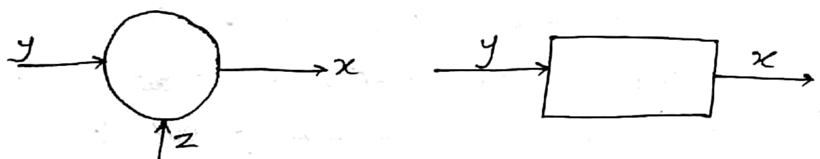
q = out flow of liquid

h = fluid storage hight.

Block Diagram:→

Block diagram विभिन्न element के क्रियात्मक सम्बन्ध को show करता है। Block diagram का use केवल Automatic control के लिए किया जाता है। यह सीधी लाईन द्वारा प्रदर्शित नहीं किये जा सकते हैं। Block diagram variable के मध्य relation को show करता है।

इसमें algebric function के लिए circle से तथा dynamic function के लिए आयताकार बॉक्स से show किया जाता है।



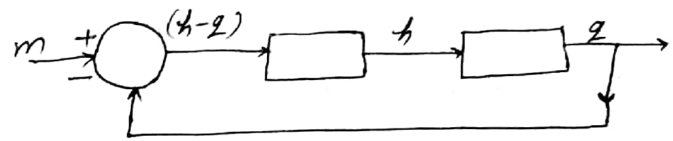
Algebric Function   Dynamic Function.

where, x = output , y = input .

जहाँ x तथा y time के साथ constant रहते हैं।

Block diagram for Process:→

चित्र में एक Block diagram for process को show किया गया है जो selection on view पर depend करता है। चित्र में output flow को important variable माना गया है जो केवल in flow द्वारा प्रभावित होता है।



इस case में variable के relation को निम्न प्रकार दिखाया गया है।

$$q = F(h, t) \quad \text{---} \quad (1)$$
$$h = g(m-q, t) \quad \text{---} \quad (2)$$

overall relationship.

$$q = F[g(m-q, t), t]$$

यहाँ पर block diagram in variable में relation को show करता है।

Inflow को Important variable माना गया है, इस case में variable का relation निम्न प्रकार लिखा जाता है।

$$q = F(h, t) \quad \text{---} \quad (1)$$
$$h = g(m-q, t) \quad \text{---} \quad (2)$$

अतः इसका mean यह है कि block diagram को अनेक प्रकार से show कर सकते हैं।

Input Variable:→

वह variable जो process पर surrounding के प्रभाव को प्रदर्शित करता है; Input variable कहलाता है।

Input variable को दो भागों में devide किया गया है —
(a) Manipulated variable   (b) Disturbance variable.

(a) Manipulated Variable:→

वह variable जिसकी value change कर सकते हैं, 'Manipulated Variable' कहलाते हैं।

OR

वह variable जो automatic controller द्वारा control variable में adjustment के रूप में चुने जा सकते हैं, 'Manipulated variables' कहलाते हैं।

(b) Disturbance variable:→

वह variable जिसकी value change नहीं कर सकते हैं, 'disturbance variable' कहलाते हैं।

OR

वह variable जो automatic controller के द्वारा control variable में adjustment के रूप में नहीं चुने जाते हैं disturbance variable कहलाते हैं।

Output variable:→

output variable को दो भागों में बाँटा गया है —

(a) Measured output (b) Unmeasured output

(a) Measured output:→

वह output जिसे हम measure कर सकते हैं, वे measured output कहलाते हैं।

(b) Unmeasured Variable output:→

वह output जिसे हम measure नहीं कर सकते हैं, unmeasured output कहलाते हैं।

G.L.C. → Gas-Liquid cromotagraph

H.P.C. → High pressure cromotograph.

Transducers:→

Many measurement can not be used for control untill they are converted physical quantity which can be Transducers.

Example:→

current, voltage etc.

Element of control system:→

(i) Actual value की तुलना desired value (set point) से करता है।

(ii) Measuring means के साथ control variable का मान ज्ञात करता है।

(iii) Small position deviation के लिए (विपरीत) counter action उत्पन्न करता है।

(iv) अन्त में इन सभी के द्वारा एक समीकरण drive करता है।

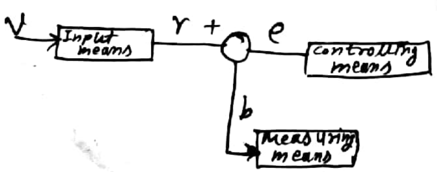
(control action का तरीका) Mode of Control Action:→

वह विधि जिसके द्वारा automatic controller में counter action produce होता है, mode of control system कहलाता है।

एक controller में निम्न function होते हैं —

(1.) Measuring Means (c–b)
(2.) Input means (v–r)
(3.) Actuating means (r–b)
(4.) Controlling means (e–m)
(5.) Final control means (m–c)

where,
c = control variable
b = feed back variable
v = set point
r = Reference Input
m = Manipulated variable
e = actual signal.



(1.) Input Means:→

It means, set point (temp., Press) को reference input में convert करता है। जिसकी unit feed back variable के समान होती है।

$$\boxed{\begin{aligned}\text{Inpt means} &= v - r \\ &= \text{set point} \rightarrow \text{(निर्देश)(अनुमोदक) Reference Point}\end{aligned}}$$

(2.) Measuring Means:→

यह control variable (temp., Press) को Indicating variable (Displacement, pressure, electrical signal) में change करता है।

$$\boxed{\text{Measuring Means} = c \rightarrow b}$$

(3.) Actuating Means:→

यह सामान्यत: substracting device है जो निम्न नियम को follow करती है।

$$\boxed{\begin{aligned}&e = r - b \\ &\text{Actuating means} = \text{Reference input point} - \text{feed back variable}\end{aligned}}$$

(4.) Controlling Means:→

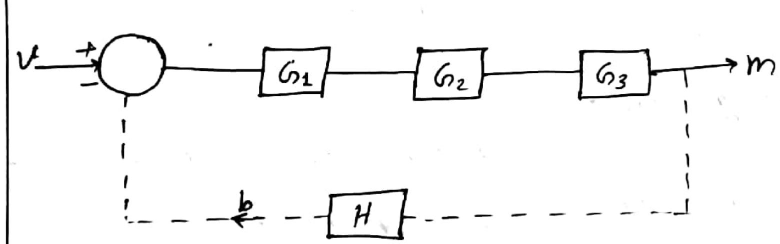
यह actuating signal को amplification (उद्धरण), differential integrating आदि के द्वारा परिवर्तित करती है। जिसमें Con...

output प्राप्त होता है। जो कि manipulated variable के परिमाण में परिवर्तन के लिए final control element को operate करता है।

$\boxed{\text{Controlling means} = e - m}$

(5) **Final Control Element :→**

Final control element एक mechanism है जो कि automatic controller से प्राप्त output signal के आधार पर manipulated variable के मान में change करता है। चित्र में final control element (G_2) को automatic control loop में दिखाया गया है।



where,
G_1 = controlling element
G_2 = final control element
G_3 = Measuring element.

Final control element के निम्न दो भाग होते हैं—

(1) Actuator (2) Device.

(1) **Actuator :→**

इसका use automatic control में output signal को उच्च शक्ति युक्त सदस्य की स्थिति में परिवर्तन करना होता है।

(2) **Device :→**

Device का use manipulated variable में adjust करने में होता है। Actuator एक Accurate output position को provide करता है जो input signal के समानुपाती होता है। Input signal के बजाय बहुत से force output member पर कार्य करते हैं।

Most Important force निम्न हैं—

(i) गतिशील भागों के द्रव्यमान के कारण एक जड़त्व बल लगता है।

(ii) दो surface के adjustment में static friction force लगता है।

(iii) weight and unbalance के कारण धक्के वाले बल लगते हैं।

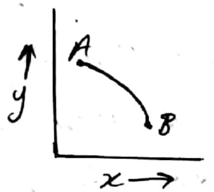
CHAPTER No: 3 PROCESS CHARACTERSTICS

Variable:→

Variable वह physical या chemical property quantity है जो control system के लिए measurement का कार्य करती है। 'Variable' कहलाती है।

Process:→

वह परिवर्तन जो किसी वस्तु के या पदार्थ के physical एवं chemical गुणों में होता है। Process कहलाता है।



Example:→

चित्र में दर्शाया गया है कि कोई पदार्थ Point A पर था, तथा process के दौरान वह Point A से Point B पर गया, इस दौरान उसके physical तथा chemical property में change हुआ, जिसे 'Process' कहते हैं।

Process Variables:→

वह variable जो process के दौरान effect डालते हैं, Process variables कहलाते हैं। इनका use process control के लिए किया जाता है। Process variable का मापन एवं नियंत्रण अनेक प्रकार से किया जाता है। इनका use निम्न के लिए भी किया जाता है।

(i) Product की quality में सुधार।

(ii) Product की दक्षता में सुधार।

(iii) Material की saving।

(iv) Product की testing के लिए।

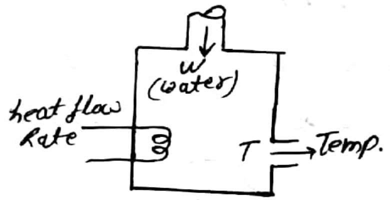
(v) Cost की accounting में।

Example of Process Variables:→ Pressure, Temperature, volume concentration etc.

Controlled Variable:→

वह variable जो किसी भी process की desired form या Product की अवस्था को show करता है, 'Controlled' 'Variable' कहलाता है।

जैसा कि Fig. में एक heater को दिखाया गया है। जिसका काम heat water की supply को maintain करता है इसमें output का temp. 'Controlled Variable' है।



Manipulated Variable:→

वह variable जो किसी automatic Controller के द्वारा Control variable में adjustment के रूप में चुना जाता है। 'manipulated variable' कहलाता है।

इसका use control variable को desired value पर maintain करना है। इसको हम Process Variable भी मान सकते हैं जिसके कारण control variable के response में तेजी से परिवर्तन होता है।

जैसा कि पीछे fig. में दिखाया गया है कि fluid का flow rate तथा temp. manipulated variable है।

Load Variables:→ वह variables जो manipulated variable एवं control variable को छोड़कर शेष बचते हैं load variables कहलाते हैं।

किसी भी process के सभी variable एक-दूसरे से स्वतंत्र होते हैं।

Process degree of freedom:→ Process degree of freedom को निम्न प्रकार define कर सकते हैं —

किसी भी process की state का निर्धारण हम उसकी degree of freedom के द्वारा कर सकते हैं।

किसी भी process की degree of freedom की eqn. को हम निम्न प्रकार define कर सकते हैं।

$$\boxed{n = n_v - n_c}$$

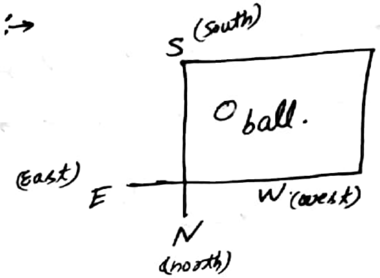
where,
n = no. of freedom.
n_v = no. of variables
n_c = no. of defining component.

इसको phase eqation द्वारा भी define किया जा सकता है।

$$\boxed{F = C - P + 2}$$

where,
F = no. of freedom
P = Phase of any substance
C = Component of any substance.

Example:→



माना किसी मेज पर कोई गेंद स्थित है इसकी position को ज्ञात करने के लिए हमें तीन co-ordinate की आवश्यकता पड़ती है। East-west, North-south व गेंद की ऊँचाई तब,

no. of freedom,

$n_v = 3$, $n_c = 1$

$n = n_v - n_c$

$n = 3 - 1$

$\boxed{n = 2.}$

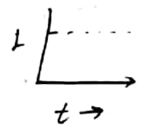
Forcing Function:→ वह function जो किसी भी system को प्रभावित करता है. 'forcing function' कहलाता है। Laplace transformation equation को हम निम्नरूप में लिख सकते हैं —

$$a_n \frac{d^n x}{ds^n} + a_{n-1} \frac{d^{n-1} x}{ds^{n-1}} + \cdots + a_1 \frac{dx}{ds} + a_0 x = 0.$$

जहाँ $a_n, a_{n-1} \ldots\ldots a_1, a_0$ constant हैं तथा $f(t)$ को forcing function कहते हैं।

Step Function:→ यदि Input value में change हेतु समय नगण्य है तो इसे 'Step Function' कहते हैं।

$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



then,

$$L f(t) = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$L f(t) = \int_0^\infty e^{-st} \cdot 1\, dt$$

$$L f(t) = \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$L f(t) = -\frac{1}{s}\left[e^{-s(\infty)} - e^{-s(0)}\right]$$

$$L f(t) = -\frac{1}{s}[0-1]$$

$$\boxed{L f(t) = \frac{1}{s}}$$

Response of first order step input for first order:→

We know that,

$$\frac{\text{output}}{\text{input}} = \frac{Y(s)}{X(s)} = \frac{1}{\tau s+1}$$

$$Y(s) = X(s) \frac{1}{\tau s+1}$$

$$Y(s) = \frac{1}{s} \cdot \frac{1}{\tau s+1} \quad \text{——— (1)} \quad \left\{\because X(s) = \frac{1}{s}\right\}$$

Let $\frac{1}{s(\tau s+1)} = \frac{A}{s} + \frac{B}{(\tau s+1)}$ ——— (i)

$1 = A(\tau s+1) + Bs$ ——— (ii)

Put $s = 0$ {Putting equal to zero, factor of the 'B' i.e. $s = 0$.}

$1 = A(\tau \cdot 0 + 1) + B \times 0$

$\boxed{A = 1.}$

{Putting equal to zero the factor of 'A', i.e. $(\tau s+1) = 0$, $s = -1/\tau$}

Put $s = -\frac{1}{\tau}$

$1 = A(0) + B\left[-\frac{1}{\tau}\right]$

$\boxed{B = -\tau}$

Putting the values of A & B in eq^n (i).

$$\frac{1}{s(\tau s+1)} = \frac{1}{s} - \frac{\tau}{(\tau s+1)} \quad \{\text{from eq}^n\ (1)\ \&\ (i)\}$$

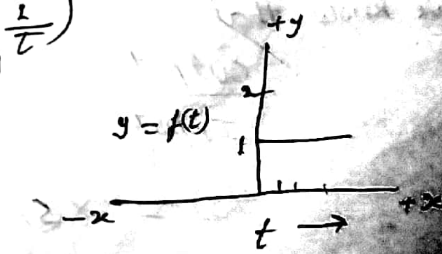
$$\therefore Y(s) = \frac{1}{s} - \frac{\tau}{\tau s+1}$$

Taking inverse laplace transformation.

$$y(t) = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{\tau}{\tau s+1}\right)$$

$$y(t) = 1 - L^{-1}\left(\frac{1}{s+\frac{1}{\tau}}\right)$$

$$\boxed{y(t) = 1 - e^{-t/\tau}}$$



Ramp Function:→

यदि input में change time के सापेक्ष linear हो तो उसे 'Ramp function' कहते हैं Ramp function को mathematically रूप में निम्न प्रकार show करते हैं

$$f(t) = \begin{cases} 0 & t < 0 \\ t & t > 0 \end{cases}$$

then

$$L f(t) = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$L f(t) = \int_0^\infty \underset{II}{e^{-st}} \cdot \underset{I}{t}\, dt$$

$$L f(t) = (t)\left[\frac{e^{-st}}{-s}\right]_0^\infty - \int_0^\infty 1 \cdot \frac{e^{-st}}{(-s)}\, dt$$

$$L f(t) = [t \times 0] - \left[\frac{e^{-st}}{s^2}\right]_0^\infty$$

$$L f(t) = 0 - \frac{1}{s^2}[0-1]$$

$$\boxed{L f(t) = \frac{1}{s^2}}$$

Response of Ramp function Input for first order:→

we know that,

$$\frac{\text{output}}{\text{Input}} = \frac{Y(s)}{X(s)} = \frac{1}{\tau s+1}$$

$$Y(s) = X(s) \cdot \frac{1}{\tau s+1}$$

$$Y(s) = \frac{1}{s^2} \cdot \frac{1}{\tau s+1} \quad ---(1) \qquad \left\{\because X(s) = \frac{1}{s^2}\right\}$$

Let,

$$\frac{1}{s^2(\tau s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(\tau s+1)} \quad \text{———} (i)$$

$$1 = As(\tau s+1) + B(\tau s+1) + Cs^2 \quad ----(ii)$$

Put $s=0$,

$$\boxed{1 = B}$$

Put $s = -1/\tau$

$$1 = C\left(\frac{1}{\tau}\right)^2$$

$$\boxed{C = \tau^2}$$

compairing coefficient of s,

$$A + \tau B = 0$$

$$A + \tau = 0$$

$$\boxed{A = -\tau}$$

Putting the values of A, B & C in eqⁿ (i).

$$\frac{1}{s^2(\tau s+1)} = \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{(\tau s+1)}$$

$$\therefore Y(s) = -\frac{\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{(\tau s+1)}$$

Taking Inverse Laplace transformation,

$$y(t) = L^{-1}\left[-\frac{\tau}{s}\right] + L^{-1}\left[\frac{1}{s^2}\right] + L^{-1}\left(\frac{\tau^2}{\tau s+1}\right)$$

$$\boxed{y(t) = -\tau(1) + t + \tau e^{-t/\tau}}$$

Impulse Function:→

This is a special case of pulse function is which $t_0 = 0$ but the area Ai under the impulse function remain constant and finite.

Thus for the impulse function —

$$f(t) = \left[\frac{m}{s}\left(t - e^{-st_0}\right)\right]$$

$$= \lim_{t_0 \to 0} \frac{m}{s}\left(t - e^{-st}\right)$$

$$= \lim_{t_0 \to 0} \frac{Ai}{s}\left[\frac{0 + e^{-st_0} + s}{t}\right]$$

Sinusiodal Function:→

वह Input change जिसमें cyclic properties होती है, sinusiodal function कहलाता है। इसे mathematically रूप में निम्न प्रकार लिखते हैं :—

$$f(t) = \begin{cases} 0 & t < 0 \\ A\sin at & t > 0 \end{cases}$$

then,

$$Lf(t) = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^\infty e^{-st} A\sin At\, dt \qquad \left\{\because \sin at = \frac{e^{iat} - e^{-iat}}{2i}\right\}$$

$$= A\int_0^\infty e^{-st} A\left(\frac{e^{iat} - e^{-iat}}{2i}\right) dt$$

$$= \frac{A}{2i}\int_0^\infty \left\{e^{-t(-ia+s)} - e^{-t(ia+s)}\right\} dt$$

$$= \frac{A}{2i}\left[\int_0^\infty e^{-(ia+s)t}\, dt - \int_0^\infty e^{-(ia+s)t}\, dt\right]$$

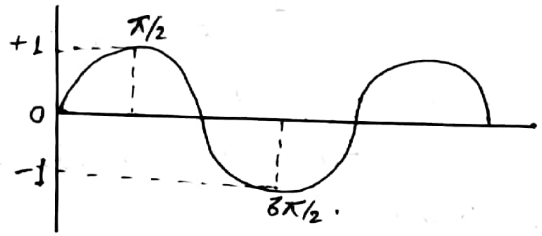
$$= \frac{A}{2i}\left[\frac{e^{-(ia+s)t}}{(ia-s)} + \frac{e^{-(ia+s)t}}{(ia+s)}\right]_0^\infty$$

$$= \frac{A}{2i}\left[0 + 0 - \frac{1}{(ia-s)} - \frac{1}{(ia+s)}\right]$$

$$= \frac{A}{2i}\left[\frac{1}{s-ia} - \frac{1}{s+ia}\right]$$

$$= \frac{A}{2i}\left[\frac{s+ia-s+ia}{s^2+a^2}\right]$$

$$\boxed{Lf(t) = \frac{Aa}{s^2+a^2}}$$



Responce of a sinusiodal function for first order:→

we know that,

$$\frac{\text{output}}{\text{Input}} = \frac{Y(s)}{X(s)} = \frac{1}{(\tau s+1)}$$

$$Y(s) = X(s)\cdot\frac{1}{(\tau s+1)}$$

$$Y(s) = \frac{Aa}{s^2+a^2}\cdot\frac{1}{(\tau s+1)} \quad \text{--- (1)}$$

$$\left\{\because X(s) = \frac{Aa}{s^2+a^2}\right\}$$

Let, $$\frac{Aa}{(s^2+a^2)(\tau s+1)} = \frac{A's+B}{s^2+a^2} + \frac{c}{(\tau s+1)} \quad \text{--- (i)}$$

$$Aa = A's(\tau s+1) + B(\tau s+1) + c(s^2+a^2) \quad \text{--- (ii)}$$

Put $s = -1/\tau$ we get

$$Aa = c\left[\frac{1}{\tau^2}+a^2\right]$$

$$\boxed{c = \frac{Aa\tau^2}{1+a^2\tau^2}}$$

Compairing the coefficient of s^2

$$A'\tau + c = 0$$

$$A' = -\frac{Aa\tau^2}{\tau(1+a^2\tau^2)}$$

$$\boxed{A' = -\frac{Aa\tau}{1+a^2\tau^2}}$$

Campairing the coefficient of s.

$$B\tau + A' = 0$$

$$\boxed{B = +\frac{Aa}{1+a^2\tau^2}}$$

Putting the values of A', B & C in equation(i).

$$\frac{Aa}{(s^2+a^2)(\tau s+1)} = -\frac{Aa\tau s}{(s^2+a^2)(1+a^2\tau^2)} + \frac{Aa}{(s^2+a^2)(1+a^2\tau^2)} + \frac{Aa\tau^2}{(\tau s+1)(1+a^2\tau^2)}$$

$$\therefore Y(s) = -\frac{Aa\tau s}{(s^2+a^2)(1+a^2\tau^2)} + \frac{Aa}{(s^2+a^2)(1+a^2\tau^2)} + \frac{Aa\tau^2}{(\tau s+1)(1+a^2\tau^2)}$$

Taking inverse Laplace transformation,

$$L^{-1}Y(s) = -\frac{Aa\tau}{(1+a^2\tau^2)}L^{-1}\left(\frac{s}{s^2+a^2}\right) + \frac{A}{(1+a^2\tau^2)}L^{-1}\left(\frac{a}{s^2+a^2}\right) + \frac{Aa\tau}{(1+a^2\tau^2)}\cdot L^{-1}\left(\frac{1}{s+\frac{1}{\tau}}\right)$$

$$\boxed{y(t) = -\frac{Aa\tau}{(1+a^2\tau^2)}\cos at + \frac{A\sin at}{(1+a^2\tau^2)} + \frac{Aa\tau\cdot e^{-t/\tau}}{(1+a^2\tau^2)}}$$

Exponential Function:→

$$f(t) = \begin{cases} 0 & t < 0 \\ e^{-at} & t > 0 \end{cases}$$

then,

$$Lf(t) = \int_0^\infty e^{-st} \cdot f(t)\,dt.$$

$$= \int_0^\infty e^{-st} \cdot e^{-at}\,dt.$$

$$= \int_0^\infty e^{-(s+a)t}\,dt$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)}\right]_0^\infty$$

$$Lf(t) = \frac{1}{(s+a)}\left[-(0-1)\right]$$

$$\boxed{Lf(t) = \frac{1}{s+a}}$$

Responce of Exponential Function Input for first order:→

we know that;

$$\frac{\text{Output}}{\text{Input}} = \frac{Y(s)}{X(s)} = \frac{1}{(\tau s+1)}$$

$$Y(s) = X(s) \cdot \frac{1}{(\tau s+1)}$$

$$Y(s) = \frac{1}{(s+a)} \cdot \frac{1}{(\tau s+1)} \text{ ——— } (1)$$

$$\left[\because X(s) = \frac{1}{s+a}\right]$$

Let $\frac{1}{(s+a)(\tau s+1)} = \frac{A}{(s+a)} + \frac{B}{(\tau s+1)}$ --- (i)

$1 = A(\tau s+1) + B(s+a)$ ---- (ii)

Put $s = -a$ {Putting equal to zero, factor of the 'B' i.e. $s+a = 0$, $s = -a$}

$1 = A(-a\tau + 1)$

$$\boxed{A = \frac{1}{1-a\tau}}$$

Put $s = -1/\tau$ {Putting equal to zero the factor of 'A', i.e. $(\tau s+1) = 0$, $s = -\frac{1}{\tau}$}

$1 = B\left[-\frac{1}{\tau} + a\right] \Rightarrow \boxed{B = \frac{\tau}{(a\tau-1)}}$

Putting the values of A and B in eqn (1)

$$\frac{1}{(s+a)(\tau s+1)} = \frac{1}{(1-a\tau)(s+a)} + \frac{\tau}{(a\tau-1)(\tau s+1)}$$

$$\therefore y(s) = \frac{1}{(1-a\tau)(s+a)} + \frac{\tau}{(a\tau-1)(\tau s+1)}$$

Taking Inverse Laplace transformation,

$$L^{-1}y(s) = \frac{1}{(1-a\tau)} L^{-1}\left(\frac{1}{s+a}\right) + \frac{\tau}{(a\tau-1)} L^{-1}\frac{1}{(\tau s+1)}$$

$$\boxed{Y(t) = \frac{e^{-at}}{1-a\tau} + \frac{e^{-t/\tau}}{a\tau-1}.}$$

Laplace Transformation:→

Laplace Transformation एक ऐसी गणितीय विधि है, जिसके द्वारा differential Equations को आसानी से हल किया जा सकता है। Differential equation को transfer करने के लिए algebric equation प्राप्त करनी होती है। जिससे complex variable, स्वतंत्र चर के रूप में line को respect करता है। फिर इस algebric equation को हल किया जाता है। तथा फिर उसका inverse transfer ज्ञात करके शुद्ध हल प्राप्त किया जाता है।

Some Important formulas:→

(1.) $L(1) = \frac{1}{s}$

(2) $L(t) = \frac{1}{s^2}$

(3.) $L(t^2) = \frac{2}{s^3}$

4) $L(t^n) = \frac{n!}{s^{n+1}}$

(5) $L(e^{at}) = \frac{1}{s-a}$

(6) $L(e^{-at}) = \frac{1}{s+a}$

(7.) $L(\sin at) = \frac{a}{s^2+a^2}$

(8) $L(\cos at) = \frac{s}{s^2+a^2}$.

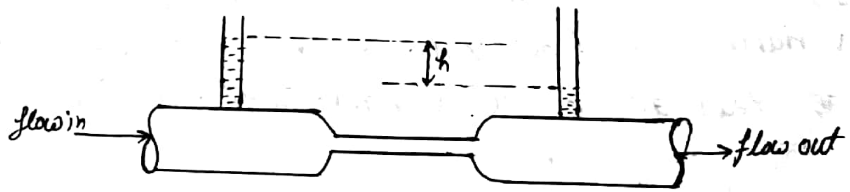
Elements of Process dynamics:→

Automatic controller के लिए किसी भी process का dynamic analysis करना आवश्यक होता है। क्योंकि automatic controller, process की dynamic action पर depend करता है। process की analysis, process dynamic के भिन्न भिन्न Element द्वारा की जाती है। ये element mainly चार प्रकार के होते हैं।

(1.) Proportional element (2.) capacitance Element
(3.) Time Constant Element (4.) Oscillatory Element

(1.) Proportional Element:→

चित्र में एक proportional Element को तथा Physical diagram में एक capillary tube को show किया गया है।



यहाँ पर fluid के flow rate को input variable माना गया है और इस variable q के कारण head h उत्पन्न होता है जिसको output variable कहते हैं। Proportional element में flow rate को इस प्रकार लिख सकते हैं—

$$q = \frac{h}{R}$$

$$h = qR \quad \text{——— (1)}$$

At study state,

$$h_s = q_s R \quad (2)$$

∴ Deviation form.

$$h - h_s = R(q - q_s)$$

$$H = RQ \text{ ——— } (3)$$

Taking laplace transformation,

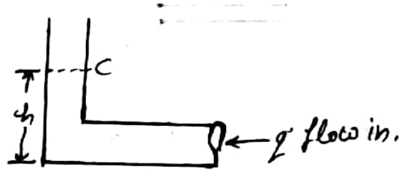
$$H_s = RQ_s \qquad \left\{ \begin{array}{l} \because HL(L) = RQ\,L(1) \\ H/s = RQ\frac{1}{s} \Rightarrow Hs = RQs \end{array} \right\}$$

$$\therefore R = \frac{H_s}{Q_s}$$

where, R = Resistance of capallary.

(2) Capacitance: Element :→

Capacitance Element को figure में show किया गया है। यहाँ rate q को input variable तथा flow Rate के कारण उत्पन्न liquid के head को output variable माना गया है।



यदि output system capacitance पर depend करता है तो इसे capacitance Element कहते हैं। यदि system के output में परिवर्तन की दर input के समानुपाती हो तो ऐसे Element को capacitance element कहते हैं।

Taking material balance,

$$q = c\frac{dh}{dt} \text{ ---- } (1)$$

where,
c = capacitance
h = output variable
t = time
q = input variable

At study state,

$$q_s = c\frac{dh_s}{dt} \text{ ——— } (2)$$

Deviation form.

$$q - q_s = c\left[\frac{dh}{dt} - \frac{dh_s}{dt}\right]$$

~~Deviation form.~~

$$q - q_s = c\frac{d}{dt}(h - h_s)$$

$$Q = c\frac{dH}{dt} \text{ ——— } (3)$$

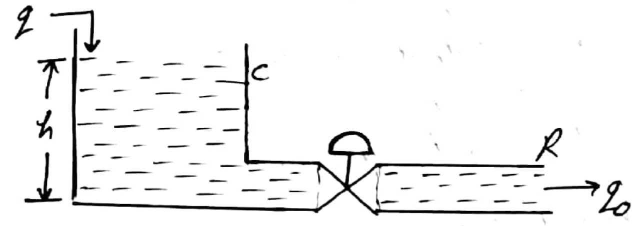
Taking Laplace Transformation,

$$Q_s = C_s H_s$$

$$\boxed{\frac{1}{C_s} = \frac{H_s}{Q_s}}$$

(3) Time Constant Element :→

Time Constant Element को figure में show किया गया है।



Input variable q तथा output variable q_0 माना गया है। capacitance C तथा tank का प्रतिरोध R है। Per unit time flow rate, head पर depend करता है। इसे इस प्रकार परिभाषित किया जा सकता है —

$$q_0 = \frac{h}{R} \quad \text{——— (1)}$$

Taking material balance around tank

Mass flow rate in - mass flow rate (out)
= Accumulation.

$$\rho q - \rho q_0 = \frac{d}{dt}(\rho A h) \qquad \left\{ \begin{array}{l} \because \dot{m} = \rho V A, \; VA = q \\ \dot{m} = \rho q \end{array} \right\}$$

$$\rho(q - q_0) = \rho \frac{dh}{dt} \cdot A$$

$$\left\{ \begin{array}{l} A = C, \cancel{\text{Accumulation}} = ? \\ \text{Area} = \text{capacitance} \end{array} \right\}$$

$$q - q_0 = C \frac{dh}{dt} \quad \text{——— (2)}$$

At study state,

$$q_s - q_{0s} = C \frac{dh_s}{dt} \quad \text{——— (3)}$$

In deviation form.

$$(q - q_s) - (q_0 - q_{0s}) = C \frac{d}{dt}(h - h_s) \qquad \left\{ \begin{array}{l} \because q_0 = \frac{h}{R} \\ \therefore Q_0 = \frac{H}{R} \end{array} \right\}$$

$$Q - Q_0 = C \frac{dH}{dt}$$

$$Q - \frac{H}{R} = C \frac{dH}{dt} \quad \text{——— (4)}$$

Taking Laplace transformation,

$$Q_s - \frac{H_s}{R} = CsH_s$$

$$RQ_s - H_s = CsH_sR$$

$$RQ_s = H_s + CsH_sR$$

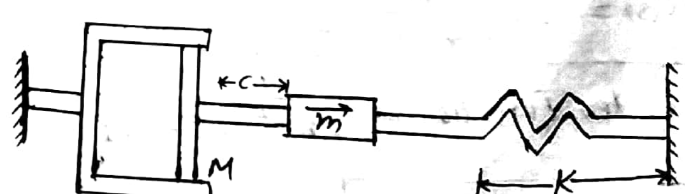
$$RQ_s = H_s(RCs + 1)$$

$$\frac{H_s}{Q_s} = \frac{R}{(RCs + 1)}$$

$$\boxed{\frac{H_s}{Q_s} = \frac{R}{(\tau s + 1)}} \qquad \left\{ \begin{array}{l} \because \tau = RC \\ \therefore \tau s = RCs \end{array} \right\}$$

(4) Oscillatory Element :→

Oscillatory Element को नीचे चित्र में दिखाया है। इस system में mass spring तथा damping system को show किया गया है।



(black weight)

According to Newton's second law,

$$M\frac{d^2c}{dt^2} = -B\frac{dc}{dt} - KC + m$$

$$M\frac{d^2c}{dt^2} + B\frac{dc}{dt} + KC = m \quad \text{——— (1)}$$

where,

C = output variables.

M = Mass of system (block)

m = Input variable force.

K = Hook's constant (lbs/ft)

B = viscous damping coefficient.

Taking laplace transformation.,

$$MS^2 C_{(s)} + BS C_{(s)} + K C_{(s)} = m_{(s)}$$

$$C_{(s)} = \frac{m_{(s)}}{K\left[\frac{MS^2}{K} + \frac{BS}{K} + 1\right]} \quad \text{——— (2)}$$

where, characteristics time $(T) = \sqrt{\frac{m}{K}}$

$$T^2 = \frac{m}{K}$$

Damping ratio. $(F) = \sqrt{\frac{B^2}{4Km}}$

from equation (2)

$$C_{(s)} = \left[\frac{1}{K}\,\frac{1}{T^2S^2+2FTS+1}\right] m_{(s)}$$

$$\boxed{C_{(s)} = \frac{\frac{1}{K}\cdot m_{(s)}}{T^2S^2+2FTS+1}}$$

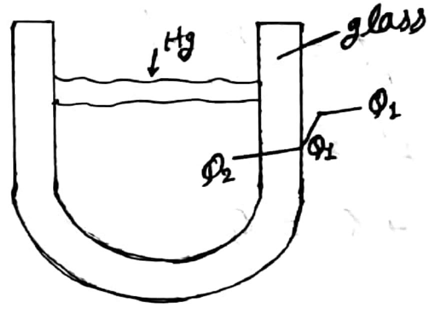
where, system function $= \dfrac{\frac{1}{K}}{T^2S^2+2FTS+1}$

$$\boxed{\frac{\frac{1}{K}}{T^2S^2+2FTS+1}}$$

Determination of system function or transfer function of the following:→

(A) First order system or time constant element:→

(i) Nacked bulb thermometer:→ यह एक first order system है जिसके response को first order linear differential equation द्वारा व्यक्त किया जाता है।



$\theta_1 > \theta_2$.

माना mercury thermometer को अचानक θ_1 temp के water में डुबो दिया जाता है, माना glass की thermal capacity नगण्य तथा mercury का temp. θ_2°C है।

The unsteady steady balance —

Heat input − Heat output = Heat accumulation.

$$UA(\theta_1 - \theta_2) - 0 = mc_p\frac{d\theta_2}{dt}$$

$$UA\theta_1 - UA\theta_2 = mc_p\frac{d\theta_2}{dt} \quad \text{----(1)}$$

where, m = mass of Hg.

c_p = Heat capacity of Hg.

U = overall coefficient.

t = time

A = heat transferred area.

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उपरोक्त समी० को इस प्रकार arrange करेंगे ताकि θ_2 के सभी term L.H.S. में θ_1 के सभी term R.H.S. में आ जाएं

$$mC_p\frac{d\theta_2}{dt} + UA\theta_2 = UA\theta_1$$

$$\frac{d\theta_2}{dt}\cdot\frac{mC_p}{UA} + \theta_2 = \theta_1 \quad \text{—— (2)}$$

group $\frac{mC_p}{UA}$ time की unit है। अतः $\frac{mC_p}{UA}$ को system का time constant कहते हैं।

$$\boxed{\tau = \frac{mC_p}{UA}}$$

Now from Eq^n. (2)

$$\tau\frac{d\theta_2}{dt} + \theta_2 = \theta_1 \quad \text{—— (3)}$$

Taking Laplace transformation.

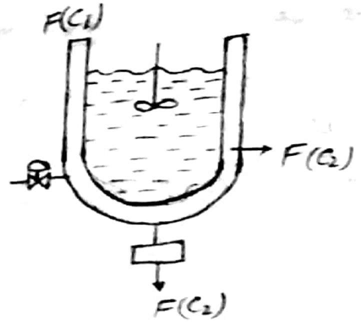
$$\tau\left[S\theta'_{2(s)} + \theta'_{2(0)}\right] + \theta'_{2(s)} = \theta'_{1(s)}$$

$$\tau S\,\theta'_{2(s)} + \theta'_{2(s)} = \theta'_{1(s)}$$

$$\theta'_{2(s)}\left[\tau S + 1\right] = \theta'_{1(s)}$$

$$\boxed{\frac{\theta'_{2(s)}}{\theta'_{1(s)}} = \frac{1}{(\tau S+1)}}$$

(ii) Stirred tank heater :→ चित्र में stirred tank heater को दिखाया गया है जिसका use process की steam को heat करने में किया जाता है।



mass balance.

$$FC_1 - FC_2 = V\frac{dC_2}{dt} \quad \text{—— (1)}$$

At steady state

$$F(C_1)_s - FC_{2(s)} = 0$$

माना F input और output का volumetric flow rate है। ρ और C_p liquid की density & specific heat तथा C_p constant है।

Taking deviation equation,

$$F(C_1 - C_2) = V\frac{dC_2}{dt}$$

$$C_1 - C_2 = \frac{V}{F}\frac{dC_2}{dt} \qquad \left\{\because \frac{V}{F} = \tau\right\}$$

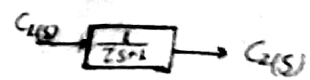
$$C_1 - C_2 = \tau\frac{dC_2}{dt}$$

Taking Laplace transformation.

$$C_{1(s)} - C_{2(s)} = \tau S\, C_{2(s)}$$

$$\tau S = \frac{C_{1(s)}}{C_{2(s)}} - 1$$

$$\tau S + 1 = \frac{C_{1(s)}}{C_{2(s)}}$$



$$\boxed{\frac{C_{2(s)}}{C_{1(s)}} = \frac{1}{\tau S+1}}$$

(iii) **Mixing Process :→**

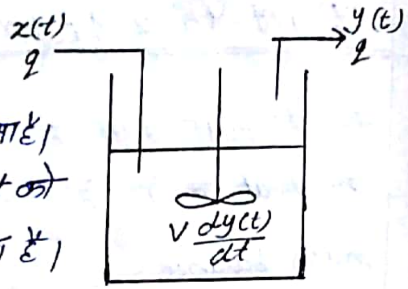


figure में mixing process को दिखाया गया है। इसमें agitator का use salt को dissolve करने में किया जाता है। इसमें solution को q volumetric flow rate पर प्रवेश कराया जाता है तथा solution की density constant रहते है। agitator के कारण vaccum उत्पन्न हो जाता है। Density constant होने के कारण inflow rate, outflow rate के बराबर होगा जिसके कारण volume में भी परिवर्तन नहीं होगा।

Taking material balance arround the tank,

Deviation = unsteady state − steady state

Input rate of salt − output rate of salt = Accumulation

$$q[x(t)] - q[y(t)] = \frac{d}{dt}[V \cdot y(t)]$$

$$q\,x(t) - q\,y(t) = \frac{d}{dt}[V \cdot y(t)] \quad \text{---} \; ①$$

At steady state

∴ Rate of accumulation = 0

$$q\,x_s(t) - q\,y_s(t) = 0 \quad \text{---} \; ②$$

from ① & ②

Deviation form,

$$q[x(t) - x_s(t)] - q[y(t) - y_s(t)] = V\frac{d}{dt}y(t).$$

$$q\,x(t) - q\,y(t) = V\frac{d}{dt}y(t)$$

$$x(t) - y(t) = \frac{V}{q}\frac{d}{dt}y(t) \qquad \left\{\because \frac{V}{q} = \tau\right\}$$

$$= \tau\frac{d}{dt}y(t)$$

$$x(t) = \tau\frac{d}{dt}y(t) + y(t) \quad \text{---} \; ③$$

Taking Laplace transformation,

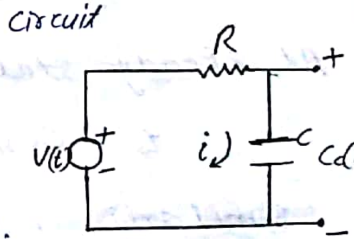
$$X(s) = \tau(s)\,Y(s) + Y(s)$$

$$X(s) = Y(s)[\tau s + 1]$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s + 1}$$

$$\boxed{G(s) = \frac{1}{(\tau s + 1)}}$$

(iv) **R.C. Circuit :→**



चित्र में R.C. circuit को दिखाया गया है। Voltage को $V(t)$ को series में जुड़े Resistance R and capacitance C के circuit में supply की जाती है।

जब $t < 0$ अर्थात् steady state voltage then., $V(t) = V(s)$

Applying Kirchaff's law that states, in any loop voltage rises equal to the voltage drop at, gives,-

~~V(t)~~ $V_{(t)} = Ri_{(t)} + \frac{1}{C}\int i\,dt$

$= Ri_{(t)} + \frac{1}{C} i_{(t)}$ —— (1)

But we know that,

$q = CV$

$V = \frac{q}{C}$

$V = \frac{it}{C}$

$\left\{ \because q = it \quad i = \frac{q}{t},\ i = \frac{dq}{dt} \right\}$

i का मान समी० (1) में रखने पर,

$V_{(t)} = R\frac{dq}{dt}(t) + \frac{1}{C} q(t)$ —— (2)

Since the voltage across the capacitance,

from, $q = CV$

$C_c = \frac{q}{C}$ $\quad \{\because V = C_c\}$

$q = C_c \cdot C$

At steady state

$q_s = C_{c(s)} \cdot C$

Similarly $V_s = \frac{q_s}{C}$

$V_s = \frac{C_{cs} \cdot C}{C}$ $\quad \left\{ \because V = \frac{q}{C},\ q_s = C_{c(s)} C \right\}$

$\boxed{V_s = C_{cs}}$

deviation form,

$V' = V - V_s$

$dq_{(t)} = \phi = q - q_s$

$E_c = C_c - C_{cs} = \frac{\phi}{C}$ $\quad \left\{ \text{where, } E = E.M.F. \right\}$

$\phi = C \cdot E_c$

तब समीकरण (2) से,

$V - V_s = \frac{Rd[q - q_{(s)}]}{dt} + \frac{C_c + C_{c(s)}}{C} = R\frac{d\phi}{dt} + \frac{\phi}{C}$

$V' = RC\frac{dE_c}{dt} + E_c$ —— (3)

Taking Laplace transformation,

$V(s) = RCSE_{c(s)} - RCE_c(0) + E_{c(s)}$

$V(s) = RCSE_{c(s)} + E_{c(s)}$

$V(s) = E_{c(s)} \cdot [RCS + 1]$

$V(s) = E_{c(s)} [RCS + 1]$

$\frac{E_{c(s)}}{V(s)} = \frac{1}{(RCS + 1)}$

$\boxed{\frac{E_{c(s)}}{V(s)} = \frac{1}{(\tau S + 1)}}$ $\quad \{\because \tau = RC\}$

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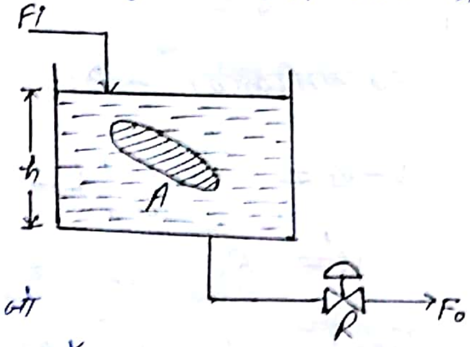
(V) **Liquid Level System:→**

चित्र में एक liquid Level system को दर्शाया गया है जिसकी height 'h' तथा tank का uniform cross-sectional area A एवं liquid का input flow rate 'F_i' तथा output flow rate F_o है। इसमें एक valve लगा है जो Resistance का कार्य करता है।तब,



$$F_o = \frac{\text{Deriving force to flow}}{\text{Resistance to flow}}$$

$$F_o = \frac{h}{R} \quad \text{———} \ (1)$$

mass balance arround the tank.

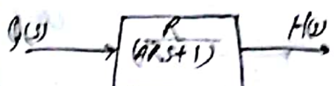
Input − output = Accumulation.

$$F_i - F_o = A\frac{dh}{dt}$$

$$F_i - \frac{h}{R} = A\frac{dh}{dt}$$

$$RF_i - h = AR\frac{dh}{dt}$$

$$RF_i = AR\frac{dh}{dt} + h \quad \text{———} \ (2)$$



Block diagram

At steady state condition.

$$RF_{i(s)} = AR\frac{dh_s}{dt} + h_{(s)} \qquad \left\{\because \frac{dh}{dt} = 0\right\}$$

$$RF_{i(s)} = 0 + h_{(s)}$$

$$RF_{i(s)} = h_{(s)} \quad \text{———} \ (3)$$

Taking deviation,

$$RF_i - RF_{i(s)} = AR\frac{d}{dt}[h - h_{(s)}] + [h - h_s]$$

$$RF_i' = AR\frac{dh'}{dt} + h' \qquad \left\{\begin{matrix}\because AR = \tau_p \\ R = K_p\end{matrix}\right\}$$

$$K_pF_i' = \tau_p\frac{dh'}{dt} + h' \quad \text{———} \ (4)$$

Taking Laplace transformation,

$$K_pF_i'(s) = \tau_p s\, h'_{(s)} + \tau_p h_{(0)} + h'_{(s)}$$

$$K_pF'_{i(s)} = \tau_p s\, h'_{(s)} + h'_{(s)}$$

$$K_pF_{i(s)} = h'_{(s)}[\tau_p s + 1]$$

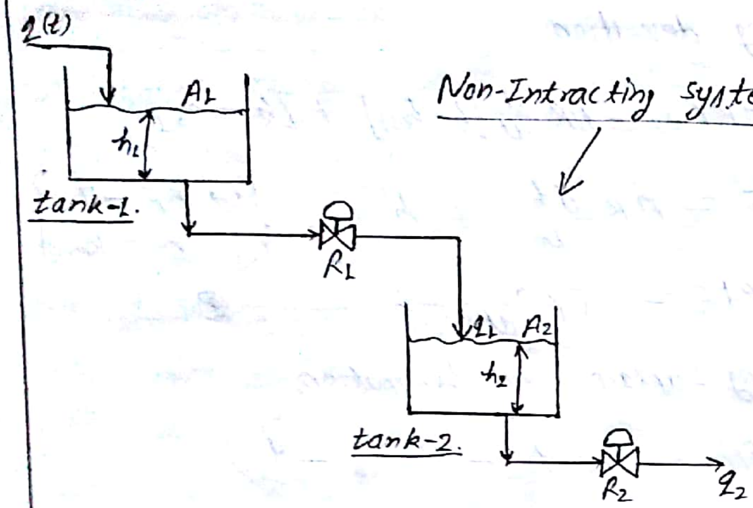
$$\boxed{\frac{h'_{(s)}}{F_{i(s)}} = \frac{K_p}{(\tau_p s+1)}}$$

Two time constant type liquid vessel Cascaded i.e:→

(a) **Non-Intracting System:→**

Non-Intracting system को figure में दिखाया गया है। इस system में tank first का outflow दूसरे tank में R_1 के द्वारा flow करता है तथा वह head h_1 पर depend करता है तथा इसी प्रकार, दूसरे tank में outlet flow का मान h_2 पर depend करता है। इसलिए head (h_1) का प्रभाव h_2 पर निर्भर नहीं करता है। इसलिए इस system को non-Intracting system कहते हैं।

Non-Intracting system



माना चित्र में एक जल प्रवाह cascade system है। q volumetric flow Rate से कोई system stream tank first में enter कर रही है। तथा liquid की density constant है। Tank-1 और Tank-2 के cross-sectional area A_1 व A_2 uniform है। व resistance R_1 व R_2 हैं।

mass balance arround tank-1,

$$q(t) - q_1 = A_1 \frac{dh_1}{dt} \quad \text{--- (1)}$$

mass balance arround tank-2,

$$q_1 - q_2 = A_2 \frac{dh_2}{dt} \quad \text{--- (2)}$$

Linear Resistance,

$$q_1 = \frac{h_1}{R_1}$$

व $$q_2 = \frac{h_2}{R_2}$$

q_1 का मान समी॰ (1) में रखने पर,

$$q(t) - \frac{h_1}{R_1} = A_1 \frac{dh_1}{dt}$$

$$R_1 q(t) - h_1 = A_1 R_1 \frac{dh_1}{dt}$$

$$R_1 q(t) = A_1 R_1 \frac{dh_1}{dt} + h_1 \quad \text{--- (3)}$$

At steady state condition.

$$R_1 q_{(s)} = 0 + h_{1(s)} \quad \text{--- (4)}$$

Deviation form,

$$R_1 [q(t) - q(s)] = A_1 R_1 \frac{dh_1'}{dt} + h_1'$$

$$R_1 Q_1 = A_1 R_1 \frac{dh_1'}{dt} + h_1' \quad \text{--- (5)}$$

Taking Laplace transformation,

$$R_1 Q_1(s) = A_1 R_1 H_1(s) + H_1(s)$$

$$= H_1(s) \cdot [A_1 R_1 + 1]$$

$$\frac{H_1(s)}{Q_1(s)} = \frac{R_1}{A_1 R_1 s + 1}$$

$$\boxed{\frac{H_1(s)}{Q_1(s)} = \frac{R}{(\tau s + 1)}} \quad \left\{ \because AR = \tau,\ R = K_p \right\}$$

or $$\boxed{\frac{H_1(s)}{Q_1(s)} = \frac{K_p}{(\tau s + 1)}}$$

we know that,

$$q_1 = \frac{h_1}{R_1}$$

$$\Rightarrow Q_1 = \frac{H_1}{R_1}$$

$$H_1 = Q_1 R_1$$

or

$$h_1' = q_1' R_1$$

$$\frac{q_1(s)}{q'(s)} = \frac{1}{\tau s+1}$$

or

$$\frac{Q_1(s)}{Q(s)} = \frac{1}{\tau s+1} \quad \text{——— (6)}$$

Taking equation (2)

$$q_1 - q_2 = A_2 \frac{dh_2}{dt}$$

$$\frac{h_1}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt} \quad \text{——— (7)}$$

At seady state,

$$\frac{h_1(s)}{R_1} - \frac{h_2(s)}{R_2} = A_2 \times 0$$

$$\frac{h_1(s)}{R_1} - \frac{h_2(s)}{R_2} = 0 \quad \text{——— (8)}$$

at Deviation form,

$$\frac{1}{R_1}[h_1 - h_1(s)] - \frac{1}{R_2}[h_2 - h_2(s)] = A_2 \frac{dh_2}{dt}$$

$$\frac{H_1}{R_1} - \frac{H_2}{R_2} = A_2 \frac{dH_2}{dt}$$

$$\frac{H_1}{R_1} = A_2 \frac{dH_2}{dt} + \frac{H_2}{R_2} \quad \text{——— (9)}$$

Taking Laplace transformation,

$$\frac{H_1(s)}{R_1} = A_2 s H_2(s) + \frac{H_2(s)}{R_2}$$

$$\frac{H_1(s)}{R_1} = \frac{1}{R_2}[R_2 A_2 s H_2(s) + H_2(s)]$$

$$\frac{R_2}{R_1} H_1(s) = \tau s H_2(s) + H_2(s) \qquad \{\because R_2 A_2 = \tau\}$$

$$\frac{R_2}{R_1} H_1(s) = H_2(s)[\tau s + 1]$$

$$\frac{H_2(s)}{H_1(s)} = \frac{R_2}{R_1(\tau s+1)}$$

$$\frac{H_2(s)}{R_1 Q_1(s)} = \frac{R_2}{R_1(\tau s+1)} \qquad \{\because H_1 = Q_1 R_1\}$$

$$\frac{H_2(s)}{Q_1(s)} = \frac{R_2}{(\tau s+1)} \quad \text{——— (10)}$$

from equation (6) $\cancel{x}$ 10.

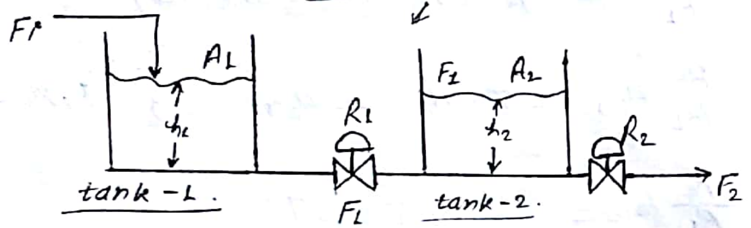
$$\frac{H_2(s)}{Q_1(s)} \times \frac{Q_1(s)}{Q(s)} = \frac{R_2}{(\tau s+1)} \times \frac{1}{(\tau s+1)}$$

$$\boxed{\frac{H_2(s)}{Q(s)} = \frac{R_2}{(\tau s+1)} \cdot \frac{1}{(\tau s+1)}}$$

Two time constant type liquid vessel non-cascaded i.e. →

Interacting System:→ इस system में R_1 से निकलने वाला flow head h_1 व h_2 दोनों पर depend करता है। इसलिए इसे 'interacting system' कहते हैं।

Intracting System



Tank-1 and Tank-2 को figure में show किया गया है। tank-1 का area A_1 तथा hight h_1 है। material balance arround the the tank-1.

$$F_i - F_1 = A_1 \frac{dh_1}{dt} \quad \text{--- (1)}$$

material balance arround the tank-2.

$$F_1 - F_2 = A_2 \frac{dh_2}{dt} \quad \text{--- (2)}$$

Assume linear resistance to flow.

$$F_1 = \frac{h_1 - h_2}{R_1}, \quad F_2 = \frac{h_2}{R_2}.$$

F_1 का मान समी० (1) में रखने पर,

$$F_i - \frac{(h_1 - h_2)}{R_1} = A_1 \frac{dh_1}{dt}$$

$$F_i R_1 \overset{\text{(minus)}}{-} (h_1 - h_2) = A_1 R_1 \frac{dh_1}{dt}$$

$$F_i R_1 = A_1 R_1 \frac{dh_1}{dt} + (h_1 - h_2) \quad \text{--- (3)}$$

At steady state, $\left\{\because \frac{dh_1}{dt} = 0\right\}$

$$h_1 - h_2 = R_1 F_i$$

$$h_{1(s)} - h_{2(s)} = R_1 F_{i(s)} \quad \text{--- (4)}$$

Taking deviation form,

$$A_1 R_1 \frac{dH_1}{dt} + H_1 - H_2 = R_1 F_i'$$

where,

$$H_1 = h_1 - h_{1(s)}$$
$$H_2 = h_2 - h_{2(s)}$$
$$F_i' = F_i - F_{i(s)}$$

Taking Laplace transformation,

$$A_1 R_1 S H_1(s) + H_1(s) - H_2(s) = R_1 F_i'(s)$$

$$H_1(s)[A_1 R_1 S + 1] - H_2(s) = R_1 F_i'(s) \quad \text{--- (5)}$$

Now,

F_1 व F_2 का मान समी० (2) में रखने पर,

$$\frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

$$\frac{h_1}{R_1} - h_2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = A_2 \frac{dh_2}{dt}$$

$$\frac{R_2}{R_1} h_1 - h_2\left[\frac{R_2}{R_1} + 1\right] = A_2 R_2 \frac{dh_2}{dt}$$

$$A_2R_2 \frac{dh_2}{dt} + h_2\left[1+\frac{R_2}{R_1}\right] - \frac{R_2}{R_1} h_1 = 0 \quad \text{——— (6)}$$

At steady state,

$$h_{2(s)}\left[1+\frac{R_2}{R_1}\right] - \frac{R_2}{R_1} h_{1(s)} = 0 \quad \text{——— (7)}$$

At Deviation form,

$$A_2R_2 \frac{d[h_2 - h_{2(s)}]}{dt} + [h_2 - h_{2(s)}]\left[1+\frac{R_2}{R_1}\right] - \frac{R_2}{R_1}[h_1 - h_{1(s)}] = 0$$

$$A_2R_2 \frac{dH_2}{dt} + H_2\left[1+\frac{R_2}{R_1}\right] - \frac{R_2}{R_1} H_1 = 0 \quad \text{——— (8)}$$

Taking Laplace transformation.

$$A_2R_2\, S H_{2(s)} + H_{2(0)} + H_{2(s)}\left[1+\frac{R_2}{R_1}\right] - \frac{R_2}{R_1} H_{1(s)} = 0$$

$$A_2R_2\, S H_{2(s)} + H_2(s)\left[1+\frac{R_2}{R_1}\right] - \frac{R_2}{R_1} H_1(s) = 0$$

$$H_2(s)\left[A_2R_2 S + 1 + \frac{R_2}{R_1}\right] = \frac{R_2}{R_1} H_1(s)$$

$$H_2(s)\left[A_2R_1R_2 S + R_1 + R_2\right] = R_2 H_1(s)$$

$$H_1(s) = \left[A_2R_1R_2 S + R_1 + R_2\right] \frac{H_2(s)}{R_2} \quad \text{——— (9)}$$

$H_1(s)$ का मान समी० ⑤ में रखने पर,

$$\frac{H_2(s)}{R_2}\left[A_2R_1R_2 S + R_1 + R_2\right]\left[A_1R_1 S + 1\right] - H_2(s) = R_1 F_i(s).$$

$$H_2(s)\left[\frac{(A_2R_1R_2 S + R_1 + R_2)(A_1R_1 S + 1)}{R_2} - 1\right] = R_1 F_i(s)$$

$$H_2(s)\left[(A_2R_1R_2 S + R_1 + R_2)(A_1R_1 S + 1) - R_2\right] = R_1 R_2 F_i'(s)$$

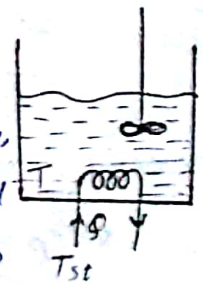
$$\frac{H_2(s)}{F_i(s)} = \frac{R_1R_2}{\left[(A_2R_1R_2 S + R_1 + R_2)(A_1R_1 S + 1) - R_2\right]}$$

$$\boxed{\frac{H_2(s)}{F_i'(s)} = \frac{R_1R_2}{\left[(SR_1\tau_2 + R_1 + R_2)(\tau_1 S + 1) - R_2\right]}} \quad \left\{ \begin{array}{l} \because \tau_1 = A_1R_1 \\ \tau_2 = A_2R_2 \end{array} \right\}$$

Continuous Stirred tank chemical reactor (CSTR) with Ist Order chemical Reaction:→

or Heating system:→

Or CSTR Heater:→



The liquid in the Tank to heated to the Saturated steam, Steam flow by coil. The coil immersed in the liquid. The temp. of liquid is the constant that the volume of liquid is constant.

where,

T = Temp of liquid

T_{st} = Temp. of Saturated steam.

From, energy balance.

$$Q = mC_p \frac{dT}{dt} \qquad \left\{ \because \rho = \frac{m}{V},\ m = \rho V \right\}$$

$$Q_1 = V \rho C_p \frac{dT}{dt} \quad \text{——— (1)}$$

& $$Q_2 = UA\,(T_{st} - T) \quad \text{——— (2)}$$

from energy balance

$$Q_1 = Q_2$$

$$V\rho C_p \frac{dT}{dt} = UA\,(T_{st} - T) \quad \text{——— (3)}$$

where, V = Volume of fluid in the tank.

ρ = density of fluid

C_p = Heat capacity

T_{st} = Temp of saturated steam.

At steady state. $\frac{dT}{dt} = 0$

$$UA(T_{st(s)} - T_{(s)}) = 0$$

$$T_{st(s)} - T_{(s)} = 0 \quad \text{——— (4)}$$

At deviation form

$$V\rho C_p \frac{dT'}{dt} = UA\,(T_{st}' - T') \quad \text{——— (5)}$$

where,

$$T' = T - T_{(s)}$$

$$T_{st}' = T_{st} - T_{st(s)}$$

Taking Laplace transformation of Eqn (5)

$$V\rho C_p \left[sT'_{(s)} - T'_{(0)} \right] = UA \left[T'_{st(s)} - T'_{(s)} \right]$$

$$V\rho C_p\, sT'_{(s)} + UA\,T'_{(s)} = UA\,T'_{st(s)}$$

$$T'_{(s)} \left[V\rho C_p s + UA \right] = UA\,T'_{st(s)}$$

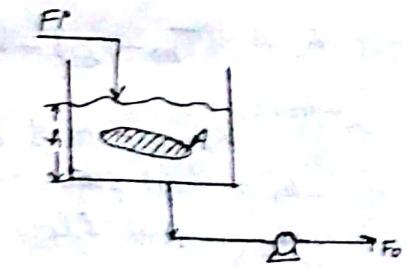
$$\frac{T'_{(s)}}{T'_{st(s)}} = \frac{UA}{(V\rho C_p s + UA)}$$

$$\frac{T'_{(s)}}{T'_{st(s)}} = \frac{1}{1 + \frac{V\rho C_p s}{UA}}$$

$$\boxed{\frac{T'_{(s)}}{T'_{st(s)}} = \frac{1}{(\tau s + 1)}} \qquad \left\{ \because \tau = \frac{V\rho C_p}{UA} \right\}$$

<u>Pure capacitive system:→</u>

mass balance,



$$F_i - F_o = A \frac{dh}{dt} \quad \text{——— (1)}$$

At steady state.

$$F_{i(s)} - F_o = 0 \quad \text{——— (2)}$$

At deviation form.

$$(F - F_{i(s)}) - (F_o - F_o) = A \frac{dh'}{dt}$$

$$F_i' = A \frac{dh'}{dt}$$

Taking Laplace transformation

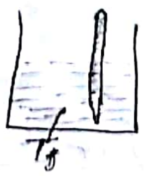
$$F_{i(s)}' = As\,h'_{(s)} - Ah'_{(0)}$$

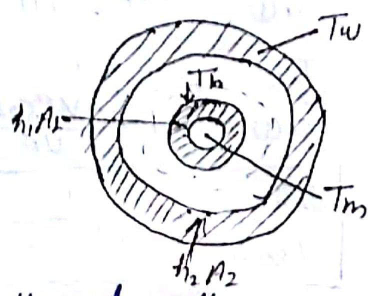
$$\boxed{\frac{h'_{(s)}}{F_{i(s)}} = \frac{1}{As}}$$

B. Second Order System or Oscillatory type element :→

(i) Bulb in thermowell :→

यदि thermometer को है thermowell में रखा जाये तो इसका response second order differential equation द्वारा indicate किया जाता है। इस condition में दो capacitance तथा दो resistance से होकर heat का transmission होगा।





Taking Heat balance for thermal wall–

Input – output = Accumulation.

$$h_1 A_1 (T_m - T_b) - h_2 A_2 (T_b - T_w) = m_1 c_1 \frac{dT_b}{dt} \quad \text{—— (1)}$$

where,

A_1 = Area of bulb, A_2 = Area of thermal wall.

h_1 = heat flow coefficient of thermal wall

h_2 = Heat flow coefficient of mercury

T_b = Temp. of bulb.

T_m = Temp. of mercury

T_w = Temp. of Thermal wall

c_1 = thermal coefficient of thermal wall

c_2 = thermal coefficient of thermal mercury.

Taking Heat Balance of Mercury,

$$h_2 A_2 T_b - h_2 A_2 T_w - 0 = m_2 c_2 \frac{dT_w}{dt} \quad \text{—— (2)}$$

$$h_2 A_2 (T_b - T_w) = m_2 c_2 \frac{dT_w}{dt}$$

$$T_b - T_w = \frac{m_2 c_2}{h_2 A_2} \cdot \frac{dT_w}{dt}$$

$$T_b = T_w + \frac{m_2 c_2}{h_2 A_2} \cdot \frac{dT_w}{dt} \quad \text{—— (3)}$$

T_b का मान समीकरण (1) में रखने पर,

$$h_1 A_1 \left[T_m - T_w - \frac{m_2 c_2}{h_2 A_2} \cdot \frac{dT_w}{dt}\right] - h_2 A_2 \left[T_w + \frac{m_2 c_2}{h_2 A_2} \frac{dT_w}{dt} - T_w\right]$$
$$= m_1 c_1 \left(\frac{dT_w}{dt} + \frac{m_2 c_2}{h_2 A_2} \cdot \frac{d^2 T_w}{dt^2}\right)$$

$$h_1 A_1 T_m - h_1 A_1 T_w - \frac{h_1 A_1 m_2 c_2}{h_2 A_2} \cdot \frac{dT_w}{dt} - h_2 A_2 T_w - \frac{h_2 A_2 m_2 c_2}{h_2 A_2} \cdot \frac{dT_w}{dt} + h_2 A_2 T_w = m_1 c_1 \frac{dT_w}{dt} + \frac{m_1 c_1 m_2 c_2}{h_2 A_2} \cdot \times \frac{d^2 T_w}{dt^2}$$

$$h_1 A_1 h_2 A_2 T_m - h_1 A_1 h_2 A_2 T_w - h_1 A_1 m_2 c_2 \frac{dT_w}{dt} - h_2 A_2 m_2 c_2 \frac{dT_w}{dt} = h_2 A_2 m_2 c_1 \frac{dT_w}{dt} + m_1 c_1 m_2 c_2 \frac{d^2 T_w}{dt^2}$$

$$m_1 m_2 c_1 c_2 \frac{d^2 T_w}{dt^2} + \frac{dT_w}{dt}\left[h_2 A_2 m_1 c_1 + h_1 A_1 m_2 c_2 + h_2 A_2 m_2 c_2\right]$$
$$= h_1 A_1 h_2 A_2 T_m - h_1 A_1 h_2 A_2 T_w \quad \text{—— (4)} \rightarrow$$

Let, $\alpha_1 = m_1 m_2 c_1 c_2$

$\alpha_2 = m_1 c_1 h_2 A_2 + m_2 (2 h_1 A_1 + m_2) c_2 h_2 A_2$.

$\alpha_0 = h_1 A_1 \cdot h_2 A_2$.,

Putting the values in equ ④

$$\alpha_1 \frac{d^2 T_\omega}{dt} + \alpha_2 \frac{dT_\omega}{dt} = \alpha_0 (T_m - T_\omega)$$

$$\alpha_1 \frac{d^2 T_\omega}{dt^2} + \alpha_2 \frac{dT_\omega}{dt} + \alpha_0 T_\omega = \alpha_0 T_m .$$

Let, $T^2 = \frac{\alpha_1}{\alpha_0}$ & $\frac{\alpha_2}{\alpha_0} = 2 \xi T$.

$$\frac{\alpha_1}{\alpha_0} \cdot \frac{d^2 T_\omega}{dt^2} + \frac{\alpha_2}{\alpha_0} \frac{dT_\omega}{dt} + T_\omega = T_m .$$

$$T^2 \frac{d^2 T_\omega}{dt^2} + 2 \xi T \frac{dT_\omega}{dt} + T_\omega = T_m \quad \text{—⑤}$$

Taking Laplace Transformation,

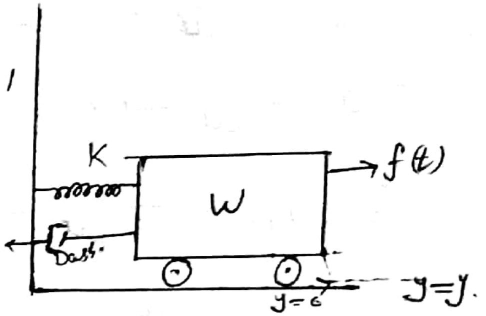
$$T^2 S^2 T_{\omega(s)} + 2 \xi T \times S\, T_{\omega(s)} + T_{\omega(s)} = T_{m(s)}$$

$$[T_{\omega(s)} [T^2 S^2 + 2 S \xi T + 1] = T_{m(s)}$$

$$\boxed{\frac{T_{\omega(s)}}{T_{m(s)}} \quad \frac{1}{(T^2 s^2 + 2 \xi T S + 1)}}$$

(ii) Mechanical Damper (damped variable:→

चित्र में W kg द्रव्यमान का एक block frictionless Horizontal spring से जुडा है। Block में एक viscous damper जुडा है। Block Horizontal plane पर forcing function F(t) के अन्तर्गत दोलन करने के लिए free है। माना प्रारम्भिक स्थिति में



zero time में block zero position deviation में आगे बढ़ रहा है तथा position y और velocity $\frac{dv}{dt}$ दोनो positive है।

Block पर निम्न बल कार्य करते है —

(1) The force exerced by the spring.—

$$F_1 = -Ky$$

where, K = Hook's constant

y = Displacement.

(2.) The viscous friction—

$$F_2 = -c \frac{dy}{dt}$$

where, c = damping coefficient.

(3) Forcing Function—

F(t).

According to Newton's law of motion.

Sum of all forces = Rate of change of momentum

$$-Ky - c\frac{dy}{dt} + F(t) = ma \qquad \left\{ \because m = w \ \& \ a = \frac{d^2y}{dt^2} \right\}$$

$$-Ky - c\frac{dy}{dt} + F(t) = w\frac{d^2y}{dt}$$

$$\frac{w}{K}\frac{d^2y}{dt^2} + \frac{c}{K}\frac{dy}{dt} + y = \frac{F(t)}{K} \quad \text{—— (1)}$$

Here, $\tau^2 = w/K$

$$\frac{c}{K} = 2\xi\tau$$

$$K_p = \frac{1}{K}$$

Now,

$$\tau^2\frac{d^2y}{dt^2} + 2\xi\tau\frac{dy}{dt} + y = K_p F(t) \quad \text{—— (2)}$$

At steady state condition,

$$y_{(s)} = K_2 F(s)$$

At Deviation form,

$$\tau^2\frac{d^2y}{dt^2} + 2\xi\tau\frac{dy}{dt} + y = K_p F(t) \quad \text{—— (3)}$$

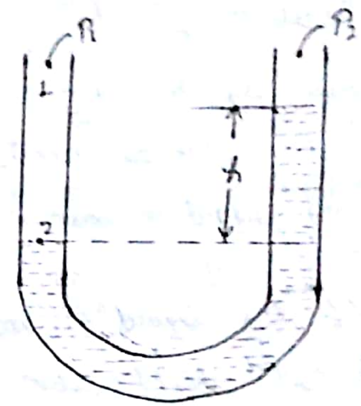
Taking Laplace transformation,

$$\tau^2 s^2 y_{(s)} - \tau^2 s y(0) - \tau^2 y'(0) + 2\xi\tau s y(s) - 2\xi\tau y(0) + y_{(s)} = K_2 F(s)$$

$$y(s)\,[\tau^2 s^2 + 2\xi\tau s + 1] = K_p F(s)$$

$$\boxed{\frac{y(s)}{F(s)} = \frac{K_p}{(\tau^2 s^2 + 2 s\xi\tau + 1)}}$$

(iii) Fluid manometer or U-tube :→



Taking force balance :-

(Force due to press P_1) – (Force due to Press P_2) – (Force due to liquid level difference) – (Force due to liquid friction) = (mass of liquid in tube) × Acceleration

$$P_1A_1 - P_2A_2 - A_2\rho g(2h) - \frac{8A\mu L}{R^2}\frac{dh}{dt} = m\frac{d^2h}{dt^2}$$

If $A_1 = A_2 = A$

$$(P_1 - P_2)A - 2\rho g h A - \frac{8A\mu L}{R^2}\frac{dh}{dt} = \rho A L\frac{d^2h}{dt^2}$$

$$\Delta P A - 2\rho g h A - \frac{8A\mu L}{R^2}\frac{dh}{dt} = \rho A L\frac{d^2h}{dt^2}$$

$$\frac{\Delta P}{2\rho g} - h - \frac{4\mu L}{\rho g R^2}\frac{dh}{dt} = \frac{L}{2g}\frac{d^2h}{dt^2}$$

$$\frac{\Delta P}{2\rho g} = \frac{L}{2g}\frac{d^2h}{dt^2} + \frac{4\mu L}{\rho g R^2}\frac{dh}{dt} + h = 0$$

Here, $\frac{1}{2\rho g} = K_p$, $\frac{L}{2g} = \tau^2$ & $\frac{4\mu L}{\rho g R^2} = 2\xi\tau$.

Now,

$$\tau^2 \frac{d^2h}{dt^2} + 2\xi\tau \frac{dh}{dt} + h = K_p \Delta P \quad \text{—— (1)}$$

where, ρ = density of mercury.

g = acceleration due to gravity.

m = mass of liquid in manometer.

v = average velocity in tube.

L = length of the liquid in manometers.

h = hight of liquid level from initial plane ~~to~~ of the final plane.

Taking Laplace Transformation;-

$$\tau^2 s^2 h(s) - \tau^2 s h(0) + \tau^2 h'(0) + 2s\xi\tau h(s) - 2\xi\tau L(0) + h(s) = K_p \Delta P(s).$$

$$\tau^2 s^2 h(s) + 2s\xi\tau h(s) + h(s) = K_p \Delta P(s)$$

$$h(s)\,[\tau^2 s^2 + 2s\xi\tau + L] = K_p \Delta P(s).$$

$$\boxed{\frac{h(s)}{\Delta P(s)} = \frac{K_p}{(\tau^2 s^2 + 2s\xi\tau + L)}}$$

Mode of Control Action:→

वह Method जिससे automatic controller के द्वारा error को reduce करने के लिए control action Produce किया जाता है; Mode of control या control action कहलाता है।

ये निम्न प्रकार के होते हैं -

(1) Proportional Control

(2) Proportional Derivative control.

(3) Proportional Integral Control.

(4) Proportional Intigral Derivative Control.

(1) Proportional Control :→

Proportional controller Produce an output signal that is proportional to the error E.

This action may be expressed as.

$$P = K_c E + P_s \quad \text{—— (1)}$$

where,

P = output signal

K_c = gain or sensitivity

E = error (set point – measured variable).

P_s = a constant.

we introduce the deviation variable.

$$P' = P - P_s$$

In equation (1) at time $t=0$ तो $E=0$

then, $P'_{(t)} = K_c E_{(t)}$

Taking Laplace transformation.,

$$P'_{(s)} = K_c E_{(s)}$$

$$\boxed{\frac{P'_{(s)}}{E_{(s)}} = K_c.}$$

Proportional band is used for proportional control action of engineers in place of term gain.

For this ~~action~~ control. action,

$$K_c = \frac{100}{PD}\ \%$$

NOTE:→ on/off control - proportional control का ही एक mode है।

(2) Proportional Derivative Control :→

Proportional control तथा derivative control को जोड़ने पर 'Proportion derivative control' प्राप्त होता है।

This mode of control may be represented by,

$$P = K_c E + K_c \tau_D \frac{dE}{dt} + P_s \quad \text{——— (1)}$$

where, K_c = gain or sensitivity

τ_D = Derivative time

P_s = a constant.

from equation (1)

$$P - P_s = K_c E + K_c \tau_D . \frac{dE}{dt}$$

$$P' = K_c E + K_c \tau_D \frac{dE}{dt}$$

Taking Laplace transformation.

$$P'_{(s)} = K_c E_{(s)} + K_c \tau_D S E_{(s)}$$

$$P'_{(s)} = E_{(s)} [K_c + K_c \tau_D S]$$

$$\boxed{\frac{P'_{(s)}}{E_{(s)}} = K_c [1 + \tau_D . S]}$$

(3) Proportional Integral Control (PIC) :→

Integral control action तथा proportional control action को जोड़ने पर proportional Integral control प्राप्त किया जाता है। इस combination का मुख्य aim दोनों control action से निश्चित लाभ का प्राप्त करना है। इस control के mode को निम्न relation से समझ सकते हैं

$$P = K_c E + \frac{K_c}{\tau_I} \int_0^t E\,dt + P_s . \quad \text{——— (1)}$$

where, K_c = gain or sensitivity

τ_{I} = Integral time.

P_s = a constant.

when, $E = 1$.

$$P = K_c + \frac{K_c}{\tau_I} t + P_s \quad \text{——— (2)}$$

from equation (1)

$$P - P_s = K_c E + \frac{K_c}{\tau_I} \int_0^t E\,dt$$

$$P' = K_c E + \frac{K_c}{\tau_I} \int_0^t E\,dt$$

Taking Laplace Transformation,

$$P'_{(s)} = K_c E_{(s)} + \frac{K_c}{\tau_I} \frac{E_{(s)}}{s}$$

$$P'_{(s)} = E_{(s)} \left[K_c + \frac{K_c}{\tau_I \cdot s}\right]$$

$$\boxed{\frac{P'_{(s)}}{E_{(s)}} = K_c \left[1 + \frac{1}{\tau_I \cdot s}\right]}$$

(4) Proportional Integral Derivative (PID) Control :→

This mode of control is conbination of the previous modes and given by the expression—

$$P = K_c E + K_c \tau_D \frac{dE}{dt} + \frac{K_c}{\tau_I} \int_0^t E\,dt + P_s \quad \text{——— (1)}$$

$$P - P_s = K_c E + K_c \tau_D \frac{dE}{dt} + \frac{K_c}{\tau_I} \int_0^t E\,dt$$

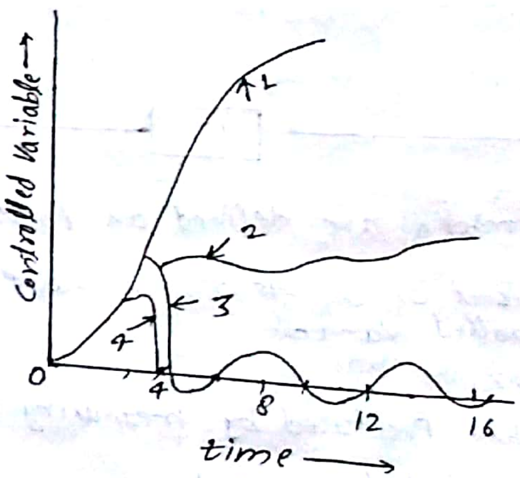
$$P' = K_c E + K_c \tau_D \frac{dE}{dt} + \frac{K_c}{\tau_I} \int_0^t E\,dt . \longrightarrow$$

Taking Laplace transformation

$$P'_{(s)} = K_c E_{(s)} + K_c \tau_D s E_{(s)} + \frac{K_c}{\tau_I} \frac{E_{(s)}}{s}$$

$$P'_{(s)} = E_{(s)} \left[K_c + K_c \tau_D s + \frac{K_c}{\tau_I \cdot s}\right]$$

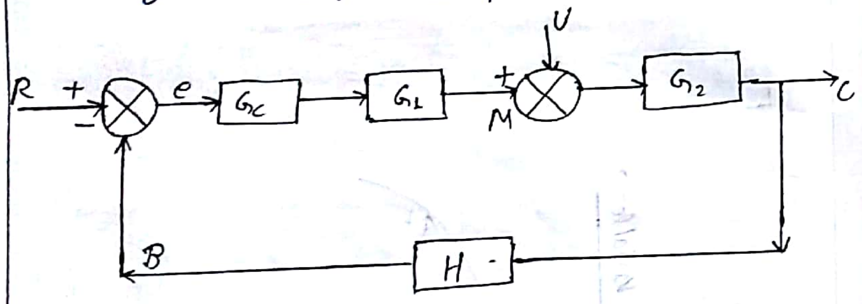
$$\boxed{\frac{P'_{(s)}}{E_{(s)}} = K_c \left[1 + \tau_D \cdot s + \frac{1}{\tau_I \cdot s}\right]}$$



Response of typical control system showing the effects of various modes of control.

Standard block diagram symbol:→

A block diagram was developed for the control of stirred tank heater, The block diagram has been redrawn and ~~incompressed~~ incorporates some standards symbols for the variable and transfer function. which are widely used in the control litrature. These symbols define as follows—

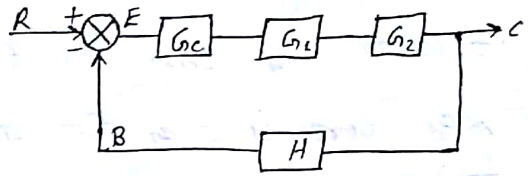


These symbols are defined as follows:→

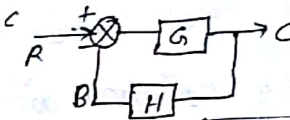
R = Set Point or ~~deside~~ desired value.
C = Controlled variable
E = error.
B = Variable Produced by measuring element.
M = Manipulated variable.
U = Load variable or disturbance.
G_c = Transfer function of controller.
G_1 = Transfer function of Final Control element
G_2 = Transfer function of Process.
H = Transfer function of measuring element.

Overall transfer function for a single loop system:→

एक control system को एक block diagram के द्वारा परिभाषित किया गया है जिसे चित्र में दिखाया गया है —



(a)



(b) $\{G = G_c G_1 G_2\}$

हम इस transfer function को overall transfer function में Refer कर सकते हैं क्योंकि ये entire system को apply करते हैं। overall transfer function, control system के बारे में information को प्राप्त कर प्रयोग कर सकते हैं।

set point R में Response को change कर setting $U=0$ के द्वारा प्राप्त करते हैं। Load variable U में response को change कर $R=0$ के द्वारा प्राप्त करते हैं।

Overall transfer function for change in set point:→

For this case $U=0$ हम block diagram reduction को एक simple rule का use कर बना सकते हैं।—

हम figure (b) से direct Equation निम्न प्रकार लिख सकते हैं —

$$C = GE$$
$$B = HC$$
$$E = R-B$$

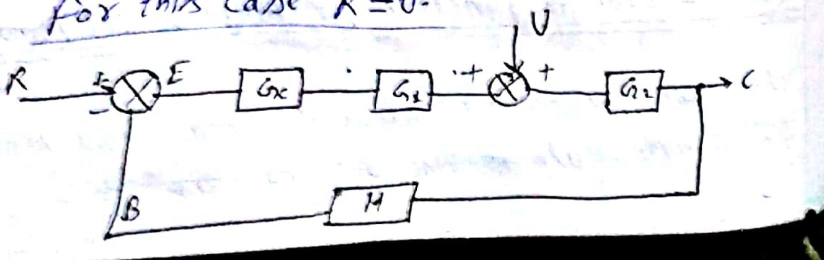
इस प्रकार Four variable तथा three equ^n हैं। हम equations को R के term में C के लिए निम्न प्रकार solve कर सकते हैं —

$$C = GE$$
$$C = G(R-B)$$
$$C = G(R-HC)$$
$$C = GR - GHC$$
$$C + GHC = GR$$
$$C(1+HG) = GR$$
$$\boxed{\frac{C}{R} = \frac{G}{1+HG}}$$

This is the ~~ovel~~ overall transfer function relating $C \rightleftharpoons R$.

overall transfer function for charge in Load(U):

For this Case $R = 0$.



From diagram

$$C = G_2(U+M) \text{ ——— } (1)$$
$$M = G_c G_1 E \text{ ——— } (2)$$
$$B = HC \text{ ——— } (3)$$
$$E = R - B \text{ ——— }$$
$$E = -B \text{ ——— } (4)$$

{∵ R=0 for this case}

Eq^n (2) का मान Eq^n (1) में रखने पर.

$$C = G_2(U + G_c G_1 E) \text{ ——— } (5)$$

we know that,

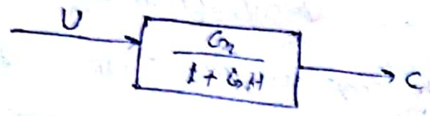
$$E = -B$$
$$E = -HC.$$

E का मान समी० (5) में रखने पर.

$$C = G_2(U - G_c G_1 HC)$$
$$C = G_2 U - G_c G_1 G_2 HC$$
$$C + G_c G_1 G_2 HC = G_2 U$$
$$C[1 + G_c G_1 G_2 H] = G_2 U$$
$$C[1 + GH] = G_2 U$$

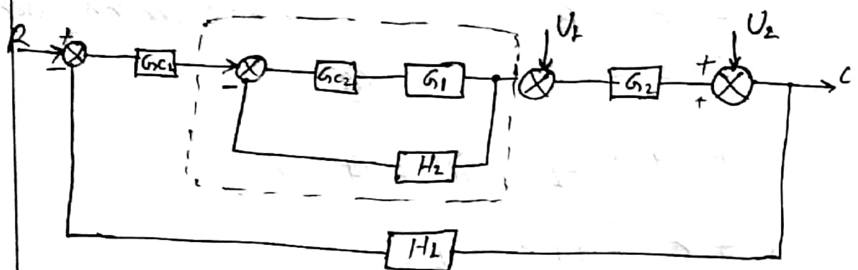
{∵ $G = G_c G_1 G_2$}

$$\boxed{\frac{C}{U} = \frac{G_2}{1+G_1 H}}$$

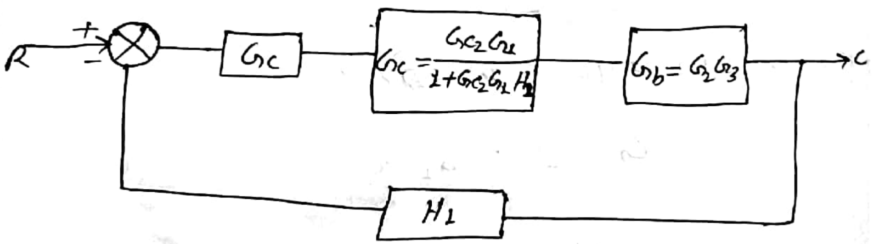


Overall transfer Function for Multiloop Control System:→

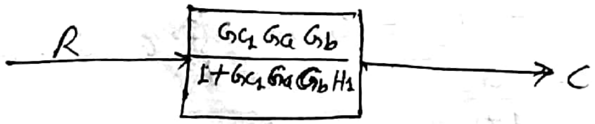
overall control system C/R system को represent करता है, हम एक single loop में G_2 और G_3 को हमेशा combine कर सकते हैं।



(a) (Original diagram)



(b) (First Reduction)

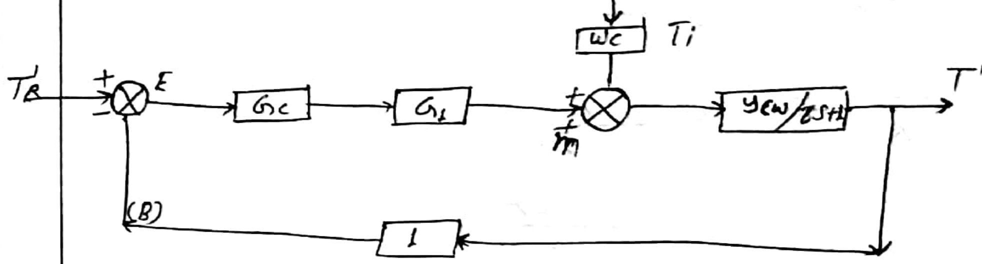


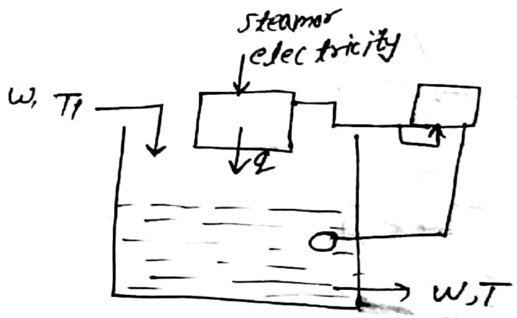
(C) (Final signal block diagram)

Figure (b) में एक sigle loop diagram है जो एक block को reduce करती है।

Proportional control at stirred tank heater for set point change (Servo Problem):→

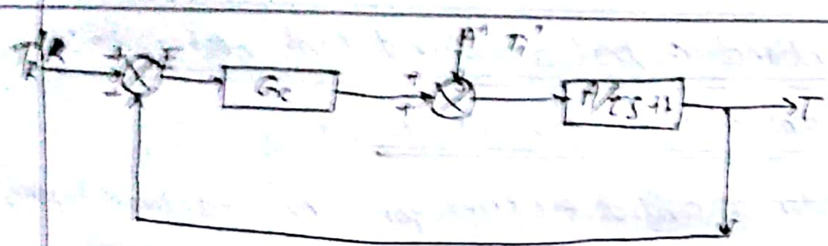
Consider a control system for stirred tank heater,





~~Hear~~ Here measuring element is a Thermometer (Thermocouple) whose response is very fast that for all particle porpose it is assumed to follow the slowly changes bath. without lag. when the feedback Transfer function is unilty (1) the system is called the unity feed back system.

These →
$K_c = G_c$
$G_1 = 1$
$H = 1$
$W_c = A^1$



then $T_i = 0$

$$\frac{T'}{T_R'} = \frac{K_c \frac{A}{1+\tau s}}{1+\frac{K_c A}{1+\tau s}}$$

$$\boxed{\frac{T'}{T_R'} = \frac{K_c A}{\tau s + 1 + K_c A}}$$

$$\because \tau_1 = \frac{\tau}{1+K_c A}$$

$$A_1 = \frac{K_c A}{1+K_c A}$$

$$\boxed{\frac{T'}{T_R'} = \frac{A_1}{\tau_1 s+1}}$$

According to this result the response of tank temperature to change in set point is first order.

NOTE:→ The time constant for the control system (τ_1) is less than that of stirred tank itself τ.

We know. offset is equal to the difference between ultimate value of temp. T' and set point T_R'.

$$\text{offset} = T'_{R(\infty)} - T'_{(\infty)} \quad \text{——— (1)}$$

$$\frac{T'}{T_R'} = \frac{A_1}{\tau_1 s+1}$$

$\because T_R' = \frac{1}{s}$ $\qquad$ $\because T_R' = \frac{1}{s}$

$T'_{R(t)} = 1$

from Laplace transfer function. $\qquad T'_{R(\infty)} = 1$.

$$T'_{(s)} = \frac{1}{s} \cdot \frac{A_1}{(\tau_1 s+1)}$$

final value theorem,

$$\lim_{t\to\infty}[f(t)] = \lim_{s\to 0}[s f(s)]$$

$$\lim_{t\to\infty}[T'_{(\infty)}] = \lim_{s\to 0}\left[\cancel{s} \cdot \frac{1}{\cancel{s}} \frac{A_1}{\tau_1 s+1}\right]$$

$$= \lim_{s\to 0}\left[\frac{A_1}{\tau_1 s+1}\right]$$

$$T'_{(\infty)} = A_1$$

from equation (1)

$$\text{offset} = T'_{R(\infty)} - T'_{(\infty)}$$

$$= 1 - A_1$$

$$\because A_1 = \frac{K_c A}{1+K_c A}$$

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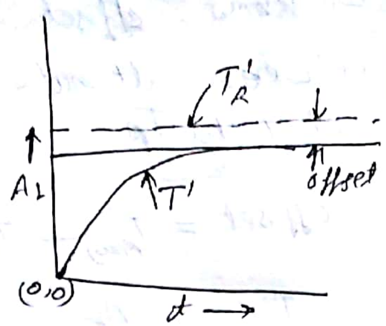
$$\text{offset} = 1 - \frac{K_cA}{1+K_cA}$$

$$= \frac{1 + K_cA - K_cA}{1+K_cA}$$

$$\boxed{\text{offset} = \frac{1}{1+K_cA}}$$



Unit step response for set point change [P-control]

Or

(Servo Problem)

P C at stirred tank heater for set point change:→

for Proportional control

$$\frac{T'}{T_R'} = \frac{\frac{K_cA}{\tau s+1}}{1+\frac{K_cA}{\tau s+1}}$$

$$= \frac{K_cA}{\tau s+1+K_cA}$$

This equation may be arrange in the form of first order,

$(1+K_cA)$ से प्रत्येक पद में भाग देने पर,

$$\frac{T'}{T_R'} = \frac{K_cA/1+K_cA}{\frac{\tau s}{1+K_cA} + \frac{1+K_cA}{1+K_cA}}$$

where, $A_1 = \frac{K_cA}{1+K_cA}$, $\tau_1 = \frac{\tau}{1+K_cA}$.

इस result के according, set point में tank के response से temp. change होता है। control system के लिए time constant τ_1, stirred tank से कम होता है। system के response को set point T_R' में एक unit step को change करते हैं। इस प्रकार temp. $T'(\infty)$ की ultimate value desired change से match नहीं करती है। इसे offset कहते हैं।

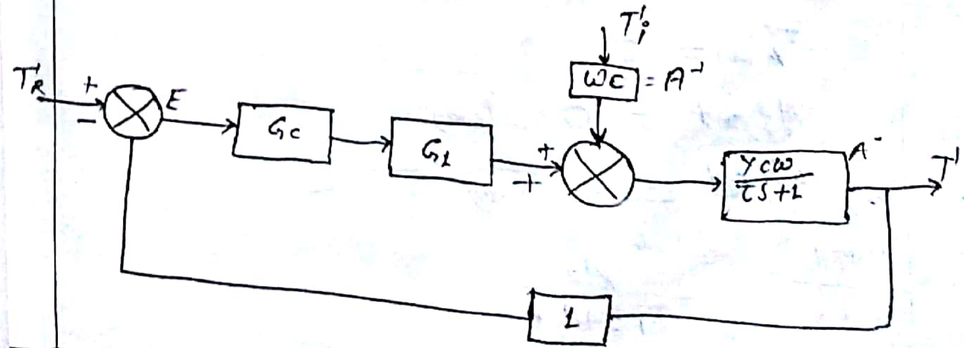
$$\text{offset} = T_R'(\infty) - T'(\infty)$$

$$= 1 - \frac{K_cA}{1+K_cA}$$

$$= \frac{1+\cancel{K_cA} - \cancel{K_cA}}{1+K_cA}$$

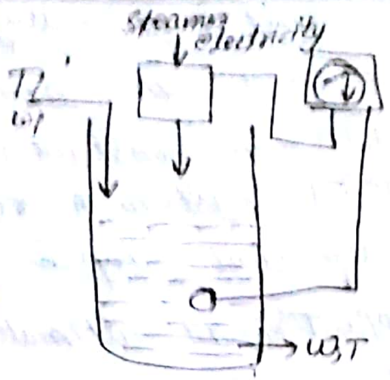
$$\boxed{\text{offset} = \frac{1}{1+K_cA}}$$

Proportional control at stirred tank heater for load change (Regular Problem):→



when $T'_R = 0$

The overall transfer function, because



$$\frac{T'}{T_i'} = \frac{AA'/(\tau s+1)}{1+K_cA/(\tau s+1)}$$

$$\frac{T'}{T_i'} = \frac{AA'}{K_cA+(\tau s+1)}$$

$$\frac{T'}{T_i'} = \frac{1}{\tau s+K_cA+1}$$

$$\tau_1 = \frac{\tau}{1+K_cA}$$

$$A_2 = \frac{1}{1+K_cA}$$

$$\boxed{\frac{T'}{T_i'} = \frac{A_2}{\tau_1 s+1}}$$

we know that offset is equal to the difference between ultimate value of temp. T and set point

offset $= T'_{R(\infty)} - T'_{(\infty)}$

$T'_{R(\infty)} = 0$

offset $= 0 - T'_{(\infty)}$

offset $= -T'_{(\infty)}$ —— ①

$$\frac{T'}{T_i'} = \frac{A_2}{\tau_1 s+1}$$

$T' =$ for step input

$$T'_{(s)} = \frac{1}{s}\,\frac{A_2}{\tau_1 s+1}$$

using final value theorem

$$\lim_{t\to\infty}[T'(t)] = \lim_{s\to 0}[s\,T(s)]$$

$$= \lim_{s\to 0}\left[\cancel{s}\cdot\frac{1}{\cancel{s}}\cdot\frac{A_2}{\tau_1 s+1}\right]$$

$$= \lim_{s\to 0}\left[\frac{A_2}{\tau_1 s+1}\right]$$

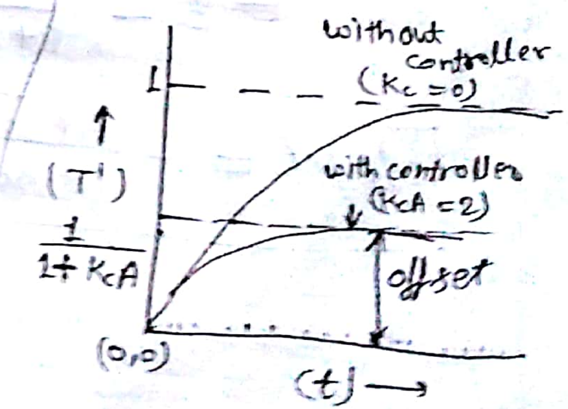
$$\lim_{t\to\infty}[T'(t)] = A_2$$

from eqn. ①

offset $= 0 - A_2$

offset $= 0 - \frac{1}{1+K_cA}$

$$\boxed{\text{offset} = -\frac{1}{1+K_cA}}$$



Or PC at stirred tank heater for load change (Regulator problem)

The overall transfer function;

$$\frac{T'}{T_L} = \frac{AA^{-1}/\tau s+1}{1+\frac{K_cA}{\tau s+1}}$$

$$\frac{T'}{T_L} = \frac{1}{\tau s+1+K_cA}$$

This may be arrange in the form of the fluid order lag.

$$\frac{T'}{T_L'} = \frac{A_2}{\tau_2 s+1}$$

where, $A_2 = \frac{1}{1+K_cA}$

$\tau_2 = \frac{\tau}{1+K_cA}$.

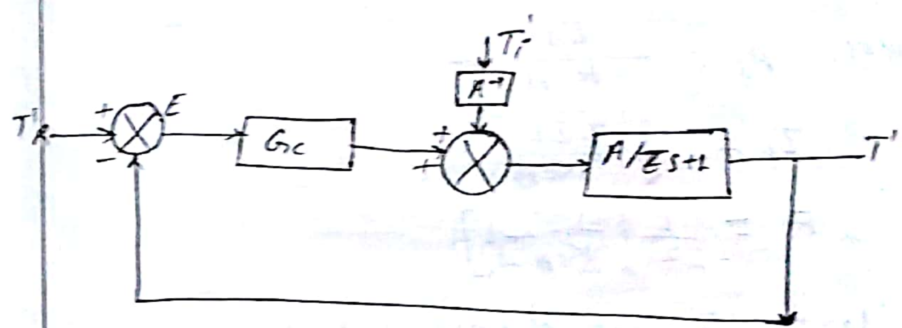
From equation the offset because.

$$\text{offset} = T'_{R(\infty)} - T'_{(\infty)}$$

$$= 0 - \frac{1}{1+K_cA}$$

$$\boxed{\text{offset} = -\frac{1}{1+K_cA}}$$

Proportional Integral control of stirred tank heater for load change:→



The overall transfer function for load change

$$\frac{T'}{T_i'} = \frac{AA^{-1}/\tau s+1}{1+\left(\frac{K_cA}{\tau s+1}\right)\left(1+\frac{1}{\tau_I s}\right)}$$

$$= \frac{1}{(\tau s+1) + K_cA\left(\frac{\tau_I s+1}{\tau_I s}\right)}$$

$$= \frac{\tau_I s}{\tau_I s(\tau s+1) + K_cA(\tau s+1)}$$

$$= \frac{\tau_I s}{\tau\tau_I s^2 + \tau_I s + K_cA\tau s + K_cA}$$

$$\frac{T'}{T_i'} = \frac{\tau_I s}{\tau\tau_I s^2 + (K_cA\tau_I + \tau_I)s + K_cA}$$

$$= \frac{\tau_I s/K_cA}{\frac{\tau\tau_I s^2}{K_cA} + \tau_I s\left(\frac{1}{K_cA}+1\right)+1}$$

$$\boxed{\frac{T'}{T_i'} = \frac{A_1 S}{\tau_1^2 S^2 + 2F\tau_1 S + 1}}$$

where, $A_1 = \frac{\tau_I}{K_c A}$

$\tau_1 = \sqrt{\frac{\tau \tau_I}{K_c A}}$

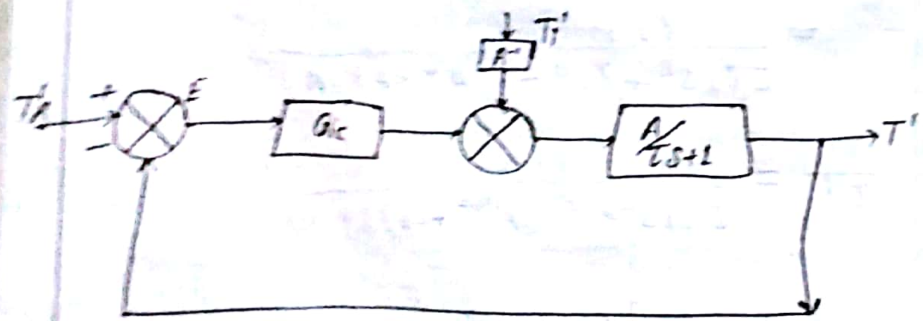
$F = \frac{1}{2}\left[\frac{1}{K_c A} + 1\right]$

for unit step change in load.

$T_i' = \frac{1}{S}$

$$\boxed{T' = \frac{A_1}{\tau_1^2 S^2 + 2F\tau_1 S + 1}}$$

Proportional Integral Control of stirred tank heater for set point change :→



$$\frac{T'}{T_R'} = \frac{\left(1 + \frac{1}{\tau_I S}\right)\left(\frac{K_c A}{\tau S + 1}\right)}{1 + K_c A\left(1 + \frac{1}{\tau_I S}\right)\left(\frac{1}{\tau S + 1}\right)}$$

$$= \frac{\frac{K_c A}{\tau S + 1}\left(\frac{\tau_I S + 1}{\tau_I S}\right)}{1 + \frac{K_c A}{\tau S + 1}\left(\frac{\tau_I S + 1}{\tau_I S}\right)}$$

$$= \frac{K_c A (\tau_I S + 1)/\tau_I S}{\tau S + 1 + K_c A\left(\frac{\tau_I S + 1}{\tau_I S}\right)}$$

$$= \frac{K_c A (\tau_I S + 1)}{\tau_I S(\tau S + 1) + K_c A(\tau_I S + 1)}$$

$$= \frac{K_c A (\tau_I S + 1)}{\tau \tau_I S^2 + \tau_I S + K_c A \tau_I S + K_c A}$$

$$\frac{T'}{T_R'} = \frac{\tau_I S + 1}{\frac{\tau \tau_I}{K_c A} S^2 + S\left(\frac{\tau_I + K_c A \tau_I}{K_c A}\right) + 1}$$

At unit step change

$T_R' = \frac{1}{S}$

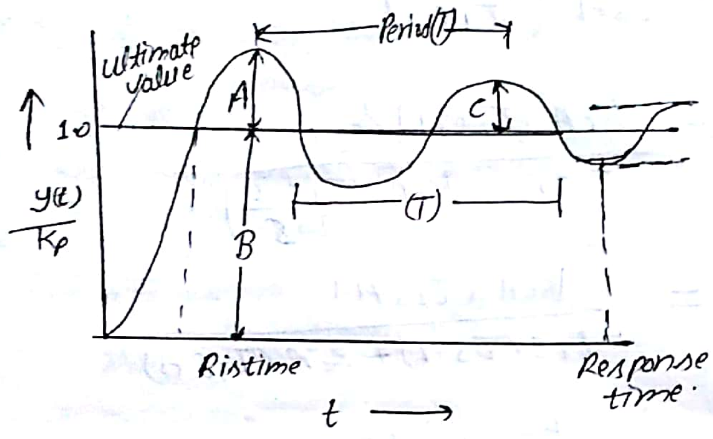
$$T' = \frac{\tau_I}{\tau_1^2 S^2 + 2F\tau S + 1} + \frac{1}{S}\cdot\frac{1}{\tau_1^2 S^2 + 2F\tau_1 S + 1}$$

Offset $= T_{R(\infty)} - T_{(\infty)}$

$= 1 - 1$

$$\boxed{\text{Offset} = 0.}$$

Characterstics of an underdamp Response :→



(1.) **overshoot :→** It is the ratio of A/B where B is the ultimate value of response and 'A' is the maximum amount by which the response exceeds its ultimate value.

$$\boxed{\text{overshoot} = \text{exponential}\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)}$$

where, $\frac{-\pi\xi}{\sqrt{1-\xi^2}}$ = Power of the exponential function.

& ξ = damping ratio.

(v) overshoot ultimate value से अधिक Response को measure करता है। इसे A/B के Ratio से Represent करते हैं।

Or

or **overshoot :→** किसी higher order system के input में unit step change करने पर response का मान ultimate value से कितना अधिक होता है इसकी माप overshoot के द्वारा की जाती है। overshoot का मान ξ के घटने पर बढ़ता है। Fig. में A/B के अनुपात को 'overshoot' कहते हैं।

(2.) **Decay Ratio :→** It is defined as of the sizes of successive peaks given by C/A.

$$\boxed{\text{Decay Ratio} = \text{exponential}\left(\frac{-2\pi\xi}{\sqrt{1-\xi^2}}\right)}$$

$$\boxed{\text{Decay Ratio} = (\text{overshoot})^2}$$

Or

दो क्रमशः Peaks की size के अनुपात को decay Ratio कहते हैं।

Or

(v) दो successive Peaks के size के अनुपात को 'decay Ratio' कहते हैं। इसे C/A के ratio से Represent करते हैं।

③ **Rise Time :→** It is defined as the time required for the response to reach its final for the first time.

Or.

① Response को अपनी ultimate value तक पहुँचने के लिए जो time लगता है उसे Rise time कहते हैं

Or.

① किसी higher order system के response को first time, ultimate value तक पहुंचने में लगे समय को 'Rise time' कहते हैं। इसे t_r से प्रदर्शित करते हैं।

Rise time का मान ξ के बढ़ने से बढ़ता है।

④ **Response time :→** This is the time required for the response to come within 5% of its ultimate value and remain there.

Or.

① Response को अपनी ultimate value के ± 5 प्रतिशत तक पहुँचने में जो time लगता है। उसे 'Response time' कहते हैं

① or किसी higher order system के Response को इसके ultimate value के ± 5 प्रतिशत तक आने तथा उसी पर स्थिर होने मे लगे समय को 'Response time' कहते हैं

⑤ **Period of Oscillation :→** Radiation frequency is defined as.

①
$$\omega = \frac{\sqrt{1-\xi^2}}{\tau}$$

If $\omega = 2\pi f$

$$f = \frac{\omega}{2\pi}$$

$$\boxed{f = \frac{1}{2\pi}\left(\frac{\sqrt{1-\xi^2}}{\tau}\right)}$$

where, f = cyclical frequency

T = Period of oscillation.

$$\boxed{T = \frac{1}{f} = 2\pi\left(\frac{\tau}{\sqrt{1-\xi^2}}\right)}$$

Or

① unit step change करने पर किसी higher order के Response को निम्न रूप में लिखा जा सकता है। (जो ऊपर हैं ?) ।

Natural Period of Oscillation :→

A Second order system with $\xi = 0$ is system free of any damping.

$$\omega_n = \frac{\sqrt{1-0}}{\tau} = \frac{1}{\tau}$$

$$\omega_n = 2\pi f_n$$

$$\therefore f_n = \frac{1}{T_n} = \frac{\omega_n}{2\pi}$$

$$\boxed{f_n = \frac{1}{2\pi\tau}}$$

where, f_n = Natural cyclical frequency

T_n = " " Period

τ = significance of undamped period.

Or

(1) undamped condition के अन्तर्गत radian frequency को Natural frequency कहते हैं। इसे ω_n से प्रदर्शित करते हैं।

NUMERICALS.

Q.1. A step change of magnitude 4 is introduces into the system having the transfer function :-

$$\frac{Y(s)}{X(s)} = \frac{10}{s^2 + 1.6s + 4}$$

Determine :→ (A) Percent overshoot (ii) Rise Time, (iii) maximum value of $Y(t)$ (iv) Ultimate value of $Y(t)$ (V) Period of oscillation.

Sol:— $$\frac{Y(s)}{X(s)} = \frac{10}{s^2 + 1.6s + 4}$$

$$\frac{Y(s)}{X(s)} = \frac{10}{4\left(\frac{s^2}{4} + \frac{1.6s}{4} + 1\right)}$$

$$\frac{Y(s)}{X(s)} = \frac{10/4}{\left(\frac{s^2}{4} + \frac{1.6s}{4} + 1\right)} \Rightarrow \frac{10/4}{(0.25s^2 + 0.4s + 1)} \quad \text{—(1)}$$

Eqn. (1) Compairing by second order eqn.

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau^2 s^2 + 2\tau\xi s + 1}$$

$$\tau^2 = 0.25 = \tau = \sqrt{0.25} \Rightarrow \boxed{\tau = 0.5}$$

$$2\tau\xi = 0.4 \Rightarrow \xi = \frac{0.4}{2\times\tau}$$

$$\xi = \frac{0.4}{2\times 0.5} \Rightarrow \boxed{\xi = 0.4}$$

% overshoot $= \left[e^{-\left(\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)}\right] 100\%$

% overshoot $= \left[e^{-\left(\frac{3.14\times 0.4}{\sqrt{1-(0.4)^2}}\right)}\right] \times 100\%$

$$= \left[e^{-\left(\frac{1.256}{\sqrt{1-0.16}}\right)} \right] \times 100\%$$

$$= \left[e^{-\left(\frac{1.256}{\sqrt{0.84}}\right)} \right] \times 100\%$$

$$= \left[e^{-\left(\frac{1.256}{.9165}\right)} \right] \times 100\%$$

$$= 0.2541 \times 100\%$$

$$\boxed{\text{overshoot} = 25.41\%}$$ Solved.

Rise time $\Rightarrow t_r = \left(\frac{\pi - \phi}{\sqrt{1-\xi^2}}\right) \times \tau$

$$\phi = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\phi = \tan^{-1} \frac{\sqrt{1-(0.4)^2}}{.4} \Rightarrow \tan^{-1}\left(\frac{0.9165}{.4}\right)$$

$$\phi = \tan^{-1}(2.29125)$$

$$\boxed{\phi = 1.1592}$$

Rise Time, $T_r = \left(\frac{\pi - \phi}{\sqrt{1-\xi^2}}\right)\tau$

$$= \left(\frac{3.14 - 1.1592}{\sqrt{1-(0.4)^2}}\right) \times 5$$

$$= \frac{1.9807}{.9165} \times .5 \Rightarrow \frac{0.99036}{.9165}$$

$$\boxed{t_r = 1.08052}$$ Solved.

Ultimate value of $y(t)$

from eq^n. (1)

$$\frac{Y(s)}{X(s)} = \frac{10/4}{(.25s^2 + .4s + 1)}$$

$\because X_s = \frac{A}{s} = \frac{4}{s}$

$\because A = 4$

$$Y(s) = \frac{10/4}{.25s^2 + .4s + 1} \times [X(s)]$$

$$Y(s) = \frac{4}{s} \times \frac{10/4}{.25s^2 + .4s + 1}$$

$$Y(s) = \frac{10}{s(.25s^2 + .4s + 1)}$$

$\because$ taking Laplace

$$y(t) = 10\left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi t/\tau} \sin\left[\sqrt{1-\xi^2}\frac{t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]\right]$$

$\because t = \infty$ रखने पर,

$$y(\infty) = 10(1-0)$$

$$\boxed{y(\infty) = 10}$$ Solved.

Maximum value of $y(t)$

$$y(t) = A + B \Rightarrow B\left[\frac{A}{B} + 1\right]$$

$\because A/B =$ overshoot

where, $B =$ ultimate value.

$$y(t) = 10\,[.2541 + 1]$$

$$= 10 \times 1.2541$$

$$\boxed{y(t) = 12.541}$$ Solved.

Period of oscillation:→

$$T = \frac{2\pi\tau}{\sqrt{1-\xi^2}}$$

$$T = \frac{2\times 3.14\times 0.5}{\sqrt{1-(0.4)^2}}$$

$$= \frac{3.14}{\sqrt{1-0.16}} \Rightarrow \frac{3.14}{\sqrt{.84}}$$

$$= \frac{3.14}{0.9165} \Rightarrow 1.0842$$

$$\boxed{T = 1.0842}$$ Solved.

Q. $\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 1 \qquad x(0) = x'(0) = 0.$

Sol:⇒

$$[s^2x(s) - s\,x(0) - x'(0)] + [s\,x(s) - x(0)] + x(s) = \frac{1}{s}$$

$$s^2x(s) + s\,x(s) + x(s) = \frac{1}{s}$$

$$x(s)[s^2+s+1] = \frac{1}{s}$$

$$x(s) = \frac{1}{s^2+s+1}\cdot\frac{1}{(s)}$$

Making Partial Fraction.

$$\frac{1}{s(s^2+s+1)} = \frac{A}{s} + \frac{Bs+C}{(s^2+s+1)} \quad \text{—— (1)}$$

$$1 = A(s^2+s+1) + (Bs+C)(s)$$

$$1 = As^2 + As + A + Bs^2 + Cs$$

$$1 = s^2(A+B) + s(A+C) + A$$

$$\therefore \boxed{A = 1} \qquad A+B=0 \qquad A+C=0$$

$$B = -A \qquad A = -C$$

$$\boxed{B=-1} \qquad \boxed{C=-1}$$

Putting the value of A, B, C in eqn (1)

$$x(s) = \frac{1}{s} + \frac{[s\times(-1)]-1}{(s^2+s+1)}$$

$$x(s) = \frac{1}{s} - \frac{(s+1)}{(s^2+s+1)}$$

$$x(s) = \frac{1}{s} - \frac{(s+1)}{(s^2+s+1)}$$

$$= \frac{1}{s} - \frac{(s+\frac{1}{2}+\frac{1}{2})}{(s^2+s+1+\frac{1}{4}-\frac{1}{4})}$$

$$= \frac{1}{s} - \left[\frac{(s+\frac{1}{2})+\frac{1}{2}}{(s+\frac{1}{2})^2+(1-\frac{1}{4})}\right]$$

$$= \frac{1}{s}\left[\frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2+\frac{3}{4}} + \frac{1/2}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right]$$

$$= \frac{1}{s} - \left[\frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} + \frac{\frac{1}{2}\times\frac{\sqrt{3}}{2}\times\frac{2}{\sqrt{3}}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right]$$

Taking Laplace Inverse,

→

$$x(t) = 1 - \left[e^{-t/2} \cos\frac{\sqrt{3}}{2}t + e^{-t/2}\frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}\right)t\right]$$

$$\boxed{x(t) = 1 - e^{-t/2}\left[\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right]}$$

Solved.

Q.2 $\frac{d^2x}{dt^2} + \frac{2dx}{dt} + x = 1 \{x(0) = x'(0) = 0\}$

Sol:→

$$s^2 x(s) - s\,x(0) - x'(0) + 2[s\,x(s) - x(0)] + x(s) = \frac{1}{s}$$

$$\Rightarrow s^2 x(s) + 2s\,x(s) + x(s) = \frac{1}{s}$$

$$x(s) = (s^2+2s+1) = \frac{1}{s}$$

$$x(s) = \left[\frac{1}{s(s^2+2s+1)}\right]$$

~~Making Partial Prod...~~.

Making Partial Fraction,

$$\frac{1}{s(s^2+2s+1)} = \frac{A}{s} + \frac{Bs+C}{(s^2+2s+1)} \quad \text{—①}$$

$$1 = A(s^2+2s+1) + (Bs+C)s$$

$$1 = As^2 + 2As + A + Bs^2 + Cs$$

$$1 = (A+B)s^2 + (2A+C)s + A$$

$\boxed{A = 1}$ $\quad 2A + C = 0$

$A + B = 0$ $\quad 2 + C = 0$

$\boxed{B = -1}$ $\quad \boxed{C = -2}$.

from eqn ①

$$x(s) = \frac{1}{s} + \frac{(-s-2)}{s^2+2s+1}$$

$$x(s) = \frac{1}{s} - \frac{s+2}{(s^2+2s+1)}$$

$$x(s) = \frac{1}{s} - \frac{(s+2+1-1)}{(s^2+2s+1+1-1)}$$

$$x(s) = \frac{1}{s} - \left[\frac{(s+1)+1}{(s^2+1)^2}\right.$$

Q.4 $\frac{dx}{dt} = \int_0^t x(t)\,dt \cancel{- t} \quad [x(0) = 3]$

$$\frac{dx}{dt} = \int_0^t 1\cdot x(t)\,dt - t$$

Taking laplace both side.

$$s\,x(s) - x(0) = \frac{x(s)}{s} - \frac{1}{s^2}$$

$$s\,x(s) - 3 = \frac{x(s)}{s} - \frac{1}{s^2}$$

~~$s x(s)$~~

$$s\,x(s) - 3 = \frac{x(s)}{s} - \frac{1}{s^2}$$

$$sx(s) - \frac{x(s)}{s} = 3 - \frac{1}{s^2}$$

$$\frac{s^2 x(s) - x(s)}{\cancel{s}} = \frac{3s^2 - 1}{s^{\cancel{2}}}$$

$$x(s)[s^2 - 1] = \frac{(3s^2 - 1)}{s}$$

$$x(s) = \frac{(3s^2-1)}{s(s^2-1)} \Rightarrow x(s) = \frac{(3s^2-1)}{s(s+1)(s-1)}$$

$$x(s) = \frac{(3s^2-1)}{s(s+1)(s-1)}$$

Making Partial Fraction.

$$\frac{3s^2-1}{s(s+1)(s-1)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s-1)} \quad \text{——— (1)}$$

$$3s^2 - 1 = A(s+1)(s-1) + Bs(s-1) + C(s+1)s$$

$$3s^2 - 1 = As^2 - A + Bs^2 - Bs + Cs^2 + Cs$$

$$3s^2 - 1 = (A+B+C)s^2 - A + s(C-B)$$

$$3s^2 - 1 = (A+B+C)s^2 - A + (C-B)s$$

$A = 1$ $\qquad C - B = 0$

$A + B + C = 3$ $\qquad B = C$

$1 + 2B = 3$

$2B = 2$ $\quad \boxed{C = 1}$ $\quad \boxed{A = 1}$

$\boxed{B = 1}$

from eqn. (1)

$$x(s) = \frac{1}{s} + \frac{1}{(s+1)} + \frac{1}{(s-1)}$$

Taking laplace inverse.

$$\boxed{x(t) = 1 + e^{-t} + e^{t}}$$

(8.5) $\dfrac{3s}{(s^2+1)(s^2+4)}$

Sol:- Let $x(s) = \dfrac{3s}{(s^2+1)(s^2+4)}$

making partial fraction.

$$\frac{3s}{(s^2+1)(s^2+4)} = \frac{As+B}{(s^2+1)} + \frac{(s+1)}{(s^2+4)} \quad \text{——— (1)}$$

$$= (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

$$3s = As^3 + Bs^2 + 4As + 4B + Cs^3 + Cs + Ds^2 + D$$

$$3s = (A+C)s^3 + (D+B)s^2 + (4A+C)s + 4B + D$$

comparing both side with coefficient of (s^3, s^2, s etc). and (A, B, C, D)

$A + C = 0$	$4B + D = 0$	$D = -4B$
$B + D = 0$	$\because A + C = 0$	$B + D = 0$
$4A + C = 3$	$\frac{3-C}{4} + C = 0$	$B - 4B = 0$
$4A = 3 - C$	$3 - C + 4C = 0$	$-3B = 0$
$A = \frac{3-C}{4}$	$3 + 3C = 0$	$\boxed{B = 0}$
	$\cancel{3}C = -\cancel{3}1$	$\boxed{D = 0}$
	$\boxed{C = -1}$	
	$\boxed{A = -1}$	

$\longrightarrow$

From eqⁿ ①

$$x(s) = \frac{s+0}{(s^2+1)} - \frac{s+0}{(s^2+4)}$$

$$x(s) = \frac{s}{(s^2+1)} - \frac{s}{(s^2+4)} \Rightarrow \frac{s}{(s^2+1)} - \frac{s}{(s^2+2^2)}$$

Taking laplace Inverse,

$$x(t) = \cos t - \cos 2t$$

$$\boxed{x(t) = \cos t - \cos 2t}$$ Solved

Q.(1) $z = 0.1$ minute $\theta_i = 90°F$

$\theta_1 = 100°F$ $\theta_2 = 98°F$

$$98 - 90 = 100 - 90\left[1 - e^{-t/0.1}\right]$$

$$8 = 10\left(1 - e^{-t/0.1}\right)$$

$$\frac{8}{10} = \left(1 - e^{-t/0.1}\right)$$

$$\frac{\cancel{8}-10}{10} = -e^{-t/0.10}$$

$$\cancel{-}e^{-t/0.1} = \cancel{-}\frac{\cancel{2}}{\cancel{10}_5} \Rightarrow e^{-t/0.1} = \frac{1}{5}$$

Formula → $Y(t) = A(1 - e^{-t/z})$

$$\theta_2 - \theta_i = \theta_1 - \theta_i\,(1 - e^{-t/z})$$

$$e^{-t/0.1} = \frac{1}{5}$$

Taking log/ln both side.

$$\frac{-t}{0.1} = \log(.2)$$

$$-t = 1 \times \log(.2)$$

$$\boxed{-t = 0.1609 \text{ minute}}$$ Solved

Q(2) $z = 1$ min. $\theta_1 = 100°C$

$\theta_i = 50°C$ $t = 1.2$ min

$\theta_2 = ?$

Sol:→

$$\theta_2 - \theta_i = \theta_1 - \theta_i\,(1 - e^{-t/z})$$

$$\theta_2 - 50 = 100 - 50\,(1 - e^{-1.2/1})$$

$$\theta_2 = 50\,(1 - e^{-1.2/1}) + 50$$

$$\theta_2 = (50 - 50 \times 0.301) + 50$$

$$\theta_2 = 100 - 15.05$$

$$\boxed{\theta_2 = 84.95°C}$$ Solved

Q5 $\theta_i = 30^\circ C$ $\qquad z = 0.1$ minute

$\theta_1 = 100^\circ C$ $\qquad \theta_2 = 85^\circ C$

(A) $t = ?$

$(\theta_2 - \theta_i) = (\theta_1 - \theta_i)(1 - e^{-t/z})$

$85 - 30 = 100 - 30\,(1 - e^{-t/0.1})$

$0.2142 = e^{-t/.1}$

Taking ln both side

$\log(0.2142) = -\frac{t}{0.1}$

$\boxed{t = 0.15408 \text{ minute}}$ Solved

(B) Time required for 90% response,

Response = 90%

$z =$ 0.1 minute

$Y(t) = A(1 - e^{-t/z})$

$.90\cancel{A} = \cancel{A}(1 - e^{-t/.1})$

$.90 = 1 - e^{-t/.1}$

$e^{-t/.1} = .1$

Taking ln both side.

$\ln(.1) = -t/.1$

$\boxed{t = 0.2302 \text{ minutes}}$ Solved

(C) $t = 12$ sec

$z = 0.1$ min
$= 6$ sec.

$\theta_2 = ?$

$\boxed{\theta_2 = 90.52^\circ C}$

Solved

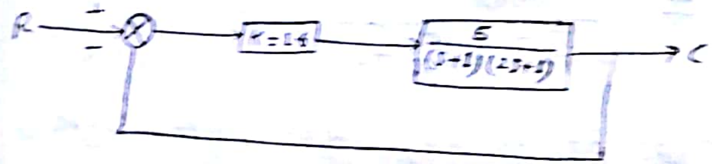
Q(3.1) The set point of the control system shown in fig is given a step change of 0.1 unit

(A) The maximum value of c and the time at which it occurs.

(B) The off set.

(C) The period of oscillation.

Draw a sketch of c(t) as a function of time.



Sol :→ $K = 1.6$

Thus:- $\frac{C(s)}{R(s)} = \frac{K \cdot \frac{5}{(s+1)(2s+1)}}{1 + K\frac{5}{(s+1)(2s+1)}}$

$\frac{C(s)}{R(s)} = \frac{5K}{(s+1)(2s+1) + 5K}$

$= \frac{5}{2s^2 + s + 2s + 1 + 5K}$

$= \frac{5}{2s^2 + 3s + 5K + 1}$

Put the value of $K = 1.6$

$\frac{C(s)}{R(s)} = \frac{5 \times 1.6}{(s+1)(2s+1) + 5 \times 1.6}$

$= \frac{8.0}{2s^2 + 2s + s + 1 + 8.0}$

$$\frac{C(s)}{R(s)} = \frac{8}{2s^2+3s+1+8}$$

$$\frac{C(s)}{R(s)} = \frac{8}{2s^2+3s+9}$$

$$\frac{C(s)}{R(s)} = \frac{8/9}{\frac{2s^2}{9}+\frac{3s}{9}+1}$$

$$\frac{C(s)}{R(s)} = \frac{8/9}{\frac{2s^2}{9}+\frac{s}{3}+1}$$

comparing by second order system eq^n.

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau^2 s^2 + 2\tau\xi s+1}$$

$$\tau^2 = \frac{2}{9} \Rightarrow \tau = \frac{\sqrt{2}}{3}$$

$$2\tau\xi = \frac{1}{3}$$

$$2\times\frac{\sqrt{2}}{3}\times\xi = \frac{1}{3} \Rightarrow \xi = \frac{1}{2\sqrt{2}} = 0.3535$$

$$\tau = \frac{\sqrt{2}}{3} = 0.4714$$

$$\xi = \frac{1}{2\sqrt{2}} = 0.3535$$

for unit step change,

$$R(s) = \frac{.1}{s}$$

$$\frac{C(s)}{R(s)} = \frac{8/9}{\tau^2 s^2+2\xi\tau s+1}$$

$$C(s) = \frac{R(s)\ 8/9}{\tau^2 s^2+2\tau\xi s+1}$$

$$C(s) = \frac{.1}{(s)}\frac{8/9}{(\tau^2 s^2+2\tau\xi s+1)}$$

$\because \xi < 1$ for under damped.

$$C(s) = \frac{.8}{9}\left[\frac{1}{\tau^2 s^2+2\tau\xi s+1}\right]$$

Taking laplace Transformation,

$$C(t) = \frac{.8}{9}\left[1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi t/\tau}\sin\left(\sqrt{1-\xi^2}\frac{t}{\tau}+\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

$$C(t) = \frac{.8}{9}[1-0]$$

$$C(t) = \frac{0.8}{9}$$

$$\boxed{C(\infty) = 0.0888}$$ Solved

(B) Off set $= R(\infty) - C(\infty)$

$\because R(s) = \frac{.1}{s}$

$R(t) = .1, \Rightarrow R(\infty) = .1$

$C(\infty) = 0.0888$

off set $= R(\infty) - C(\infty)$

$= 0.1 - 0.0888$

$$\boxed{\text{off set} = 0.012}$$ Solved

(c) Period of oscillation.

$$T = \frac{2\pi\tau}{\sqrt{1-\xi^2}}$$

$$T = \frac{2 \times 3.14 \times 0.4714}{\sqrt{1-(0.3535)^2}}$$

$$\boxed{T = 3.382 \text{ sec}}$$ solved

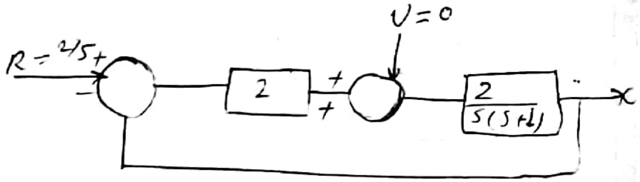
Q.13) For the control system shown in fig. determine.

(a) $C(s)/R(s)$

(b) $C(\infty)$

(c) off set

(d) $C(0.5)$



Sol :→

(a) $$\frac{C(s)}{R(s)} = \frac{2 \times \frac{2}{s(s+1)}}{1 + 2 \cdot \frac{2}{s(s+1)}}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{4}{\cancel{s(s+1)}}}{\frac{s(s+1)+4}{\cancel{s(s+1)}}} \Rightarrow \frac{4}{s(s+1)+4}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2+s+4} \Rightarrow \frac{\cancel{4}/\cancel{4}}{\frac{s^2}{4}+\frac{s}{4}+1}$$

$$\frac{C(s)}{R(s)} = \frac{1}{\frac{s^2}{4}+\frac{s}{4}+1}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{1}{0.25s^2+0.25s+1}}$$ Solved

(b) For step change. $R(s) = \frac{2}{s}$

$R(t) = 2.1$

$R(\infty) = 2.$

$$C(s) = \frac{R(s)}{0.25s^2+0.25s+1}$$

$$C(s) = \frac{2}{s[.25s^2+.25s+1]}$$

Comparing by second order eqn.

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau^2 s^2 + 2\tau\xi s + 1}$$

$\tau^2 = \frac{1}{4} \Rightarrow \boxed{0.5 = \tau}$

$2\tau\xi = 0.25$

$\boxed{\xi = 0.25}$

$\because \xi < 1$ for under damped system.

$$C(s) = \frac{2}{s}\left[\frac{1}{.25s^2+.25s+1}\right]$$

Taking Laplace,

$$C(t) = 2\left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi t/2} \sin\left(\sqrt{1-\xi^2}\,\frac{t}{\tau} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

Putting $t = \infty$.

$C(\infty) = 2[1-0]$

$\boxed{C(\infty) = 2}$ solved.

(c) offset $= R(\infty) - C(\infty)$

$= 2-2$

$\boxed{\text{off set} = 0}$ solved

(d) $C(0.5) = ?$

$$C(t) = \left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi t/2} \sin\left(\sqrt{1-\xi^2}\,\frac{t}{\tau} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

t = 0.5 putt.

$C(0.5) = [1 - 0.9375 \times 0.7788 \times 0.9952118]$

$\boxed{C(0.5) = 0.2733}$ solved

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