# MASS TRANSFER
# Theory and Practice

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# 1

# INTRODUCTION TO MASS TRANSFER

## 1.1 INTRODUCTION

A number of unit operations are carried out in Chemical Engineering applications which do not involve chemical reactions. These operations are carried out for separating either a component by mechanical means like screening, filtration and salting or increasing its concentration in a mixture. The latter is called mass transfer operation.

Frequently, these mass transfer operations are used for the separation of a product from the by-products formed and also from the unreacted raw materials. The separation technique plays a vital role in fixing the cost of final product.

## 1.2 CLASSIFICATION OF MASS TRANSFER OPERATIONS

It is classified as gas-liquid, liquid-liquid and fluid-solid operations.

### 1.2.1 Gas–Liquid

**Absorption:** Transfer of a solute from a gas mixture to a solvent is known as absorption. For example, (i) removal of ammonia gas from by-product coke ovens using water, (ii) removal of $H_2S$ from naturally occurring hydrocarbon gases by alkali solutions.

**Desorption:** This is reverse of absorption, i.e. removal of a solute in a solution using a gas. For example, removal of $NH_3$ from $NH_3$-water solution using air.

**Humidification:** Transfer of a liquid to a gas phase containing one or more components by contacting dry gas with pure liquid is known as humidification.

**Dehumidification:** Transfer of a vapour component from a gas-vapour mixture to a liquid phase by contacting them is known as dehumidification. For example, transferring water vapour from air-water vapour mixture to liquid water.

**Distillation:** Method of separating the components in a liquid mixture by the distribution of substances between a gas and a liquid phase is known as distillation. The method of separation depends on their relative volatility and applied to cases where all the components are present in both the phases. Here a new phase is created from the original solution itself. For example, separation of petroleum fractions by the application of heat, separation of high boiling water insoluble mixtures using steam.

### 1.2.2 Liquid–Liquid

**Extraction:** Separation of a component (solute) from a liquid mixture using another insoluble or partially miscible solvent is known as extraction. The separation depends on the distribution of solute between the two phases based on its physico-chemical characteristics. The two phases are solvent rich phase (extract) and residual liquid phase (raffinate). For example, (i) separation of acetic acid from acetic acid-water mixture using isopropyl ether as solvent, (ii) separation of dioxane from waterdioxane solution using benzene as solvent.

### 1.2.3 Solid–Liquid/Gas

**Leaching:** Separating a soluble solute from a solid mixture by contacting it with a solvent is known as leaching. For example, (i) separation of oil from oil seeds using hexane, (ii) separation of sugar from sugar beets using hot water, (iii) removal of copper from its ore using sulphuric acid.

**Adsorption:** Adsorption involves contact of solid with either a liquid or a gaseous mixture in which a specific substance from the mixture concentrates on the solid surface. For example, (i) removal of colour from solutions using activated carbon, (ii) removal of moisture from air by silica gel.

**Desorption:** It is the reverse of adsorption operation.

**Drying:** Drying refers to the removal of moisture from a substance. For example, (i) removal of water from a cloth, wood or paper, (ii) removal of water from solution (manufacture of spray dried milk).

**Crystallization:** The process of forming solid particles within a homogeneous phase is called crystallization. For example, (i) the homogenous phase could be a vapour as in the formation of snow, (ii) the formation of crystals of sugar from a concentrated sugar solution.

# 2

# DIFFUSION

## 2.1 INTRODUCTION

Separation of components in a mixture is achieved by contacting it with another insoluble phase. When transfer of the component from one phase to the other occurs due to concentration gradient, the phenomenon is called diffusion. The diffusion stops once equilibrium is attained. There are two types of diffusion,

1. Molecular diffusion
2. Eddy diffusion or Turbulent diffusion

## 2.2 MOLECULAR DIFFUSION AND EDDY DIFFUSION

Molecular diffusion can be defined as the movement of individual molecules in a highly zigzag manner through another fluid. The movement of molecules is imagined to be in a straight line at uniform velocity. However, the velocity and direction change when they are bombarded with other molecules. Molecular diffusion can also be called as Random-Walk process since the molecular movement is in a random path.

The phenomenon of molecular diffusion can be explained by a simple illustration, i.e. if a coloured solution is introduced in a pool of water, it begins slowly to diffuse into the entire liquid which is termed as molecular diffusion. To enhance its rate of mixing, a mechanical agitation is provided and this will cause a turbulent motion. This method of mass transfer is known as eddy or turbulent or convective diffusion.

## 2.3 DIFFUSIVITY OR DIFFUSION COEFFICIENT

Diffusion mainly depends upon the concentration gradient. In other words, the driving force for diffusion to occur is concentration gradient. This mass transfer phenomenon is defined by Fick's first law of diffusion, which states that *molar flux is directly proportional to the concentration gradient.* Mathematically,

$$J_A = -D_{AB}\left(\frac{\partial C_A}{\partial Z}\right) = -C\,D_{AB}\left(\frac{\partial x_A}{\partial Z}\right) \tag{2.1}$$

where $J_A$ is molar flux in moles/(area)(time), $D_{AB}$ is diffusion coefficient or diffusivity in area/time, $\partial C_A/\partial Z$ is concentration gradient, $C$ is molar concentration of constituents A and B in moles/vol. and $x_A$ is mole fraction of A in the mixture. The –ve sign indicates the drop in concentration with respect to distance (the movement from high concentration to low concentration).

Consider two gases A and B of equal volume placed in two boxes connected by a tube and maintained at a constant total pressure. Now molecular diffusion of both gases occurs. Since the total pressure $P$ remains constant throughout the process, the net moles of A diffused in one direction must be equal to the net moles of B diffused in opposite direction. So,

$$J_A = -J_B \tag{2.2}$$

Since the pressure is constant,

$$P = p_A + p_B = \text{constant} \tag{2.3}$$

and

$$C = C_A + C_B = \text{constant} \tag{2.4}$$

Differentiating Eq. (2.4) on both the sides,

$$dC_A = -dC_B \tag{2.5}$$

Writing Fick's law for component B,

$$J_B = -D_{BA}\frac{\partial C_B}{\partial Z} \tag{2.6}$$

Substituting for flux in Eq. (2.2) gives

$$-D_{AB}\left(\frac{\partial C_A}{\partial Z}\right) = -(-)D_{BA}\frac{\partial C_B}{\partial Z} \tag{2.7}$$

Substituting Eq. (2.5) in Eq. (2.7) and on simplification

$$D_{AB} = D_{BA} \tag{2.8}$$

This shows that the diffusivity is same for diffusion of A in B or B in A.

## 2.4 STEADY STATE MOLECULAR DIFFUSION IN FLUIDS

In the above discussion, we considered Fick's law for diffusion in a stationary fluid, i.e. there is no convective flow or bulk flow of the mixture. A general expression for flux $N_A$ will consider the whole fluid moving in bulk with its average molar velocity and its diffusional flux. Hence, the molar flux $N_A$ can be expressed as the sum of molar average velocity and diffusional flux ($J_A$)

$$N_A = (N_A + N_B)\, x_A - D_{AB}\left(\frac{\partial C_A}{\partial Z}\right) \tag{2.9}$$

For steady state molecular diffusion between two gases A and B, the net flux is given by

$$N = N_A + N_B \tag{2.10}$$

Applying Eq. (2.9) to the case of diffusion in $Z$ direction between the diffusional path $Z_1$ and $Z_2$, where the concentrations are $C_{A1}$ and $C_{A2}$ respectively. Equation (2.9) can be also written as,

$$N_A = (N_A + N_B)\frac{C_A}{C} - D_{AB}\left(\frac{\partial C_A}{\partial Z}\right),\ \text{since } x_A = \frac{C_A}{C} \tag{2.11}$$

Rearranging the above Eq. (2.11) and integrating we get

$$\int_{C_{A1}}^{C_{A2}} \frac{-dC_A}{N_A C - C_A(N_A + N_B)} = \frac{1}{CD_{AB}}\int_{Z_1}^{Z_2} dZ \tag{2.12}$$

$$\left(\frac{CD_{AB}}{N_A + N_B}\right)\ln\left\{\left[C_{A2} - \frac{C\left(\frac{N_A}{N_A + N_B}\right)}{C_{A1}}\right] - C\left(\frac{N_A}{N_A + N_B}\right)\right\} = Z_2 - Z_1 = Z \tag{2.13}$$

or

$$N_A = \left(\frac{N_A}{N_A + N_B}\right)\left(\frac{CD_{AB}}{Z}\right)\ln\left[\frac{\left(\frac{N_A}{N_A + N_B}\right) - \frac{C_{A2}}{C}}{\left(\frac{N_A}{N_A + N_B}\right) - \frac{C_{A1}}{C}}\right] \tag{2.14}$$

## 2.4.1 Molecular Diffusion in Gases

It is more convenient to use ideal gas law for gaseous mixtures. Hence,

$$\frac{C_A}{C} = \frac{p_A}{P_t} = y_A \tag{2.15}$$

where $p_A$ is the partial pressure of component A, $P_t$ is the total pressure and $y_A$ is mole fraction of component A. Further,

$$C = \frac{n}{V} = \frac{P_t}{RT} \tag{2.16}$$

Now substituting Eq. (2.16) in Eq. (2.14) gives

$$N_A = \left[\frac{N_A}{N_A + N_B}\right]\left[D_{AB}\frac{P_t}{RTZ}\right]\ln\left[\frac{\left\{\left(\frac{N_A}{N_A+N_B}\right)P_t - p_{A2}\right\}}{\left\{\left(\frac{N_A}{N_A+N_B}\right)P_t - p_{A1}\right\}}\right] \tag{2.17}$$

or

$$N_A = \left[\frac{N_A}{N_A + N_B}\right]\left[D_{AB}\frac{P_t}{RTZ}\right]\ln\left[\frac{\frac{N_A}{N_A+N_B} - y_{A2}}{\frac{N_A}{N_A+N_B} - y_{A1}}\right] \tag{2.18}$$

### 2.4.1.1 Case 1—Steady state diffusion of gas A through a stagnant gas B

In this case,

$$N_A = \text{constant and } N_B = 0,$$

$$\text{Hence, } \frac{N_A}{N_A + N_B} = 1 \tag{2.19}$$

Substituting Eq. (2.19) in Eq. (2.17) gives

$$N_A = \left(\frac{D_{AB}P_t}{RTZ}\right)\ln\left[\frac{P_t - p_{A2}}{P_t - p_{A1}}\right] \tag{2.20}$$

Since $P_t - p_{A2} = p_{B2}$, $P_t - p_{A1} = p_{B1}$, $p_{B2} - p_{B1} = p_{A1} - p_{A2}$.

Equation (2.20) becomes

$$N_A = \left(\frac{D_{AB}P_t}{RTZ}\right)\left[\frac{p_{A1} - p_{A2}}{p_{B2} - p_{B1}}\right]\ln\left(\frac{p_{B2}}{p_{B1}}\right) \tag{2.21}$$

Let,

$$p_{B,M} = \frac{p_{B2} - p_{B1}}{\ln\left(\frac{p_{B2}}{p_{B1}}\right)} \tag{2.22}$$

Then

$$N_A = \left[\frac{D_{AB}\,P_t}{RTZ\,p_{B,M}}\right](p_{A1} - p_{A2}) \tag{2.23}$$

or

$$N_A = \left[\frac{D_{AB}P_t}{RTZ}\right]\ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right] \tag{2.24}$$

#### 2.4.1.2 Case 2—Equimolar counter diffusion

In this case, $N_A = -N_B$. Then Eq. (2.17) becomes indeterminate. Hence, we can consider the general expression for flux as given in Eq. (2.9)

$$N_A = (N_A + N_B)\, x_A - D_{AB} \left(\frac{\partial C_A}{\partial Z}\right)$$

or

$$N_A = -\, D_{AB} \left(\frac{\partial C_A}{\partial Z}\right) \tag{2.25}$$

Integrating Eq. (2.25) between the respective limits

$$N_A \int_{Z_1}^{Z_2} dZ = -\, D_{AB} \int_{C_{A1}}^{C_{A2}} dC_A \tag{2.26}$$

$$N_A = \left(\frac{-D_{AB}}{Z}\right)(C_{A2} - C_{A1}) \quad \text{where } Z = Z_2 - Z_1 \tag{2.27}$$

or

$$N_A = \left(\frac{D_{AB}}{RTZ}\right)(p_{A1} - p_{A2}) \tag{2.28}$$

#### 2.4.1.3 Case 3—Steady state diffusion in multicomponent mixtures

For multicomponent mixtures, effective diffusivity ($D_{A,M}$)can be determined by using

$$D_{A,M} = \frac{N_A - y_A \sum\limits_{i=A}^{n} N_i}{\sum\limits_{i=A}^{n} \frac{1}{D_{Ai}} (y_i N_A - y_A N_i)} \tag{2.29}$$

where $D_{Ai}$ are the binary diffusivities. Here $D_{A,M}$ may vary considerably from one end of the diffusion path to the other, but a linear variation with distance can be assumed. For this situation, assume all but one component is stagnant, then Eq. (2.29) becomes,

$$D_{A,M} = \frac{1 - y_A}{\sum\limits_{i=B}^{n} y_i / D_{Ai}} = \frac{1}{\sum\limits_{i=B}^{n} y_i' / D_{Ai}} \tag{2.30}$$

where $y_i'$ is the mole fraction of component i on an A-free basis.

Substituting $D_{A,M}$ instead of $D_{A,B}$ in Eqs. (2.23) and (2.28), the mass transfer rate for multicomponent mixtures can be determined.

### 2.4.2 Diffusivity Prediction in Gases

Diffusion coefficient is a significant parameter which depends upon temperature, pressure and composition of the components. Diffusivity can be determined experimentally. For some of the systems it is given in Table 2.1 (more data is

**Table 2.1** Diffusivities of gases at standard atmospheric pressure, 101.3 kN/m$^2$

| *System* | *Temperature*, °C | *Diffusivity*, m$^2$/s × 10$^5$ |
|---|---|---|
| $H_2$–$CH_4$ | 0 | 6.25 |
| $O_2$–$N_2$ | 0 | 1.81 |
| $CO$–$O_2$ | 0 | 1.85 |
| $CO_2$–$O_2$ | 0 | 1.39 |
| Air–$NH_3$ | 0 | 1.98 |
| Air–$H_2O$ | 25.9 | 2.58 |
| | 59.0 | 3.05 |
| Air–ethanol | 0 | 1.02 |
| Air–*n*-Butanol | 25.9 | 0.87 |
| | 59.0 | 1.04 |
| Air–ethyl acetate | 25.9 | 0.87 |
| | 59.0 | 1.06 |
| Air–aniline | 25.9 | 0.74 |
| | 59.0 | 0.90 |
| Air–chlorobenzene | 25.9 | 0.74 |
| | 59.0 | 0.90 |
| Air–toluene | 25.9 | 0.86 |
| | 59.0 | 0.92 |

available in literature). In some cases, it is very difficult to determine experimentally. Hirschfelder-Bird-Spotz developed an empirical relation to determine the diffusivity for mixtures of non-polar or a polar with non-polar gas.

$$D_{AB} = \frac{\left[10^{-4}\left(1.084 - 0.249\sqrt{\frac{1}{M_A} + \frac{1}{M_B}}\right)T^{3/2}\sqrt{\frac{1}{M_A} + \frac{1}{M_B}}\right]}{\left[P_t (r_{AB})^2 \, f\left(\frac{KT}{\varepsilon_{AB}}\right)\right]} \tag{2.31}$$

where

$D_{AB}$ is the diffusivity, m$^2$/s

$T$ is the absolute temperature, K

$M_A$, $M_B$ is the molecular weight of A and B respectively, kg/kmol

$P_t$ is the absolute pressure, N/m$^2$

$r_{AB}$ is the molecular separation at collision = $\frac{r_A + r_B}{2}$, nm

$\varepsilon_{AB}$ is the energy of molecular attraction = $\sqrt{\varepsilon_A \, \varepsilon_B}$

$K$ is the Boltzmann's constant

$f\left(\frac{KT}{\varepsilon_{AB}}\right)$ is the collision function given by Fig. 2.1

The values of $r$ and $\varepsilon$ such as those listed in Table 2.2 can be calculated from other properties of gases such as viscosity. They can also be estimated empirically by

$$r = 1.18\, v^{1/3} \tag{2.32}$$

$$\varepsilon/K = 1.21\, T_b \tag{2.33}$$

where $v$ is the molal volume of liquid at normal boiling point, m$^3$/kmol and $T_b$ is the normal boiling point, K.

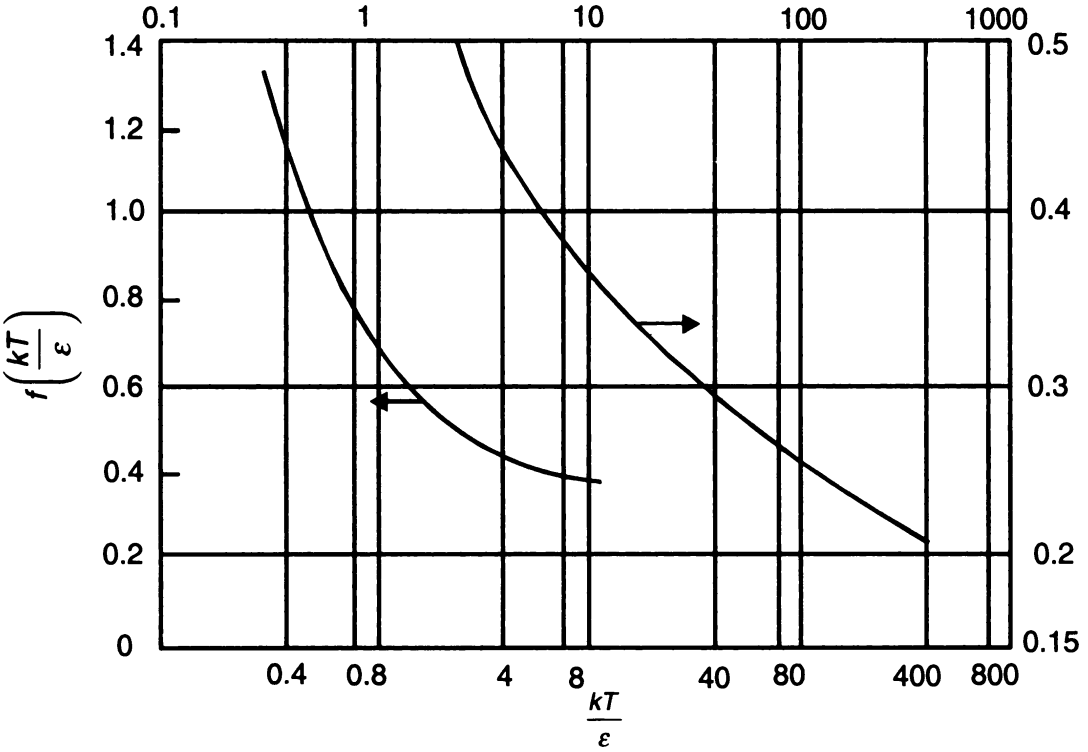


**Fig. 2.1** Collision function.

**Table 2.2** Force constants of gases as determined from viscosity data

| *Gas* | $\varepsilon/K$, K | $r$, nm |
|---|---|---|
| Air | 78.6 | 0.3711 |
| $CCl_4$ | 322.7 | 0.5947 |
| $CH_3OH$ | 481.8 | 0.3626 |
| $CH_4$ | 148.6 | 0.3758 |
| CO | 91.7 | 0.3690 |
| $CO_2$ | 195.2 | 0.3941 |
| $CS_2$ | 467 | 0.4483 |
| $C_2H_6$ | 215.7 | 0.4443 |
| $C_3H_8$ | 237.1 | 0.5118 |
| $C_6H_6$ | 412.3 | 0.5349 |
| $Cl_2$ | 316 | 0.4217 |
| HCl | 344.7 | 0.3339 |
| He | 10.22 | 0.2551 |
| $H_2$ | 59.7 | 0.2827 |
| $H_2O$ | 809.1 | 0.2641 |
| $H_2S$ | 301.1 | 0.3623 |
| $NH_3$ | 558.3 | 0.2900 |
| NO | 116.7 | 0.3492 |
| $N_2$ | 71.6 | 0.3798 |
| $N_2O$ | 232.4 | 0.3828 |
| $O_2$ | 106.7 | 0.3467 |
| $SO_2$ | 335.4 | 0.4112 |

### 2.4.3 Molecular Diffusion in Liquids

In the case of diffusion in liquids, $C$ and $D_{AB}$ may vary considerably with respect to process conditions. Hence, Eq. (2.14) can be modified to,

$$N_A = \left(\frac{N_A}{N_A + N_B}\right)\left(\frac{D_{AB}}{Z}\right)\left(\frac{\rho}{M}\right)_{av} \ln\left[\frac{\dfrac{N_A}{N_A + N_B} - x_{A2}}{\left(\dfrac{N_A}{N_A + N_B}\right) - x_{A1}}\right] \tag{2.34}$$

where $\rho$ is solution density and $M$ is solution molecular weight.

#### 2.4.3.1 Case 1—Diffusion of liquid A through a stagnant liquid B

In this case,

$N_B = 0$ and $N_A$ = constant. Hence,

$$N_A = \left(\frac{D_{AB}}{Z x_{B,M}}\right)\left(\frac{\rho}{M}\right)_{av} (x_{A1} - x_{A2}) \tag{2.35}$$

where

$$x_{B,M} = \left[\frac{x_{B2} - x_{B1}}{\ln\left(\dfrac{x_{B2}}{x_{B1}}\right)}\right] \tag{2.36}$$

or

$$N_A = \left(\frac{D_{AB}}{Z}\right)\left(\frac{\rho}{M}\right)_{av} \ln\left[\frac{1 - x_{A2}}{1 - x_{A1}}\right] \tag{2.37}$$

#### 2.4.3.2 Case 2—Equimolar counter-diffusion

In this case,

$$N_A = -N_B$$

Hence,

$$N_A = \left(\frac{D_{AB}}{Z}\right)(C_{A1} - C_{A2}) = \left(\frac{D_{AB}}{Z}\right)\left(\frac{\rho}{M}\right)_{av} (x_{A1} - x_{A2}) \tag{2.38}$$

### 2.4.4 Diffusivity Prediction in Liquids

Diffusivity has the dimension of area/time similar to that of gases. A few typical data are listed in Table 2.3 and more are available in literature. For some cases such as dilute solution of non-electrolytes, the diffusivity can be estimated by using Wilke and Chang empirical correlation.

$$D_{AB} = \frac{(117.3 \times 10^{-18})(\varphi M_B)^{0.5} T}{\mu v_A^{0.6}} \tag{2.39}$$

where

$D_{AB}$ is the diffusivity of A in very dilute solution in solvent B, $m^2/s$
$M_B$ is the molecular weight of solvent, kg/kmol.
$T$ is the absolute temperature, K
$\mu$ is the solution viscosity, kg/m-s
$v_A$ is the solute molal volume at normal boiling point, $m^3$/kmol.
= 0.0756 for water as solute.
$\varphi$ is the association factor for solvent.
= 2.26 for water as solvent
= 1.90 for methanol as solvent
= 1.50 for ethanol as solvent
= 1.00 for unassociated solvents, e.g. benzene and ethyl ether.

**Table 2.3** Liquid diffusivities

| *Solute* | *Solvent* | *Temperature*, K | *Solute concentration*, kmol/$m^3$ | *Diffusivity*, $m^2/s \times 10^9$ |
|---|---|---|---|---|
| $Cl_2$ | Water | 289 | 0.12 | 1.26 |
| HCl | Water | 273 | 9 | 2.7 |
| | | | 2 | 1.8 |
| | | 283 | 9 | 3.3 |
| | | | 2.5 | 2.5 |
| | | 289 | 0.5 | 2.44 |
| $NH_3$ | Water | 278 | 3.5 | 1.24 |
| | | 288 | 1.0 | 1.77 |
| $CO_2$ | Water | 283 | 0 | 1.46 |
| | | 293 | 0 | 1.77 |
| NaCl | Water | 291 | 0.05 | 1.26 |
| | | | 0.2 | 1.21 |
| | | | 1.0 | 1.24 |
| | | | 3.0 | 1.36 |
| | | | 5.4 | 1.54 |
| Methanol | Water | 288 | 0 | 1.28 |
| Acetic acid | Water | 285.5 | 1.0 | 0.82 |
| | | | 0.01 | 0.91 |
| | | 291 | 1.0 | 0.96 |
| Ethanol | Water | 283 | 3.75 | 0.50 |
| | | | 0.05 | 0.83 |
| | | 289 | 2.0 | 0.90 |
| *n*-Butanol | Water | 288 | 0 | 0.77 |
| $CO_2$ | Ethanol | 290 | 0 | 3.2 |
| Chloroform | Ethanol | 293 | 2.0 | |

The value of $v_A$ can be estimated from the data of atomic volumes added together. Typical data on atomic and molecular volume is available in Table 2.4.

**Table 2.4** Atomic and molecular volumes

| *Gas* | *Atomic volume,* $m^3/katom \times 10^3$ | *Gas* | *Molecular volume,* $m^3/kmol \times 10^3$ |
|---|---|---|---|
| Carbon | 14.8 | $H_2$ | 14.3 |
| Hydrogen | 3.7 | $O_2$ | 25.6 |
| Chlorine | 24.6 | $N_2$ | 31.2 |
| Bromine | 27.0 | Air | 29.9 |
| Iodine | 37.0 | CO | 30.7 |
| Sulphur | 25.6 | $CO_2$ | 34.0 |
| Nitrogen | 15.6 | $SO_2$ | 44.8 |
| In primary amines | 10.5 | NO | 23.6 |
| In secondary amines | 12.0 | $N_2O$ | 36.4 |
| Oxygen | 7.4 | $NH_3$ | 25.8 |
| In methyl esters | 9.1 | $H_2O$ | 18.9 |
| In higher esters | 11.0 | $H_2S$ | 32.9 |
| In acids | 12.0 | COS | 51.5 |
| In methyl ethers | 9.9 | $Cl_2$ | 48.4 |
| In higher ethers | 11.0 | $Br_2$ | 53.2 |
| Benzene ring subtract | 15 | $I_2$ | 71.5 |
| Naphthalene ring subtract | 30 | | |

## 2.4.5 Pseudo Steady State Diffusion

In many mass transfer operations, one of the boundaries between the fluids may move with time. If the length of the diffusion path changes over a period of time, a pseudo steady state develops. Here, the molar flux is related to the amount of A leaving the liquid by,

Flux = rate of change of liquid level × molar concentration of A in liquid phase

$$N_{AZ} = \frac{dZ}{dt} \times C_{A,L} \tag{2.40}$$

$$N_{AZ} = \left(\frac{C\,D_{AB}}{Zx_{B,M}}\right) \times (x_{B2} - x_{B1}) = \left(\frac{dZ}{dt}\right) C_{A,L} \tag{2.41}$$

Integrating Eq. (2.41) between $t = 0$, $Z = Z_{t0}$ and $t = t$, $Z = Z_t$

$$\int_{Z_{t0}}^{Z_t} z dz = \left[\frac{C\,D_{AB}(x_{B2} - x_{B1})}{x_{B,M}\,C_{A,L}}\right] \int_0^t dt \tag{2.42}$$

After integration and simplification,

$$\frac{t}{Z_t - Z_{t0}} = \frac{x_{B,M}\,C_{A,L}(Z_t + Z_{t0})}{2\,C\,D_{AB}(x_{A1} - x_{A2})} \tag{2.43}$$

i.e.

$$\frac{t}{Z_t - Z_{t0}} = \frac{x_{B,M}\,C_{A,L}\,(Z_t + Z_{t0} + Z_{t0} - Z_{t0})}{2\,C\,D_{AB}\,(x_{A1} - x_{A2})} \tag{2.44}$$

i.e.

$$\frac{t}{Z_t - Z_{t0}} = \frac{x_{B,M} C_{A,L} (Z_t - Z_{t0})}{2\, C\, D_{AB} (x_{A1} - x_{A2})} + \frac{2\, Z_{t0} x_{B,M} C_{A,L}}{2\, C\, D_{AB} (x_{A1} - x_{A2})} \tag{2.45}$$

Equation (2.45) is of the form,

$$y = mx + C \tag{2.46}$$

$$\text{where } y = \frac{t}{Z_t - Z_{t0}} \text{ and } x = (Z_t - Z_{t0})$$

$$\text{Slope, } m = \frac{x_{B,M} C_{A,L}}{2\, C\, D_{AB} (x_{A1} - x_{A2})}$$

and

$$\text{Constant } C = \frac{Z_{t0}\, x_{B,M}\, C_{A,L}}{C\, D_{AB} (x_{A1} - x_{A2})}$$

Since Eq. (2.45) is linear, by plotting $t/(Z_t - Z_{t0})$ against $(Z_t - Z_{t0})$, from the slope of line, $D_{AB}$ can be calculated, as the other parameters of Eq. (2.45) are all known. This equation is called as Winkelmann's relation.

## 2.5 DIFFUSION IN SOLIDS

Fick's law of diffusion can be applied to the system which is under steady state condition. It is applicable when diffusivity is independent of concentration and when there is no bulk flow. So, *the rate of diffusion of substance A per unit cross section of solid is proportional to the concentration gradient in the direction of diffusion.*

$$N_A = -D_A \left( \frac{dC_A}{dZ} \right) \tag{2.47}$$

where $D_A$ is the diffusivity of A through the solid. When the diffusion is taking place through a flat slab of thickness Z, then Eq. (2.47) becomes

$$N_A = \frac{D_A (C_{A1} - C_{A2})}{Z} \tag{2.48}$$

Here $C_{A1}$ and $C_{A2}$ are concentrations at opposite sides of the slab. For solids of varying transfer area, the diffusional rate is given by,

$$W = N_A S_{av} = \frac{D_A S_{av} (C_{A1} - C_{A2})}{Z} \tag{2.49}$$

$S_{av}$ is average mass transfer area of respective solid surfaces. Hence, for radial diffusion through a solid cylinder of inner and outer radii $r_1$ and $r_2$ respectively and its length $l$,

$$S_{av} = 2\pi r l \tag{2.50}$$

$$W = -D_A\ 2\pi rl \left(\frac{dC}{dr}\right) \tag{2.51}$$

On integrating

$$W \int_{r_1}^{r_2} \frac{dr}{r} = -D_A\ 2\pi l \int_{C_{A1}}^{C_{A2}} dC \tag{2.52}$$

or

$$W \ln\left(\frac{r_2}{r_1}\right) = -D_A 2\pi l (C_{A2} - C_{A1}) \tag{2.53}$$

or

$$W = \frac{[D_A\ 2\pi l\ (C_{A1} - C_{A2})}{\ln\left(\frac{r_2}{r_1}\right)}\left(\frac{r_2 - r_1}{r_2 - r_1}\right) \tag{2.54}$$

or

$$W = \frac{D_A S_{av} (C_{A1} - C_{A2})}{Z} \tag{2.55}$$

where $S_{av} = \dfrac{2\pi l (r_2 - r_1)}{\ln\left(\frac{r_2}{r_1}\right)}$ and $Z = (r_2 - r_1)$.

Similarly for radial diffusion through a spherical shell of inner and outer radii $r_1$ and $r_2$, the surface is

$$S_{av} = 4\pi r_1 r_2 \text{ and } Z = (r_2 - r_1) \tag{2.56}$$

## 2.5.1 Types of Solid Diffusion

The nature of solids and its interaction with the diffusing substance influence the rate of mass transfer. Different types of solid diffusion are discussed below.

### 2.5.1.1 Diffusion through polymers

Diffusion through polymeric membranes, e.g. gaseous separation through a membrane, mainly depends on the pressure gradient as the driving force. *Diffusion takes place from high pressure region to low pressure region.* A particular activation energy is needed for diffusion to take place and the temperature dependency of diffusivity is given by Arrhenius type relation,

$$D_A = D_o \exp\left[\frac{-H_D}{RT}\right] \tag{2.57}$$

where $H_D$ is the energy of activation and $D_o$ is a constant. For simple gases, $D_A$ is independent of concentration but for permanent gases, diffusivity is strongly dependent on solute concentration in the solid. The diffusional flux is given by

$$V_A = \frac{D_A S_A (p_{A1} - p_{A2})}{Z} \tag{2.58}$$

where

$V_A$ is the diffusional flux, $cm^3 \cdot$ gas (STP)/$cm^2$s

$D_A$ is the diffusivity of A, $cm^2$/s

$p_A$ is the partial pressure of diffusing gas, cm Hg

$S_A$ is the solubility coefficient, $cm^3 \cdot$ gas (STP)/$cm^3$ solid $\cdot$ cm Hg

$Z$ is the thickness of polymeric membrane, cm

Permeability can be defined as

$$P = D_A \, S_A \tag{2.59}$$

where $P$ is the permeability, $cm^3$ gas $\cdot$ (STP)/$cm^2 \cdot$ s (cm Hg/cm)

The solubility is related to concentration in SI units as,

$$c_A \text{ (kmol/m}^3\text{ solid)} = Sp_A = 22.414 \tag{2.60}$$

and in CGS system as $c_A$ (g mol/$cm^3$ solid) $= Sp_A = 22414$ (2.61)

#### 2.5.1.2 Diffusion through crystalline solids

Solute nature and crystalline structure are the important parameters in this type of diffusion. Some of the mechanisms followed for diffusion through crystal geometry are given below:

1. Interstitial mechanism—Solute atoms diffuse from one interstitial site to the next in the crystal lattice.
2. Vacancy mechanism—If lattice sites are vacant, an atom in an adjacent site may jump into the vacant site.
3. Interstitialcy mechanism—A large atom occupying in an interstitial site pushes the neighbouring lattice into an interstitial position and moves into the vacancy produced.
4. Crowded-ion mechanism—An extra atom in a chain of close-packed atoms can displace several atoms in the line from their equilibrium position.
5. Diffusion along grain boundaries—Diffusion takes place in crystal interfaces and dislocations.

#### 2.5.1.3 Diffusion in porous solids

The solid may be porous in nature such as adsorbents or membrane and the diffusion takes place either by virtue of concentration gradient or by hydrodynamic flow behaviour because of pressure difference. In steady state diffusion of gases, there are two types of diffusive movement, depending on the ratio of pore diameter $d$, to the mean free path of the gas molecules, $\lambda$.

If the ratio $d/\lambda > 20$, molecular diffusion predominates

$$N_A = \left(\frac{N_A}{N_A + N_B}\right)\left(\frac{D_{AB,eff} P_t}{RTZ}\right) \ln \left[\frac{\dfrac{N_A}{N_A + N_B} - y_{A2}}{\dfrac{N_A}{N_A + N_B} - y_{A1}}\right] \tag{2.62}$$

If $d/\lambda < 0.2$, the rate of diffusion is governed by the collisions of the gas molecules within the pore walls and follows Knudsen's law.

$$N_A = \frac{D_{K,A}\,(p_{A1} - p_{A2})}{RTl} \tag{2.63}$$

where

$D_{K,A}$ is the Knudsen diffusivity, cm$^2$/s

$l$ is the length of the pore, cm

$p_A$ is the partial pressure of diffusing substance, cmHg

Knudsen diffusivity can be determined by using an empirical relation,

$$D_{K,A} = \left(\frac{d}{3}\right)\left(\frac{8\,g_c\,RT}{\pi M_A}\right)^{1/2} \tag{2.64}$$

The mean free path $\lambda$ can be estimated by

$$\lambda = \left(\frac{3.2\mu}{P_t}\right)\left(\frac{RT}{2\pi g_c M_A}\right)^{1/2} \tag{2.65}$$

If $0.2 < d/\lambda < 20$, both molecular and Knudsen diffusion take place

$$N_A = \left(\frac{N_A}{N_A + N_B}\right)\left(\frac{D_{AB,eff}\,P_t}{ZRT}\right)\ln\left[\frac{\left(\frac{N_A}{N_A + N_B}\right)\left(1 + \frac{D_{AB,eff}}{D_{KA,eff}}\right) - y_{A2}}{\left(\frac{N_A}{N_A + N_B}\right)\left(1 + \frac{D_{AB,eff}}{D_{KA,eff}}\right) - y_{A1}}\right] \tag{2.66}$$

## 2.5.2 Unsteady State Diffusion

Since solids are not readily transported, as fluids, unsteady state diffusional conditions arise more frequently in solids than in fluids. For unsteady state diffusion, Fick's second law is applied,

$$\frac{\partial C}{\partial t} = D_{AB}\left[\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right] \tag{2.67}$$

## WORKED EXAMPLES

1. Estimate the diffusivities of the following gas mixtures:
   (a) Nitrogen—carbon dioxide, 1 Standard atm., 25°C.
   (b) Hydrogen chloride—air, 200 kN/m$^2$, 25°C.

**Solution.**

**(a) System:** $N_2$ and $CO_2$ at 1 Standard atm., 25°C

Let A denote nitrogen and B denote carbon dioxide

$$r_A = 0.3798 \text{ nm}, \qquad r_B = 0.3941 \text{ nm}$$

$$r_{AB} = (0.3798 + 0.3941)/2 = 0.38695 \text{ nm}$$

$$\left\{\frac{\varepsilon}{K}\right\}_A = 71.4, \qquad \left\{\frac{\varepsilon}{K}\right\}_B = 195.2$$

$$\left\{\frac{\varepsilon}{K}\right\}_{AB} = \sqrt{[71.4 \times 195.2]} = 118.056$$

$$\frac{KT}{\varepsilon_{AB}} = \frac{298}{118.056} = 2.52$$

$$f\left(\frac{KT}{\varepsilon_{AB}}\right) = 0.5 \text{ (from Fig. 2.1)}$$

$$\sqrt{\left[\left(\frac{1}{M_A}\right)+\left(\frac{1}{M_B}\right)\right]} = \sqrt{\left[\left(\frac{1}{28}\right)+\left(\frac{1}{44}\right)\right]} = 0.242$$

$$D_{AB} = \frac{10^{-4}\left\{1.084 - 0.249\sqrt{\left[\left(\frac{1}{M_A}\right)+\left(\frac{1}{M_B}\right)\right]}\right\}T^{3/2}\sqrt{\left[\left(\frac{1}{M_A}\right)+\left(\frac{1}{M_B}\right)\right]}}{P_t(r_{AB})^2 f\left(\frac{KT}{\varepsilon_{AB}}\right)}$$

$$D_{AB} = \frac{10^{-4}\left\{1.084 - 0.249\sqrt{\left[\left(\frac{1}{M_A}\right)+\left(\frac{1}{M_B}\right)\right]}\right\}T^{3/2}\sqrt{\left[\left(\frac{1}{M_A}\right)+\left(\frac{1}{M_B}\right)\right]}}{P_t(r_{AB})^2 f\left(\frac{KT}{\varepsilon_{AB}}\right)}$$

$$= \frac{10^{-4}\{1.084 - (0.249 \times 0.242)\} \times (298)^{3/2} \times (0.242)}{1.013 \times 10^5 \times (0.38695)^2 \times 0.5}$$

$D_{AB} = 1.6805 \times 10^{-5}$ m$^2$/s **Ans.**

**(b) System:** HCl and Air at 200 kN/m$^2$, 25°C
Let A denote HCl and B denote air

$$r_A = 0.3339 \text{ nm}, \qquad r_B = 0.3711 \text{ nm}$$

$$r_{AB} = \frac{0.3339 + 0.3711}{2} = 0.3525 \text{ nm}$$

$$\left\{\frac{\varepsilon}{K}\right\}_A = 344.7, \qquad \left\{\frac{\varepsilon}{K}\right\}_B = 78.6$$

$$\left\{\frac{\varepsilon}{K}\right\}_{AB} = \sqrt{[344.7 \times 78.6]} = 164.6$$

$$\left(\frac{KT}{\varepsilon_{AB}}\right) = \frac{298}{164.6} = 1.81$$

$$f\left(\frac{KT}{\varepsilon_{AB}}\right) = 0.62 \text{ (from Chart of Fig. 2.1)}$$

$$\sqrt{\left[\left(\frac{1}{M_A}\right) + \left(\frac{1}{M_B}\right)\right]} = \sqrt{\left[\left(\frac{1}{36.5}\right)+\left(\frac{1}{29}\right)\right]} = 0.249$$

$$D_{AB} = \frac{10^{-4}\left\{1.084 - 0.249\sqrt{\left[\left(\frac{1}{M_A}\right)+\left(\frac{1}{M_B}\right)\right]}\right\}T^{3/2}\sqrt{\left[\left(\frac{1}{M_A}\right)+\left(\frac{1}{M_B}\right)\right]}}{P_t(r_{AB})^2 f\left(\frac{KT}{\varepsilon_{AB}}\right)}$$

$$D_{AB} = \frac{10^{-4}\{1.084 - (0.249 \times 0.249)\} \times (298)^{3/2} \times (0.249)}{200 \times 10^3 \times (0.3525)^2 \times 0.62}$$

$D_{AB} = 8.496 \times 10^{-6}$ m²/s **Ans.**

2. Estimate the diffusivity of isoamyl alcohol ($C_5H_{12}O$) at infinite dilution in water at 288 K.

**Solution.**

Viscosity of water = 1.145 cp,

$V_A$ (by Kopp's law) $= 5 \times 0.0148 + 12 \times 0.0037 + 1 \times 0.0074$
$= 0.1258$ m³/kmol

$\phi$ (Association parameter for solvent-water) = 2.26

$$D_{AB} = \frac{(117.3 \times 10^{-18})(\varphi M_B)^{0.5} T}{\mu V_A^{\ 0.6}}$$

$$= \frac{(117.3 \times 10^{-18})(2.26 \times 18)^{0.5} \times 288}{(0.001145) \times (0.1258)^{0.6}}$$

$= 0.653 \times 10^{-9}$ m²/s **Ans.**

3. The diffusivity of carbon tetrachloride, $CCl_4$ through oxygen $O_2$, was determined in a steady state Arnold evaporating cell. The cell, having a cross sectional area of 0.82 $cm^2$, was operated at 273 K and 755 mmHg pressure. The average length of the diffusion path was 17.1 cm. If 0.0208 cc of $CCl_4$ was evaporated in 10 hours of steady state operation, what should be the value of the diffusivity of $CCl_4$ through oxygen?

**Solution.**

Vapour pressure of $CCl_4$ at 273 K = 33 mm Hg
Density of liquid $CCl_4$ = 1.59 g/cm$^3$
Considering $O_2$ to be non-diffusing and with $T = 273$ K, $P_t = 755$ mm Hg, $Z = 17.1$ cm
0.0208 cc of $CCl_4$ is evaporating in 10 hours.

$$\text{i.e.}\quad \frac{0.0208 \times 1.59}{154 \times 10} = 2.147 \times 10^{-5} \text{ g mol/h}$$

$$\text{Flux } N_A = \frac{2.147 \times 10^{-5} \times 10^{-3}}{3600 \times 0.82 \times 10^{-4}} = 7.27 \times 10^{-8} \text{ kmol/m}^2\text{ s}$$

$$N_A = \frac{D_{AB} P_t}{ZRT} \ln\left[\frac{(P_t - p_{A2})}{(P_t - p_{A1})}\right]$$

$$D_{AB} = \frac{N_A \times Z \times RT}{P_t \ln\left[\dfrac{P_t - p_{A2}}{P_t - p_{A1}}\right]}$$

$$= \frac{7.27 \times 10^{-8} \times 17.1 \times 10^{-2} \times 8314 \times 273}{\left[\dfrac{755}{760} \times 1.013 \times 10^5\right] \ln\left[\dfrac{\dfrac{755}{760} \times 1.013 \times 10^5 - 0}{\dfrac{755}{760} \times 1.013 \times 10^5 - \dfrac{33}{760} \times 1.013 \times 10^5}\right]}$$

$D_{AB} = 6.355 \times 10^{-6}$ m$^2$/s **Ans.**

4. A crystal of copper sulphate $CuSO_4 \cdot 5H_2O$ falls through a large tank of pure water at 20°C. Estimate the rate at which the crystal dissolves by calculating the flux of $CuSO_4$ from the crystal surface to the bulk solution. Molecular diffusion occurs through a film of water uniformly 0.0305 mm thick surrounding the crystal. At the inner side of the film, adjacent to the crystal surface, the concentration of $CuSO_4$ is 0.0229 mole fraction $CuSO_4$ (solution density = 1193 kg/m$^3$). The outer surface of the film is pure water. The diffusivity of $CuSO_4$ is $7.29 \times 10^{-10}$ m$^2$/s. Temperature = 293 K. Molecular weight of $CuSO_4$= 160.

**Solution.**

$$Z = 0.0305 \times 10^{-3} \text{ m}$$

$$M_{\text{av}} = \frac{(0.0229 \times 160) + (0.9771 \times 18)}{1} = 21.2518$$

$$\left(\frac{\rho}{M}\right) = \frac{1193}{21.2518} = 58.136$$

For pure water,

$$\left(\frac{\rho}{M}\right)_2 = \frac{1000}{18} = 55.56$$

$$\left(\frac{\rho}{M}\right)_{\text{av}} = \frac{58.136 + 55.56}{2} = 56.848$$

$$D_{\text{AB}} = 7.29 \times 10^{-10} \text{ m}^2\text{/s}$$

Assuming water to be non-diffusing

$$N_{\text{A}} = \frac{D_{\text{AB}}\left(\frac{\rho}{M}\right)_{\text{av}} \ln\left[\frac{1 - x_{\text{A2}}}{1 - x_{\text{A1}}}\right]}{Z}$$

$$N_{\text{A}} = \left[\frac{7.29 \times 10^{-10}}{0.0305 \times 10^{-3}}\right] \times 56.848 \times \ln\left[\frac{1 - 0}{1 - 0.0229}\right]$$

$$N_{\text{A}} = 3.15 \times 10^{-5} \text{ kmol/m}^2 \text{ s.}$$ **Ans.**

5. Alcohol vapour is diffusing through a layer of water vapour under equimolar counter diffusion at 35°C and 1 atm. pressure. The molal concentration of alcohol on the two sides of the gas film (water vapour) 0.3 mm thick are 80% and 10% respectively. Assuming the diffusivity of alcohol–water vapour to be 0.18 $\text{cm}^2\text{/s}$, (i) calculate the rate of diffusion of alcohol and water vapour in kg/hr through an area of 100 $\text{cm}^2$ (ii) if the water vapour layer is stagnant, estimate the rate of diffusion of alcohol vapour.

**Solution.**

(i) Equimolar counter diffusion

$$T = (273 + 35) = 308 \text{ K}, \quad P_t = 1 \text{ atm}$$

$$Z = 0.3 \text{ mm}, \quad D_{\text{AB}} = 0.18 \times 10^{-4} \text{ m}^2\text{/s}$$

| | | moles | mol fraction |
|---|---|---|---|
| (Position 1) | | | |
| | Air | 80 | 0.8 |
| | Water | 20 | 0.2 |
| (Position 2) | | moles | mol fraction |
| | Air | 10 | 0.1 |
| | Water | 90 | 0.9 |

$$N_A = \frac{D_{AB}}{ZRT} \times [p_{A1} - p_{A2}]$$

$$N_A = \frac{D_{AB} \times P_t}{ZRT} [y_{A1} - y_{A2}]$$

$$N_A = \left[\frac{0.18 \times 10^{-4} \times 1.013 \times 10^{-5}}{0.3 \times 10^{-3} \times 8314 \times 308}\right] \times [0.8 - 0.1]$$

$N_A = 1.66 \times 10^{-3}$ kmol/m$^2$ s

Rate = $N_A \times 100 \times 10^{-4} \times 3600 \times 46$ kg/h

= $1.66 \times 10^{-3} \times 100 \times 10^{-4} \times 3600 \times 46$

= 2.749 kg/h **Ans.**

(ii) Diffusion through a stagnant film

$$N_A = \frac{D_{AB} \times P_t}{ZRT} \ln \left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \left[\frac{0.18 \times 10^{-4} \times 1.013 \times 10^{5}}{0.3 \times 10^{-3} \times 8314 \times 304}\right] \ln \left[\frac{1 - 0.1}{1 - 0.8}\right]$$

$N_A = 3.5706 \times 10^{-3}$ kmol/m$^2$ s

Rate = $N_A \times 100 \times 10^{-4} \times 3600 \times 46$ kg/h

= $3.5706 \times 10^{-3} \times 100 \times 10^{-4} \times 3600 \times 46$

= 5.9129 kg/h

= $1.6425 \times 10^{-3}$ kg/s. **Ans.**

**6.** Hydrogen gas at 1 standard atm. and 25°C flows through a pipe made of unvulcanised neoprene rubber with ID and OD of 25 and 50 mm respectively. If the concentration of hydrogen at the inner surface of the pipe is $2.37 \times 10^{-3}$ kmol hydrogen/m$^3$ and the diffusivity of hydrogen gas through the rubber is $1.8 \times 10^{-6}$ cm$^2$/s, estimate the rate of loss of hydrogen by diffusion through a pipe of 2 m length. The outside air may be assumed to be free from hydrogen.

**Solution.**

Given: $T = 298$ K, $P_t = 1$ atm, ID = 25 mm, OD = 50 mm,

$C_1 = 2.37 \times 10^{-3}$ kmol/m$^3$, $D_{AB} = 1.8 \times 10^{-6}$ cm$^2$/s, $L = 2$ m

This is the case of diffusion through polymers, so

$$V_A = \frac{D_A S_A [p_{A1} - p_{A2}]}{Z}$$

$$Z = \frac{50 - 25}{2} = 12.5 \text{ mm}$$

As per Eq. (2.60), we have

$$V_A = \frac{D_A(C_{A1} - C_{A2})}{Z}$$

$$= 1.8 \times 10^{-10} (2.37 \times 10^{-3} - 0)/12.5 \times 10^{-3}$$

$$= 0.3413 \times 10^{-10} \text{ kmol/m}^2 \text{ s}$$

$$S_{av} = \frac{2\pi L(\text{OD} - \text{ID})}{2 \ln\left[\dfrac{\text{OD}}{\text{ID}}\right]}$$

$$= \frac{2\pi \times 2 \times 25 \times 10^{-3}}{2 \ln\left(\dfrac{50}{25}\right)} = 0.2266 \text{ m}^2$$

$$\text{Rate} = V_A \times S_{av}$$

$$= 0.3413 \times 10^{-10} \times 0.2266$$

$$= 7.734 \times 10^{-12} \text{ kmol/s} \qquad \textbf{Ans.}$$

7. Ammonia diffuses through nitrogen gas under equimolal counter diffusion at a total pressure of $1.013 \times 10^5$ Pa and at a temperature of 298 K. The diffusion path is 0.15 m. The partial pressure of ammonia at one point is $1.5 \times 10^4$ Pa and at the other point is $5 \times 10^3$ Pa. Diffusivity under the given condition is $2.3 \times 10^{-5}$ m$^2$/s. Calculate the flux of ammonia.

**Solution.**

Equimolal counter diffusion

$P_t = 1.013 \times 10^5$ Pa, $T = 298$ K, $Z = 0.15$ m, $p_{A1} = 1.5 \times 10^4$ Pa,

$p_{A2} = 5 \times 10^3$ Pa, $D_{AB} = 2.3 \times 10^{-5}$ m$^2$/s

$$N_A = \frac{D_{AB}}{ZRT} \times [p_{A1} - p_{A2}]$$

$$N_A = \frac{2.3 \times 10^{-5} \times [1.5 - 0.5] \times 10^4}{0.15 \times 8314 \times 298}$$

$$N_A = 6.19 \times 10^{-7} \text{ kmol/m}^2 \text{ s} \qquad \textbf{Ans.}$$

8. An ethanol–water solution is in contact at 20°C with an organic liquid of film thickness 0.4 cm in which water is insoluble. The concentration of ethanol at the interface is 6.8 wt% and at the other side of film it is 10.8 wt%. The densities are 0.9881 g/cc and 0.9728 g/cc respectively for 6.8 wt% and 10.8 wt% ethanol solutions. Diffusivity of ethanol is $0.74 \times 10^{-5}$ cm$^2$/s. Calculate the steady state flux in kmol/m$^2$ s.

**Solution.**

| (Position 1) | | weight | moles | mol fraction |
|---|---|---|---|---|
| | Ethanol | 6.80 | 0.1478 | 0.02775 |
| | Water | 93.20 | 5.18 | 0.9722 |

| (Position 2) | | weight | moles | mol fraction |
|---|---|---|---|---|
| | Ethanol | 10.8 | 0.235 | 0.0453 |
| | Water | 89.20 | 4.96 | 0.9547 |

$$M_{av}\text{ (position 1)} = \frac{(0.02775 \times 46) + (0.9722 \times 18)}{1} = 18.776$$

$$M_{av}\text{ (position 2)} = \frac{(0.0453 \times 46) + (0.9547 \times 18)}{1} = 19.268$$

$$\left(\frac{\rho}{M}\right)_1 = \frac{0.9881 \times 10^3}{18.776} = 52.626$$

$$\left(\frac{\rho}{M}\right)_2 = \frac{972.8}{19.268} = 50.488$$

$$\left(\frac{\rho}{M}\right)_{av} = \frac{52.626 + 50.488}{2} = 51.557$$

Assuming the organic liquids is stagnant

$$N_A = \frac{D_{AB}\left(\frac{\rho}{M}\right)_{av} \ln\left[\frac{1 - x_{A2}}{1 - x_{A1}}\right]}{Z}$$

$$= \frac{0.74 \times 10^{-5} \times 10^{-4}}{0.4 \times 10^{-2}}\ 51.557 \times \ln\left[\frac{1 - 0.02775}{1 - 0.0453}\right]$$

$N_A = 1.737 \times 10^{-5}$ kmol/m$^2$ s. **Ans.**

9. Calculate the rate of diffusion of acetic acid (A) across a film of non-diffusing water (B) solution 2 mm thick at 17°C, when the concentrations (by weight) on opposite sides of the film are 10% and 4% acid. The diffusivity of acetic acid in the solution is $0.95 \times 10^{-9}$ m$^2$/s. Density of 10% and 4% acid (by weight) are 1013 kg/m$^3$ and 1004 kg/m$^3$ respectively.

**Solution.**

$Z = 2$ mm, $T = 290$ K, Basis: 100 kg of mixture

| (Position 1) | | weight, kg | kmol | mole fraction |
|---|---|---|---|---|
| | $CH_3COOH$ | 10 | 0.167 | 0.0323 |
| | $H_2O$ | 90 | 5 | 0.9677 |

| (Position 2) | weight, kg | kmol | mole fraction |
|---|---|---|---|
| $CH_3COOH$ | 4 | 0.067 | 0.0124 |
| $H_2O$ | 96 | 5.33 | 0.9876 |

$$M_{av} \text{ (position 1)} = \frac{(0.0323 \times 60) + (0.9677 \times 18)}{1} = 19.3566$$

$$M_{av} \text{ (position 2)} = \frac{(0.0124 \times 60) + (0.9876 \times 18)}{1} = 18.5208$$

$$\left(\frac{\rho}{M}\right)_1 = \frac{1013}{19.3566} = 52.3335$$

$$\left(\frac{\rho}{M}\right)_2 = \frac{1004}{18.5208} = 54.209$$

$$\left(\frac{\rho}{M}\right)_{av} = \frac{52.3335 + 54.209}{2} = 53.2714$$

Assuming water to be non-diffusing

$$N_A = \frac{D_{AB}}{Z}\left(\frac{\rho}{M}\right)_{av} \ln\left[\frac{1 - x_{A2}}{1 - x_{A1}}\right]$$

$$= \frac{0.95 \times 10^{-9}}{2 \times 10^{-3}} \times 53.2714 \times \ln\left[\frac{1 - 0.0124}{1 - 0.0323}\right]$$

$N_A = 0.515 \times 10^{-6}$ kmol/m$^2$ s **Ans.**

**10.** Carbon dioxide and oxygen experience equimolal counter diffusion in a circular tube whose length and diameter are 1 m and 50 mm respectively. The system is at a total pressure of 10 atm. and a temperature of 25°C. The ends of the tube are connected to large chambers in which the species concentrations are maintained at fixed values. The partial pressure of $CO_2$ at one end is 190 mm Hg while at the other end is 95 mm Hg. (i) Estimate the rate of mass transfer. (ii) Find the partial pressure of $CO_2$ at 0.75 m from the end where the partial pressure is 190 mm Hg. Diffusivity under given condition is $2.1 \times 10^{-5}$ m$^2$/s.

**Solution.**

Diffusivity of $CO_2 - O_2 = 2.1 \times 10^{-5}$ m$^2$/s.

$L = 1$ m, diameter = 50 mm, $P_t = 10$ atm, $T = 298$ K,

$p_{A1} = 190$ mm Hg, $p_{A2} = 95$ mm Hg, $D_{AB} = 2.1 \times 10^{-5}$ m$^2$/s

(i) $N_A = \dfrac{D_{AB}[p_{A1} - p_{A2}]}{ZRT}$

$$R = \frac{P_o V_o}{T_o} = \frac{760 \times 22.414}{273} = 62.4 \text{ (mm Hg)(m}^3\text{)/(K)(kmol)}$$

$$N_A = \frac{2.1 \times 10^{-5}}{1 \times 62.4 \times 298} (190 - 95)$$

$N_A = 1.073 \times 10^{-7}$ kmol/m$^2$ s.

Rate of mass transfer $= 1.073 \times 10^{-7} \times \pi r^2$

$$= 1.073 \times 10^{-7} \times \pi \left[\frac{50 \times 10^{-3}}{2}\right]^2$$

$= 2.107 \times 10^{-10}$ kmol/s. **Ans.**

(ii) $$\frac{y_A - y_{A1}}{y_{A2} - y_{A1}} = \frac{Z - Z_1}{Z_2 - Z_1}$$

$$\frac{p_A - p_{A1}}{p_{A2} - p_{A1}} = \frac{Z - Z_1}{Z_2 - Z_1} \qquad \left[\because y = \frac{p_A}{p_t}\right]$$

$$\frac{p_A - 190}{95 - 190} = \frac{0.75 - 0}{1 - 0} = p_A = 118.75 \text{ mm Hg}$$ **Ans.**

**11.** In an oxygen–nitrogen gas mixture at 1 atm. 25°C, the concentrations of oxygen at two planes 0.2 cm apart are 10% and 20% (by volume) respectively. Calculate the flux of oxygen when (i) nitrogen is non-diffusing and (ii) there is equimolar counter diffusion. Diffusivity of oxygen in nitrogen is 0.215 cm$^2$/s.

**Solution.**

$P_t = 1$ atm., $T = 298$ K, $Z = 0.2$ cm, $y_{A1} = 0.2$, $y_{A2} = 0.1$, $D_{AB} = 0.215$ cm$^2$/s

(i) When $N_2$ is non-diffusing,

$$N_A = \frac{D_{AB} \times P_t}{ZRT} \ln \left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \frac{0.215 \times 10^{-4} \times 1.013 \times 10^5}{0.2 \times 10^{-2} \times 8314 \times 298} \ln \left[\frac{1 - 0.1}{1 - 0.2}\right]$$

$N_A = 5.18 \times 10^{-5}$ kmol/m$^2$ s **Ans.**

(ii) For equimolar counter diffusion

$$N_A = \frac{D_{AB} \times P_t}{ZRT} [y_{A1} - y_{A2}]$$

$$N_A = \frac{0.215 \times 10^{-4} \times 1.013 \times 10^5}{0.2 \times 10^{-2} \times 8314 \times 298} [0.2 - 0.1]$$

$N_A = 4.395 \times 10^{-5}$ kmol/m$^2$ s **Ans.**

**12.** Ammonia is diffusing through an inert air film 2 mm thick at a temperature of 20°C and a pressure of 1 atm. The concentration of ammonia is 10% by volume on one side of the film and zero on the other side. Determine the mass flux. Estimate the effect on the rate of diffusion if the pressure is increased to 10 atm. The diffusivity of $NH_3$ in air at 20°C and 1 atm. is 0.185 cm$^2$/s.

**Solution.**

$P_t = 1$ atm, $T = 293$ K, $Z = 2$ mm, $y_{A1} = 0.1$, $y_{A2} = 0$, $D_{AB} = 0.185$ cm$^2$/s

Assuming air to be stagnant and non-diffusing,

$$N_A = \frac{D_{AB} \times P_t}{ZRT} \ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \frac{0.185 \times 10^{-4} \times 1.013 \times 10^5}{2 \times 10^{-3} \times 8314 \times 293} \ln\left[\frac{1-0}{1-0.1}\right]$$

$N_A = 4.05 \times 10^{-5}$ kmol/m$^2$ s

Mass flux = $N_A$ × Molecular weight

$= 4.05 \times 10^{-5} \times 17 = 6.89 \times 10^{-4}$ kg/m$^2$ s

When pressure is increased to 10 atm.,

For gases, $D_{AB} \propto \frac{1}{P_t}$

$$\frac{(D_{AB})_1}{(D_{AB})_2} = \frac{(P_t)_2}{(P_t)_1}$$

$$\frac{0.185}{(D_{AB})_2} = \left(\frac{10}{1}\right) \qquad (D_{AB})_2 = 0.0185 \text{ cm}^2/\text{s}$$

$$N_A = \frac{D_{AB}\, P_t}{ZRT} \ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \frac{0.0185 \times 10^{-4} \times 10^5 \times 10 \times 1.013}{2 \times 10^{-3} \times 8314 \times 293} \ln\left[\frac{1-0}{1-0.1}\right]$$

$N_A = 4.05 \times 10^{-5}$ kmol/m$^2$ s.

So, rate of diffusion remains same on increasing the pressure. **Ans.**

**13.** Calculate the rate of diffusion of acetic acid (A) across a film of non-diffusing water (B) solution 2 mm thick at 17°C, when the concentrations on the opposite sides of the film are 9% and 3% acid (by weight). The diffusivity of acetic acid in the solution is $0.95 \times 10^{-9}$ m$^2$/s. Density of 9% and 3% by weight acid are 1012 kg/m$^3$ and 1003 kg/m$^3$ respectively.

$$Z = 2 \text{ mm}, \quad T = 290°\text{C}, \quad D_{AB} = 0.95 \times 10^{-9} \text{ m}^2/\text{s}$$

**Solution.**

| | | weight | moles | mol fraction |
|---|---|---|---|---|
| (Position 1) | | | | |
| | $CH_3COOH$ | 9 | 0.15 | 0.0288 |
| | $H_2O$ | 91 | 5.056 | 0.9712 |
| (Position 2) | | weight | moles | mol fraction |
| | $CH_3COOH$ | 3 | 0.05 | 0.0092 |
| | $H_2O$ | 97 | 5.389 | 0.9908 |

$$M_{av} \text{ (Position 1)} = \frac{(0.0288 \times 60) + (0.9712 \times 18)}{1} = 19.2096$$

$$M_{av} \text{ (Position 2)} = \frac{(0.0092 \times 60) + (0.9908 \times 18)}{1} = 18.3864$$

$$\left(\frac{\rho}{M}\right)_1 = \frac{1012}{19.2096} = 52.682$$

$$\left(\frac{\rho}{M}\right)_2 = \frac{1003}{18.3364} = 54.699$$

$$\left(\frac{\rho}{M}\right)_{av} = \frac{52.682 + 54.699}{2} = 53.691$$

Assuming water to be stagnant

$$N_A = \frac{D_{AB}\left(\frac{\rho}{M}\right)_{av}}{Z} \ln\left[\frac{1 - x_{A2}}{1 - x_{A1}}\right]$$

$$= \frac{0.95 \times 10^{-9}}{2 \times 10^{-3}} \times 53.691 \times \ln\left[\frac{1 - 0.0092}{1 - 0.0288}\right]$$

$$N_A = 5.0956 \times 10^{-7} \text{ kmol/m}^2\text{ s} \qquad \textbf{Ans.}$$

**14.** In an oxygen-nitrogen gas mixture at 1 atm., 25°C, the concentrations of oxygen at two planes 0.2 cm apart are 10% and 20% volume respectively. Calculate the rate of diffusion of oxygen expressed as g mol/cm$^2$ s for the case where

(i) the nitrogen is non-diffusing.
(ii) there is equimolar counter diffusion of the two gases.

Diffusivity of oxygen in nitrogen at 25°C and 1 atm. is 0.206 cm$^2$/s.

**Solution.**

The value of gas constant is 82.06 $cm^3$ atm./(g mol) (K).

$P_t$ = 1 atm., $T$ = 293 K, $Z$ = 2 cm, $y_{A1}$ = 0.2, $y_{A2}$ = 0.1, $D_{AB}$ = 0.206 $cm^2/s$

(For ideal gases, volume fraction = mole fraction)

(i) $N_2$ is non-diffusing

$$N_A = \frac{D_{AB}P_t}{ZRT} \ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \frac{0.206 \times 10^{-4} \times 1.013 \times 10^5}{0.2 \times 10^{-2} \times 8314 \times 293} \ln\left[\frac{1-0.1}{1-0.2}\right]$$

$$N_A = 4.96 \times 10^{-5} \text{ kmol/m}^2 \text{ s.}$$

(ii) In equimolar diffusion,

$$N_A = \frac{D_{AB}P_t}{ZRT} [y_{A1} - y_{A2}]$$

$$N_A = \frac{0.206 \times 10^{-4} \times 1.013 \times 10^5}{0.2 \times 10^{-2} \times 8314 \times 293} [0.2 - 0.1]$$

$$N_A = 4.283 \times 10^{-5} \text{ kmol/m}^2 \text{ s.}$$ **Ans.**

**15.** Benzene is stored in a tank of diameter 10 m and open at the top. A stagnant air film 10 mm thick is covering the surface liquid beyond which benzene is absent. If the atmospheric temperature is 25°C and the corresponding vapour pressure is 150 mm Hg, estimate the rate of loss of benzene. Diffusivity of benzene is 0.02 $m^2/h$. Total pressure is 1.0 atm.

**Solution.**

$P_t$ = 1 atm., $T$ = 298 K, $p_{A1}$ = 150 mm Hg = $0.2 \times 10^5$ $N/m^2$, $p_{A2}$ = 0, $D_{AB}$ = 0.02 $m^2/h$

Assuming air layer to be stagnant

$$N_A = \frac{D_{AB}\ P_t}{ZRT} \ln\left[\frac{P_t - p_{A2}}{P_t - p_{A1}}\right]$$

$$= \frac{0.02 \times 1.013 \times 10^5}{3600 \times 10^{-3} \times 10 \times 8314 \times 298} \ln\left[\frac{1.013 \times 10^5 - 0}{1.013 \times 10^5 - 0.2 \times 10^5}\right]$$

$$= 4.996 \times 10^{-6} \text{ kmol/m}^2 \text{ s.}$$

Rate = $4.996 \times 10^{-6} \times \pi(10/2)^2 = 3.925 \times 10^{-4}$ kmol/s **Ans.**

**16.** Alcohol vapour is being absorbed from a mixture of alcohol vapour and water vapour by means of a nonvolatile solvent in which alcohol is soluble but water is not. The temperature is 97°C and the total pressure is 760 mm Hg. The alcohol vapour can be considered to be diffusing through a film of alcohol–water–vapour mixture 0.1 mm thick. The mole % of alcohol in the vapour at the outside of the film is 80%, and that on the inside, next to the solvent is 10%. The diffusivity of alcohol–water vapour mixtures at 25°C and 1 atm. is 0.15 $cm^2/s$. Calculate the rate of diffusion of alcohol vapour in kg per hour if the area of the film is 10 $m^2$.

**Solution.**

$P_t$ = 760 mm Hg, $T$ = 370 K, $Z$ = 0.1 mm, $y_{A1}$ = 0.8, $y_{A2}$ = 0.1,

$D_{AB}$ = $0.15 \times 10^{-4}$ $m^2/s$ at 25°C and 1 atm., area of film = 10 $m^2$

For gases,

$$D_{AB} \propto T^{3/2}$$

$$\left(\frac{D_{AB1}}{D_{AB2}}\right) = \left(\frac{T_1}{T_2}\right)^{3/2}$$

$$\left[\frac{0.15 \times 10^{-4}}{D_{AB2}}\right] = \left(\frac{298}{370}\right)^{3/2}$$

$$D_{AB2} \text{ (at 97°C)} = 2.075 \times 10^{-5} \text{ m}^2/\text{s}$$

Water is insoluble in solvent and thus non-diffusing

$$N_A = \frac{D_{AB}P_t}{ZRT} \ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \frac{2.075 \times 10^{-5} \times 1.013 \times 10^5}{0.1 \times 10^{-3} \times 8314 \times 370} \ln\left[\frac{1 - 0.1}{1 - 0.8}\right]$$

$$N_A = 1.0278 \times 10^{-2} \text{ kmol/m}^2 \text{ s.}$$

$$\text{Rate} = 1.0278 \times 10^{-2} \times 10 \times 3600 \times 46 = 1.70 \times 10^{-4} \text{ kg/hr.}$$

**17.** Ammonia is diffusing through an inert air film 2 mm thick at a temperature of 20°C and a pressure of 1 atm. The concentration of ammonia is 10% by volume on one side of the film and zero on the other side. $D_{AB}$ at 0°C and 1 atm. 0.198 $cm^2/s$. Estimate rate of diffusion if the temperature is 20°C and pressure is raised to 5 atm.

**Solution.**

$P_t$ = 1 atm., $T$ = 293 K, $Z$ = 2 mm, $y_{A1}$ = 0.1, $y_{A2}$ = 0,

$D_{AB}$ = 0.198 $cm^2/s$ at 0°C and 1 atm.

(Volume% = mole%, for ideal gases)

$$\left(\frac{D_{AB1}}{D_{AB2}}\right) = \left(\frac{T_1}{T_2}\right)^{3/2}$$

$$\left[\frac{0.198 \times 10^{-4}}{D_{AB2}}\right] = \left(\frac{273}{293}\right)^{3/2}$$

$$D_{AB2} = 2.2015 \times 10^{-5} \text{ m}^2/\text{s}$$

Assuming air film to be stagnant

$$N_A = \frac{D_{AB}P_t}{ZRT} \ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \frac{2.2015 \times 10^{-5} \times 1.013 \times 10^5}{2 \times 10^{-3} \times 8314 \times 293}\left[\frac{1-0}{1-0.1}\right]$$

$$N_A = 4.823 \times 10^{-5} \text{ kmol/m}^2\text{ s.}$$

Now pressure is increased to 5 atm.

$$D_{AB} \propto \left(\frac{T^{3/2}}{P_t}\right)$$

$$\frac{D_{AB1}}{D_{AB2}} = \left(\frac{T_1}{T_2}\right)^{3/2}\left(\frac{P_2}{P_1}\right)$$

$$\frac{0.198 \times 10^{-4}}{D_{AB2}} = \left(\frac{273}{293}\right)^{3/2} \times \left(\frac{1}{5}\right)$$

$$D_{AB2} = 0.198 \times 10^{-4} \times \left(\frac{293}{273}\right)^{3/2} \times \left(\frac{1}{5}\right)$$

$$D_{AB2} = 4.403 \times 10^{-6} \text{ m}^2/\text{s}$$

$$N_A = \frac{D_{AB}P_t}{ZRT} \ln\left[\frac{1 - y_{A2}}{1 - y_{A1}}\right]$$

$$N_A = \frac{4.403 \times 10^{-6} \times 5 \times 1.013 \times 10^5}{2 \times 10^{-3} \times 8314 \times 293}\left[\frac{1-0}{1-0.1}\right]$$

$$N_A = 4.823 \times 10^{-5} \text{ kmol/m}^2\text{ s.}$$

So, there is no change in flux when pressure is changed. **Ans.**
$(N_A)_{new} = (N_A)_{initial}$, where $P$ gets cancelled with the $P_t$ term in the equation in numerator.

18. An open bowl 0.2 m in diameter contains water at 350 K evaporating into the atmosphere. If the currents are sufficiently strong to remove the water vapour as it is formed and if the resistances to its mass transfer in air is equivalent to

that of a 2 mm layer for condition of molecular diffusion, what will be the rate of evaporation? Diffusivity is 0.2 cm²/s, vapour pressure is 41.8 kN/m².

**Solution.**

$P_t = 1$ atm. $T = 350$ K, $Z = 2$ mm, $P_{A1} = 0.1$, $P_{A2} = 0$ (pure air), $D_{AB} = 0.2$ cm²/s

Assuming air to be non-diffusing and a stagnant layer of air of 2 mm

$$N_A = \frac{D_{AB}\,P_t}{ZRT} \ln\left[\frac{P_t - p_{A2}}{P_t - p_{A1}}\right]$$

$$= \frac{0.2\times10^{-4}\times1.013\times10^{5}}{2\times10^{-3}\times8314\times350} \ln\left[\frac{101.3-0}{101.3-41.8}\right]$$

$$= 1.852 \times 10^{-4} \text{ kmol/m}^2 \text{ s.}$$

$$\begin{aligned}\text{Rate of evaporation} &= N_A \times \text{area}\\ &= 1.852 \times 10^{-4} \times \pi \times (0.2/2)^2\\ &= 5.82 \times 10^{-6} \text{ kmol/s}\\ &= 5.82 \times 10^{-6} \times 18\\ &= 1.048 \times 10^{-4} \text{ kg/s} \quad \textbf{Ans.}\end{aligned}$$

**19.** In an experimental determination of diffusivity of toluene in air, Stefan's method is being used. A vertical glass tube 3 mm in diameter is filled with liquid toluene to a depth of 20 mm from the top open end. After 275 hrs at 39.4°C and a total pressure of 1 atm., the level has dropped to 80 mm from the top. Neglecting counter diffusion of air to replace the liquid, estimate the diffusivity.

**Solution.**

Density of liquid toluene = 850 kg/m³

Vapour pressure of toluene at 39.4°C = 7.64 kN/m²

Gas law constant, $R = 8314$ Nm/kmol. K

$T = 312.4$ K, $t = 275$ hrs, $P_t = 1$ atm., $Z_{t_0} = 20 \times 10^{-3}$ m, $Z_t = 80 \times 10^{-3}$ m

Air is assumed to be stagnant or non-diffusing

$$\rho_L = 850 \text{ kg/m}^3, \qquad p_A = 7.64 \text{ kN/m}^2$$

$$C_{A,L} = \frac{\rho_{A,L}}{M_L} = \frac{850}{92} = 9.24 \text{ kmol/m}^3$$

$$C_A = P_t/RT = \left[\frac{1.013 \times 10^5}{8314 \times 312.4}\right] = 0.039 \text{ kmol/m}^3$$

$$x_{A1} = \frac{p_A}{P_t} = \frac{7.64}{101.3} = 0.0754$$

$$x_{B1} = 1 - 0.0754 = 0.9246$$

$$x_{A2} = 0, \qquad x_{B2} = 1$$

$$x_{B,M} = \frac{x_{B2} - x_{B1}}{\ln\left(\frac{x_{B2}}{x_{B1}}\right)} = \frac{(1-0.9246)}{\ln\left(\frac{1}{0.9246}\right)} = 0.9618$$

$$\frac{t}{(Z_t - Z_{t0})} = \frac{X_{B,M}C_{A,L}(Z_t + Z_{t0})}{2CD_{AB}(x_{A1} - x_{A2})}$$

$$D_{AB} = \frac{x_{B,M}C_{A,L}(Z_t^2 - Z_{t0}^2)}{2 \times C \times (x_{A1} - x_{A2}) \times t}$$

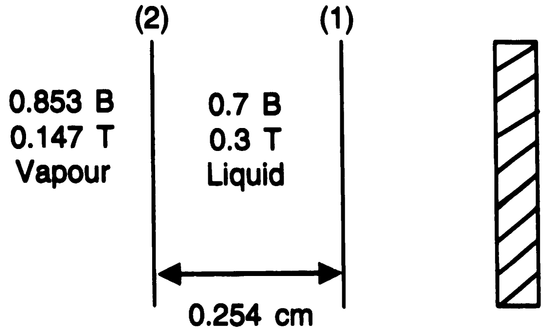
$$D_{AB} = \frac{0.9613 \times 11.81 \times [(80 \times 10^{-3})^2 - (20 \times 10^{-3})^2]}{275 \times 3600 \times 2 \times 0.039 \times (0.0754 - 0)}$$

$D_{AB} = 0.916 \times 10^{-5}$ m²/s. **Ans.**

20. A mixture of benzene and toluene is distilled in distillation unit. At one plane in the vertical tube where both benzene and toluene are condensing the vapour contains 85.3 mole% benzene and the adjacent liquid film contains 70 mole % benzene. The temperature is 360 K. Gas layer is 0.254 cm thick. The molal latent heat of vaporisation of both benzene and toluene are very close to each other. Vapour pressure of toluene is 368 mm Hg at 360 K. The system is assumed to behave ideal in liquid phase. Calculate the rate of interchange of benzene and toluene between vapour and liquid at atmospheric pressure. The diffusion coefficient is $0.0506 \times 10^{-4}$ m²/s.

**Solution.**

This is a case of equimolal counter diffusion as the latent heat of vaporisation are very close to each other.



$$N_A = \frac{D_{AB}}{ZRT}[p_{A1} - p_{A2}]$$

The partial pressure, $p_{Tol,1} = x_{Tol} \times \text{Vapour pressure}_{Tol}$

$= 0.3 \times 368/760 = 0.145$ atm

The partial pressure of toluene in vapour phase, $p_{Tol,2}$

= Mole fraction of toluene × Total pressure

$$= 0.147 \text{ atm}$$

$$N_B = \frac{D_{BA}}{ZRT}[p_{B1} - p_{B2}]$$

$$= \frac{0.0506 \times (0.145 - 0.147)}{0.254 \times 82.06 \times 360}$$

$$= -1.331 \times 10^{-8} \text{ g mol/cm}^2 \text{ s} \qquad \textbf{Ans.}$$

The negative sign indicates that the toluene is getting transferred from gas phase to liquid phase. (Hence, the transfer of benzene is from liquid to gas phase.)

**21.** A vertical glass tube 3 mm in diameter is filled with toluene to a depth of 2 cm from the top open end. After 275 hours of operation at 303 K and at a total pressure of 1 atm., the level dropped to 7.75 cm from the top. The density of the liquid is 820 kg/m$^3$ and its vapour pressure is at 57 mm Hg under the given operating conditions. Neglecting the counter diffusion of air to replace the liquid, calculate the diffusivity of toluene in air.

**Solution.**

This is a case of pseudo steady state diffusion as there is a significant change in the length of diffusion path.

$$Z_{t0} = 0.02 \text{ m}$$

$$Z_t = 0.0775 \text{ m}$$

$$t = 275 \text{ hrs.}$$

$$\text{Vapour pressure} = 57 \text{ mm Hg}$$

Molal density of liquid,

$$C_{AL} = 820/92 = 8.913 \text{ kmol/m}^3$$

$$x_{A1} = 57/760 = 0.075,\ x_{B1} = 1 - 0.075 = 0.925$$

$$x_{A2} = 0.0;\ x_{B2} = 1.0$$

$$= (x_B)_{lm} = \frac{[x_{B2} - x_{B1}]}{\ln\left(\frac{x_{B2}}{x_{B1}}\right)} = \frac{[1 - 0.925]}{\ln\left(\frac{1}{0.9250}\right)} = 0.962$$

$$= C = \frac{P}{RT} = \frac{1.0132 \times 10^{-5}}{8314 \times 303} = 0.04022 \text{ kmol/m}^3$$

$$D_{AB} = \frac{C_{AL}(x_B)_{lm}(Z_t^2 - Z_{t0}^2)}{C(x_{A1} - x_{A2})2t}$$

$$= \frac{8.913 \times 0.962 \times (0.0775^2 - 0.02^2)}{0.04022 \times (0.075 - 0.0) \times 2 \times (275 \times 3600)}$$

$$= 0.805 \times 10^{-5} \text{ m}^2\text{/s} \qquad \textbf{Ans.}$$

22. The diffusivity of the vapour of $CCl_4$ is determined by Winklemann method in which the level of liquid contained in a narrow tube maintained at a constant temperature of 321 K is continuously measured. At the top of the tube air is flowing and the partial pressure of the vapour at the top of the tube may be taken as zero at any instant. Assuming molecular mass transport, estimate the diffusivity of $CCl_4$ in air. Vapour pressure of $CCl_4$ 282 mm Hg and density of $CCl_4$ is 1540 kg/m$^3$.

The variation in liquid level with respect to time is given below:

| Time $t$, min | 0 | 26 | 185 | 456 | 1336 | 1958 | 2810 | 3829 | 4822 | 6385 |
|---|---|---|---|---|---|---|---|---|---|---|
| Liquid level, $(Z-Z_{t0})$ cm | 0 | 0.25 | 1.29 | 2.32 | 4.39 | 5.47 | 6.70 | 7.38 | 9.03 | 10.48 |

Compute $t/(Z_t - Z_{t0})$ and plot it against time, $t$

| Time $t$, min | 0 | 26 | 185 | 456 | 1336 | 1958 | 2810 | 3829 | 4822 | 6385 |
|---|---|---|---|---|---|---|---|---|---|---|
| Liquid level, $(Z_t - Z_{t0})$cm | 0 | 0.25 | 1.29 | 2.32 | 4.39 | 5.47 | 6.70 | 7.38 | 9.03 | 10.48 |
| $t/(Z_t - Z_{t0})$ | – | 104 | 143.5 | 190.5 | 304 | 357.5 | 418.5 | 514 | 533.5 | 610 |

**Solution.**

Slope = 51.4385 × 60 = 3086 s$^{-1}$/cm$^2$

$$3086 = \frac{C_{A,L}(x_B)_{lm}}{2D_{AB}C(x_{A1} - x_{A2})}$$

$$C_{A,L} = \frac{1540}{154} = 10 \text{ kmol/m}^3$$

$$C = \left(\frac{P}{RT}\right) = \frac{1}{82.06 \times 321} = 3.8 \times 10^{-5} \text{ g mol/cm}^3$$

$$(x_B)_{lm} = \frac{[x_{B2} - x_{B1}]}{\ln\left(\frac{x_{B2}}{x_{B1}}\right)} = \frac{[1 - 0.629]}{\ln\left(\frac{1}{0.629}\right)} = 0.8$$

$$D_{AB} = \frac{10 \times 0.8}{3086} \times 10^4 \times 2 \times 3.8 \times 10^{-2} \times (0.371 - 0) = 9.2 \times 10^{-6} \text{ m}^2\text{/s}$$

**Ans.**

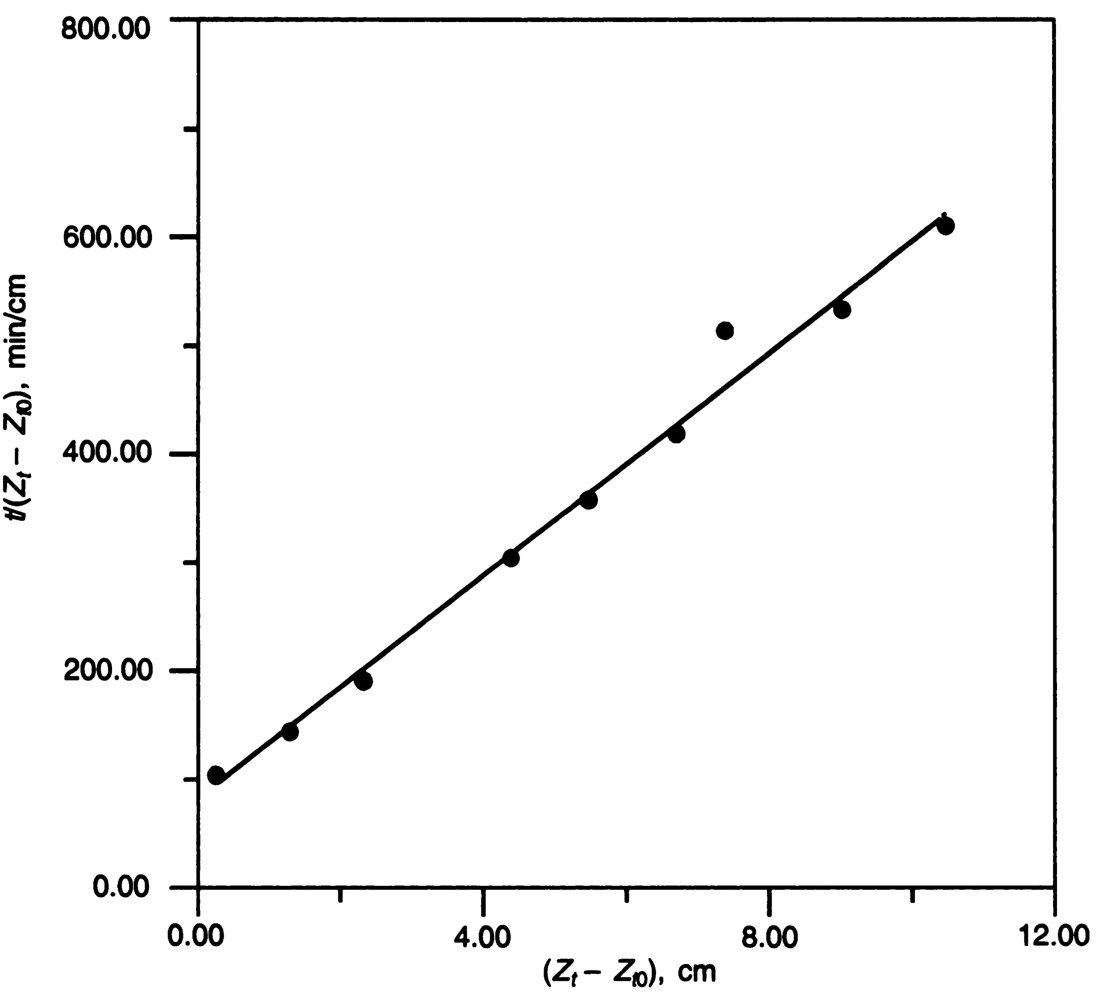


**Fig. 2.2** Example 22.

23. There are two bulbs connected by a straight tube 0.001 m in diameter and 0.15 m in length. Initially the bulb at end '1' contains nitrogen and the bulb at end '2' contains hydrogen. The pressure and temperature are maintained constant at 25°C and 1 standard atm. At a certain time after allowing the diffusion to occur between the two bulbs, the nitrogen content of the gas at end '1' of the tube is 80 mole % and at the other end is 25 mole %. If the diffusion coefficient is 0.784 $cm^2/s$, determine the rates and direction of transfer of hydrogen and nitrogen.

**Solution.**

It is a case of equimolal counter diffusion as the tube is perfectly sealed to two bulbs at the end and the pressure throughout is constant.

$$\text{Cross-sectional area of tube} = \frac{\pi}{4}D^2 = \frac{22 \times (0.001)^2}{7 \times 4} = 7.85 \times 10^{-7}\ \text{m}^2$$

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{(8314)(298)} = 0.0409\ \text{kmol/m}^3$$

$$x_{A1} = 0.8 \text{ and } x_{A2} = 0.25$$

$$\text{Rate of transfer} = \text{Area} \times N_A = \frac{\text{Area} \times D_{AB} \times C \times [x_{A1} - x_{A2}]}{Z}$$

$$= \frac{7.85 \times 10^{-7} \times 0.78 \times 10^{-4} \times 0.0409 \times (0.8 - 0.25)}{0.15}$$

$$= 0.923 \times 10^{-11} \text{ kmol/s} \qquad \textbf{Ans.}$$

**24.** Estimate the diffusivity of methanol in carbon tetrachloride at 15°C.

**Solution.**

$$D_{AB} = \frac{(117.3 \times 10^{-18})(\varphi M_B)^{0.5} T}{\mu v_A^{0.6}}$$

$M_B$ = Molecular weight of methanol 32

$\varphi = 1.9$

$v_A$ = solute volume at normal BP, $m^3$/kmol

$v_A$ for $CCl_4$ = $[14.8 + (4 \times 24.6)] \times 10^{-3}$

$= 0.1132$ $m^3$/kmol

$\mu = 0.6\ cP = 0.006\ P = 0.0006$ kg/m s

$$D_{AB} = \frac{(117.3 \times 10^{-18})(1.9 \times 32)^{0.5}(288)}{(0.0006)(0.1132)^{0.6}}$$

$$= 1.623 \times 10^{-9} \text{ m}^2\text{/s} \qquad \textbf{Ans.}$$

**25.** Estimate the diffusivity of methanol in water at 15°C.

**Solution.**

$M_B = 18, \phi = 2.26, T = 288, \mu = 0.001$ kg/m·s

$v_A$ for $C_2H_5OH$ = $[(2 \times 14.8) + (6 \times 3.7) + 7.4] \times 10^{-3}$

$= 0.0592$ $m^3$/kmol

$$D_{AB} = \frac{(117.3 \times 10^{-18})(2.26 \times 18)^{0.5}(288)}{(0.001)(0.0592)^{0.6}}$$

$$= 1.175 \times 10^{-9} \text{ m}^2\text{/s} \qquad \textbf{Ans.}$$

**26.** An unglazed porcelain plate 5 mm thick has an average pore diameter of 0.2 μm. Pure oxygen gas at 20 mm Hg (abs) 373 K on one side of the plate passes through at a rate of 0.093 cc (20 mm Hg, 373 K)/$cm^2$ s. When the pressure on the downstream side was so low as to be considered negligible. Estimate the rate of passage of Hydrogen at 298 K and 10 mm Hg abs. with negligible downstream pressure.

**Solution.**

Viscosity of oxygen = 0.02 *cp* = 20 × $10^{-6}$ Ns/m$^2$
Total pressure = 20 mmHg = 20 × 133.3 = 2666 N/m$^2$
Molecular weight of oxygen = 32
Then

$$\lambda = \frac{3.2\mu}{P_t}\left[\frac{RT}{2\pi g_c M}\right]^{0.5}$$

$$= \frac{3.2\times 20\times 10^{-6}}{2666}\left[\frac{8314\times 373}{2\times\pi\times 1\times 32}\right]^{0.5}$$

$$= 2.98\times 10^{-6}\ \text{m}$$

Pore diameter $d$ = 0.2 μm = 0.2 × $10^{-6}$ m

Therefore,

$$\frac{d}{\lambda} = \frac{0.2\times 10^{-6}}{2.98\times 10^{-6}} = 0.067$$

Hence, Knudsen diffusion occurs.

Now,

$$N_A = 0.093\ \text{cc (20 mm Hg, 373 K)/cm}^2\text{s}$$

$$= \frac{0.093\times 20\times 273}{760\times 373} = 1.79\times 10^{-3}\ \text{cm}^3/\text{cm}^2\text{s}$$

$$= \frac{1.79\times 10^{-3}}{22414} = 7.99\times 10^{-8}\ \text{g mol/cm}^2\,\text{s} = 7.99\times 10^{-7}\ \text{kmol/m}^2\,\text{s}$$

$$D_{KA} = \left[\frac{d}{3}\right]\left[\frac{8gcRT}{\pi M_A}\right]^{0.5}$$

$$= \left[\frac{0.2\times 10^{-6}}{3}\right]\left[\frac{8\times 1\times 8314\times 373}{\pi\times 32}\right]^{0.5}$$

$$= 33.11\times 10^{-6}\ \text{m}^2/\text{s}$$

$$N_A = \frac{D_{K,A}}{RTl}\left[\frac{p_{A1}-p_{A2}}{RTl}\right]$$

$$= 7.99\times 10^{-7} = 33.11\times 10^{-6}\times\left[\frac{2666-0}{8314\times 373\times l}\right]$$

Therefore,

$$l = 0.0356\ \text{m}$$

For the diffusion with Hydrogen, viscosity is 0.0085 cp, pressure is 1333 N/m$^2$, molecular weight is 2 and temperature is 298 K.
Then

$$\lambda = \frac{3.2\mu}{P_t}\left[\frac{RT}{2\pi g_c M}\right]^{0.5}$$

$$= \frac{3.2 \times 8.5 \times 10^{-6}}{1333}\left[\frac{8314 \times 298}{2 \times \pi \times 1 \times 2}\right]^{0.5}$$

$$= 9.06 \times 10^{-6} \text{ m}$$

Proe diameter $d$ = 0.2 μm = 0.2 × $10^{-6}$ m
Therefore,

$$\frac{d}{\lambda} = \frac{0.2 \times 10^{-6}}{9.06 \times 10^{-6}} = 0.022$$

Hence, Knudsen diffusion occurs.
Now,

$$[D_{K,A}]_{\text{new (Hydrogen)}} = \left[\frac{0.2 \times 10^{-6}}{3}\right] \times \left[\frac{8 \times 1 \times 8314 \times 298}{\pi \times 2}\right]^{0.5}$$

$$= 1.184 \times 10^{-4} \text{ m}^2/\text{s}$$

$$N_A = \frac{D_{K,A}}{RTl}(p_{A1} - p_{A2})$$

$$= \frac{1.184 \times 10^{-4}}{8314 \times 298 \times 0.0356}(1333 - 0)$$

$$= 1.789 \times 10^{-6} \text{ kmol/m}^2 \text{ s} \qquad \textbf{Ans.}$$

## EXERCISES

**Note:** *Any missing data may be taken from literature.*

1. Estimate the diffusivities of the following gas mixtures:
   (i) Acetone–air at STP
   (ii) Toluene–air, 1 Standard atm., 30°C.
   (iii) Aniline–air, STP.

   (**Ans:** (i) 9.2838 × $10^{-6}$ m$^2$/s (ii) 8.3186 × $10^{-6}$ m$^2$/s (iii) 6.8596 × $10^{-6}$ m$^2$/s)
2. Estimate the diffusivity of ethanol in water at 10°C.

   (**Ans:** 1.008 × $10^{-9}$ m$^2$/s)

3. Ethanol is diffusing through a layer of water of thickness 3 mm at 20°C. Diffusivity of alcohol in water is $0.52 \times 10^{-9}$ m$^2$/s. The concentrations on opposite sides of water film are 4% and 10% (by weight) of alcohol respectively are 0.99 and 0.98 g/cm$^3$. Assuming that water film is stagnant, estimate (i) the flux of alcohol and (ii) concentration of alcohol in the middle of water film.

   (**Ans:** (i) $2.432 \times 10^{-7}$ kmol/m$^2$s (ii) 0.029 (mole fraction))

4. Through the accidental opening of a valve, water has been spilled on the floor of an industrial plant in a remote, difficult to reach area. It is desired to estimate the time required to evaporate the water into the surrounding quiescent air. The water layer is 1 mm thick and may be assumed to remain at a constant temperature of 24°C. The air is also at 24°C and 1 atm pressure with an absolute humidity of 0.002 kg water vapour/kg of dry air. The evaporation is assumed to take place by molecular diffusion through a gas film 0.5 cm thick. Diffusion coefficient for water vapour in air is 0.259 cm$^2$/s.

   (**Ans:** 13.67 hours)

5. Calculate the rate of diffusion of NaCl at 18°C through a stagnant film of NaCl-water mixture 1 mm thick when the concentrations are 20% and 10% (by weight) respectively on either side of the film. Diffusivity of NaCl in water is $1.26 \times 10^{-9}$ m$^2$/s. The densities of 20% and 10% NaCl solutions are 1149 and 1067 kg/m$^3$ respectively.

   (**Ans:** $2.81 \times 10^{-6}$ kmol/m$^2$ s)

6. In an $O_2$–$N_2$ gas mixture at 1.01325 bar and 20°C, the concentration of $O_2$ at two planes 0.002 m apart are 20% and 10% volume respectively. (i) Calculate the rate of diffusion of $O_2$ expressed as kg moles of oxygen/m$^2$ s for the case where $N_2$ is non diffusing $D_{O_2-N_2} = 0.181 \times 10^{-4}$ m$^2$/s. (ii) Calculate the rate of diffusion of oxygen in kmol/m$^2$ s assuming equimolal counter diffusion.

   (**Ans:** (i) $4.4326 \times 10^{-5}$ kmol/m$^2$ s (ii) $3.763 \times 10^{-5}$ kmol/m$^2$ s)

7. A vertical glass tube of diameter 0.3 cm is filled with benzene at 30°C to a depth of 2 cm from top end. After 24 hours, the liquid level in the tube had fallen to 2.5 cm from the top end. Estimate the diffusivity of benzene into air if the air above the liquid surface in the tube is stagnant. The vapour pressure and density of benzene at 30°C are 60 mm Hg and 800 kg/m$^3$ respectively.

   (**Ans:** $0.4 \times 10^{-5}$ m$^2$/s)

8. A vertical glass tube 1 cm in diameter is filled with liquid acetone to a depth of 5 cm from the top open end. After 4 hours of operation at 303 K and at a total pressure of 1 atm., the level dropped by 2 mm. The density of the liquid is 790 kg/m$^3$ and its vapour pressure is at 288 mm Hg under the given operating conditions. Neglecting the counter diffusion of air to replace the liquid, calculate the diffusivity of acetone in air.

   (**Ans:** $0.49 \times 10^{-5}$ m$^2$/s)

9. A gas mixture containing 1/5 hydrogen and 4/5 methane by volume is prepared through which oxygen is allowed to diffuse. The total pressure is $1 \times 10^5$ N/m$^2$ and temperature is 2°C. Estimate the rate of diffusion of $O_2$ through the gas film of thickness 3 mm when concentration change across the film is 12 to 7% by volume. Diffusivity data at 1 atm., 0°C is

   (i) $D_{O_2-H_2} = 7.1 \times 10^{-5}$ m$^2$/sec.

   (ii) $D_{O_2-CH_4} = 1.88 \times 10^{-5}$ m$^2$/sec.

   (**Ans:** $1.82 \times 10^{-5}$ kmol/m$^2$ s)

10. A volatile organic compound costing Rs. 6.50 per kg is stirred in a tank 8 m in diameter and open to the atmosphere. A stagnant air film of thickness 10 mm is covering the surface of the compound, beyond which the compound is absent. If the atmospheric temperature is 27°C, vapour pressure of the compound is 160 mmHg and its diffusivity is 0.02 m$^2$/h, calculate the loss in rupees per day. Molecular weight of the organic compound is 78.

    (**Ans:** Rs. 11,745/day)

11. Estimate the rate of diffusion of chloropicrin ($CCl_3$ $NO_2$) into air, which is stagnant at 25°C and 1 atm. pressure. Diffusivity = 0.088 cm$^2$/s, vapour pressure at 25°C of $CCl_3$ $NO_2$ is 23.81 mm Hg, Density of chloropicrin = 1.65 g/cm$^3$. Surface area of liquid exposed for evaporation = 2.29 cm$^2$. Distance from liquid level to top of tube is 11.14 cm.

    (**Ans:** $0.28 \times 10^{-11}$ kmol/s)

12. A mixture of alcohol and water vapour is rectified by contact with alcohol–water liquid solution. Alcohol is transferred from gas phase to liquid phase and water from liquid to gas phase. The model flow rates are maintained equal but in opposite directions. The temperature 80°C and pressure of 1 atmosphere are maintained constant. Both components diffuse through a gas film of 0.15 mm thick. The molal concentration of alcohol on outer and inner sides of the film is 85% and 10% respectively. Calculate (i) the rate of diffusion of alcohol, (ii) rate of diffusion of water in kg per hour through a film area of one cm$^2$. The diffusivity is 0.184 cm$^2$/s.

    (**Ans:** (i) $3.17 \times 10^{-3}$ kmol/s (ii) 11.435 kg/h)

13. Ammonia is diffusing through an inert air film 2 mm thick at a temperature of 20°C and a pressure of 1 atmosphere. The concentration of $NH_3$ is 10% by volume on one side of the film and zero on the other side. Estimate the effect on the rate of diffusion of raising the total pressure to 5 atmospheres. The diffusivity of $NH_3$ in air at 0°C and 1 atm. is 0.198 cm$^2$/s.

    (**Ans:** $48.18 \times 10^{-6}$ kmol/m$^2$ s)

14. Alcohol is diffusing from gas to liquid and water from liquid to gas under conditions of equimolal counter diffusion at 35°C and 1 atmosphere pressure. The molal concentrations of alcohol on the two sides of a gas film 0.3 mm thick are 80% and 10% respectively. Assuming the diffusivity of alcohol–water vapour to be 0.18 cm$^2$/s, calculate the rate of diffusion of

alcohol and water in kilograms per hour through an area of 100 cm$^2$. Molecular weight of alcohol = 74.1; $R = 82.06$ cm$^3 \cdot$ atm g mol K.

(**Ans:** 4.43 kg/h)

**15.** Oxygen is diffusing through a stagnant layer of methane 5 mm thick. The temperature is 0°C and the pressure of 1 atmosphere. Calculate the rate of diffusion of oxygen in kilograms per hour through 1 m$^2$ of methane film when the concentration change across the film is 15% to 5% oxygen by volume. The value of diffusivity may be taken as 0.184 cm$^2$/s. $R = 82.06$ cm$^3$ atm./g mol K.

(**Ans:** 1.05 kg/hr)

# 4

# EQUIPMENT FOR GAS–LIQUID OPERATIONS

## 4.1 INTRODUCTION

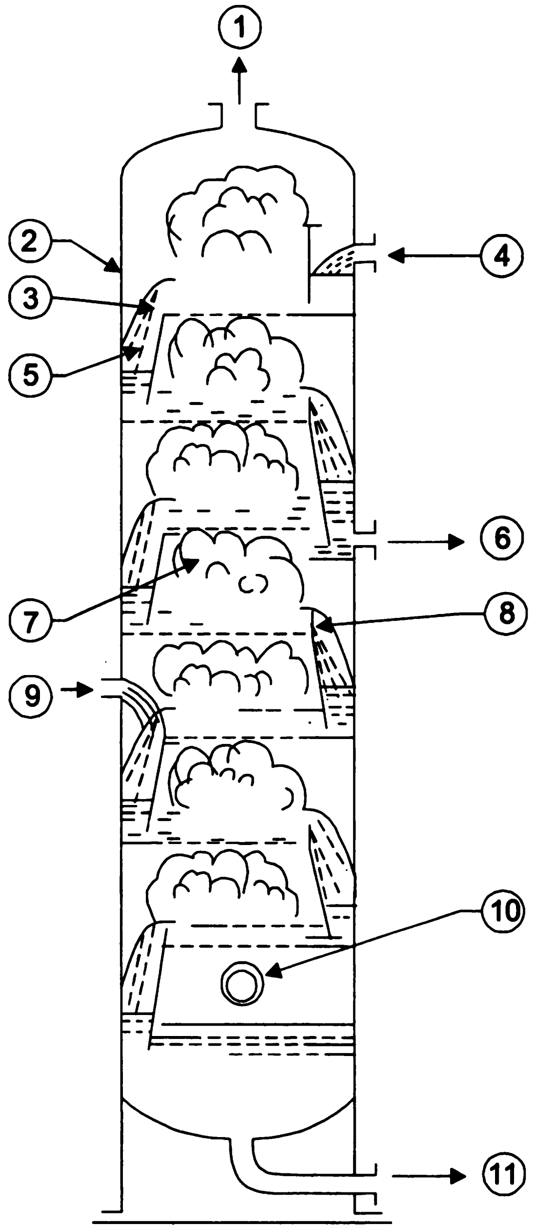
The equipment used for gas–liquid operations are classified under two types, 1. stage contactors (bubble cap, valve trays and sieve tray columns) and 2. continuous contactors (packed towers and spray towers).

## 4.2 TRAY TOWERS

A typical tray tower is shown in Fig. 4.1. These are cylindrical towers with trays or plates with a downspout to facilitate the flow of liquid from one tray to the other by gravity. The gas passes upward through the openings of one sort or another, in the trays and then passes through the liquid to form froth and subsequently discharges from it and then passes on to the next tray located above. Each tray of the tower acts as a stage, since there is an intimate contact between the gas phase and liquid phase in each tray. In order to provide a longer contact time, the liquid pool on each tray should be deep so that the gas bubbles will require relatively a longer time to rise through the column of the liquid. When the gas velocity is relatively high, it is dispersed very thoroughly into the liquid, which in turn is agitated into froth. This provides large interfacial areas.

However, these lead to certain operational difficulties like entrainment of droplets of liquid in the rising gas stream and a high-pressure drop for the gas in flowing through the trays. The higher pressure drop also results in high pumping cost and hence a higher operating cost. Especially, in the case of distillation, one may need to maintain a higher pressure in the reboiler, which also results in a higher boiling point that may lead to decomposition of heat sensitive compounds.

Sometimes a higher pressure drop also leads to a condition of *flooding* in which there will be a gradual build-up of liquid in each tray and may ultimately fill the entire space between the trays. The tower is said to be flooded and the liquid



1. Gas out
2. Shell
3. Sieve tray
4. Liquid in
5. Downspout
6. Sidestream withdrawal
7. Froth
8. Weir
9. Intermediate feed
10. Gas in
11. Liquid out

**Fig. 4.1** Schematic section of a sieve-tray tower.

may escape from the top position of the column through the gas exit, which results in lowering the efficiency.

In the case of gas–liquid systems, which tend to foam excessively, high gas velocities may lead to a condition of *priming*. In such case the foam is present in the space between trays and there is a great deal of liquid getting entrained with the gas. The liquid carried is recirculated between trays and the added liquid handling load gives rise to an increase in pressure drop leading to flooding.

If the liquid rates are too low, the gas rising through the openings of the tray may push the liquid away, a phenomenon called *coning* resulting in poor gas-liquid contact. When the gas rate is too low, much of the liquid may rain down through the opening of tray, called *weeping*, thus failing to obtain the benefit of complete flow over the trays. At very low gas rates, none of the liquid reaches the downspouts and this is known as *dumping*.

## 4.2.1 General Features

Generally the towers are made of metals depending on the nature of gas and liquid being handled. Some of them are made of glass or at times glass lined or made of plastics. To facilitate the maintenance work, smaller towers are fitted with hand holes and larger towers with manways. Trays are also made of metals or alloys and are fastened suitably to the shell to prevent their movement owing to surges of gas.

Tray spacing is chosen on the basis of expediency in construction, maintenance cost, flooding and entrainment. It varies from 15 cm. Tower diameter should be sufficiently large to handle the gas and liquid rates under satisfactory operating conditions. It can also be decreased by the use of increased tray spacing. Hence, the cost of tower, which depends also on the height, can be optimized with suitable tray spacing.

The liquid is drawn to the next lower tray by means of downcomers or downspouts. These may be circular pipes or portion of the tower cross section set aside for liquid flow by vertical plates. Since the liquid is agitated into froth on the tray, sufficient time must be provided in the downspout, so that the gas gets detached from the liquid and the liquid also flows down to the next lower tray. The legs of the downcomer will normally dip in the liquid in the next lower tray, which prevents short-circuiting of gas.

The depth of liquid on the tray required for gas contacting is maintained by overflow weir, which may or may not be a continuation of the downspout plate. Though straight weirs are common, V-notch weirs and circular weirs are also used. Weir length varies from 60 to 80% of tower diameter.

Having seen some of the constructional features of the towers let us now discuss the constructional features of trays.

## 4.3 TYPE OF TRAYS

### 4.3.1 Bubble Cap Trays

In these trays, chimneys or risers lead the gas through the tray and underneath caps surrounding the risers. The gas passes through a series of slots cut into the rim or skirt of each cap. The liquid depth is such that the caps are fully covered by them.

### 4.3.2 Sieve Trays

These are trays with perforations and the gas flows through them. The gas dispersed by the perforations, expands the liquid into a turbulent froth and results in providing enormous interfacial area for mass transfer. These trays are subject to flooding because of backup of liquid in the downspouts or excessive entrainment. In comparison to bubble caps these are quite simple and are also cost effective.

### 4.3.3 Linde Trays

These are slotted trays which show an alteration in the perforation pattern to influence the flow of liquid. The slots distributed all over the tray, not only reduce the hydraulic gradient in large trays but also influence the direction of liquid flow and eliminate stagnant areas. Thus, the efficiency of these trays are very high.

### 4.3.4 Valve Trays

These are sieve trays with large variable openings for gas flow. The perforations are covered with movable caps, which rise as the gas flow rate increases. Though the gas pressure drop is low, it is higher than sieve trays. Due to small openings the tendency to weep is also reduced.

### 4.3.5 Counter-flow Trays

In this liquid and vapour flow counter-currently through the same openings. Downspout is absent in these trays. They are more suited for vacuum distillation as the pressure drop is low.

## 4.4 TRAY EFFICIENCY

It is the fractional approach to an equilibrium stage, which is attained by a real tray. The conditions at various locations on the tray differ and are not the same. Hence, the efficiency varies at various locations.

$$\text{Point efficiency} = \left[\frac{(y_{n,\text{local}} - y_{n+1,\text{local}})}{(y^*_{n,\text{local}} - y_{n+1,\text{local}})}\right]$$

(This depends on the particular place in the tray.)

Murphree tray efficiency: This is based on the bulk average concentrations of all the local pencils of gas $= \left[\frac{(y_n - y_{n+1})}{(y^*_n - y_{n+1})}\right]$

($n$ is tray under consideration, $n + 1$ is tray below the $n$th tray)

$$\text{Overall tray efficiency} = \frac{\text{Number of ideal trays required}}{\text{Number of real trays required}}.$$

## 4.5 VENTURI SCRUBBER

This is similar to ejectors. Here a stream of absorbing liquid sprayed in the convergent duct suction draws the gas into the throat of a venturi. These devices will be useful when the liquid contains suspended solids, which may plug the plate/packed towers. The pressure drop is also low.

## 4.6 WETTED-WALL TOWERS

In these towers as shown in Fig. 4.2, a thin film of liquid flows down the inner wall of the empty vertical tube with the gas flowing co-currently or counter-currently. Generally the flow of gas is countercurrent to liquid flow. These are normally used for the measurement of mass transfer coefficient.

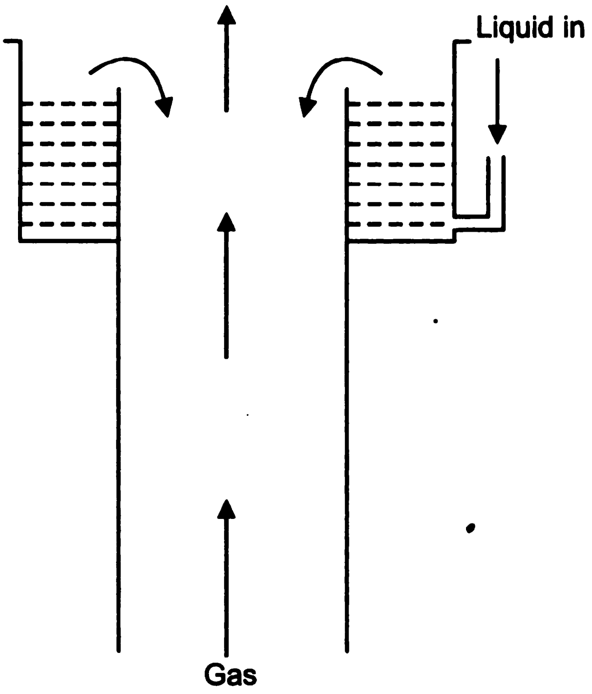


Fig. 4.2 Wetted wall column.

## 4.7 SPRAY TOWERS AND SPRAY CHAMBERS

In these units the liquid is sprayed into a gas stream by means of a nozzle as fine droplets. The flow may be counter-current as in vertical towers with the liquid flowing downward and gas upward. It can also be co-current as in the case of horizontal spray chambers. Their main feature is the low pressure drop for gas. However, it suffers from the disadvantage of high pumping cost for liquids as it has to flow out through fine nozzles and also a very high entrainment of liquids with gas, which necessitates the use of mist eliminators.

## 4.8 PACKED TOWERS

These are towers filled with packings and are used for continuous contact of liquid and gas either co-currently or countercurrently. The presence of packing gives enormous gas–liquid contact area. The liquid is distributed over the packings and trickles down through the packed bed. A typical tower is shown in Fig. 4.3.

### 4.8.1 Characteristics of Packings

1. Should provide large interfacial surface between liquid and gas.
2. Should possess desirable fluid flow characteristics like low pressure drop for gas and good enough to give high value mass transfer coefficients.
3. Chemically inert to the fluids being processed.
4. Should have good structural strength to permit easy handling and installation.
5. Cost effective.

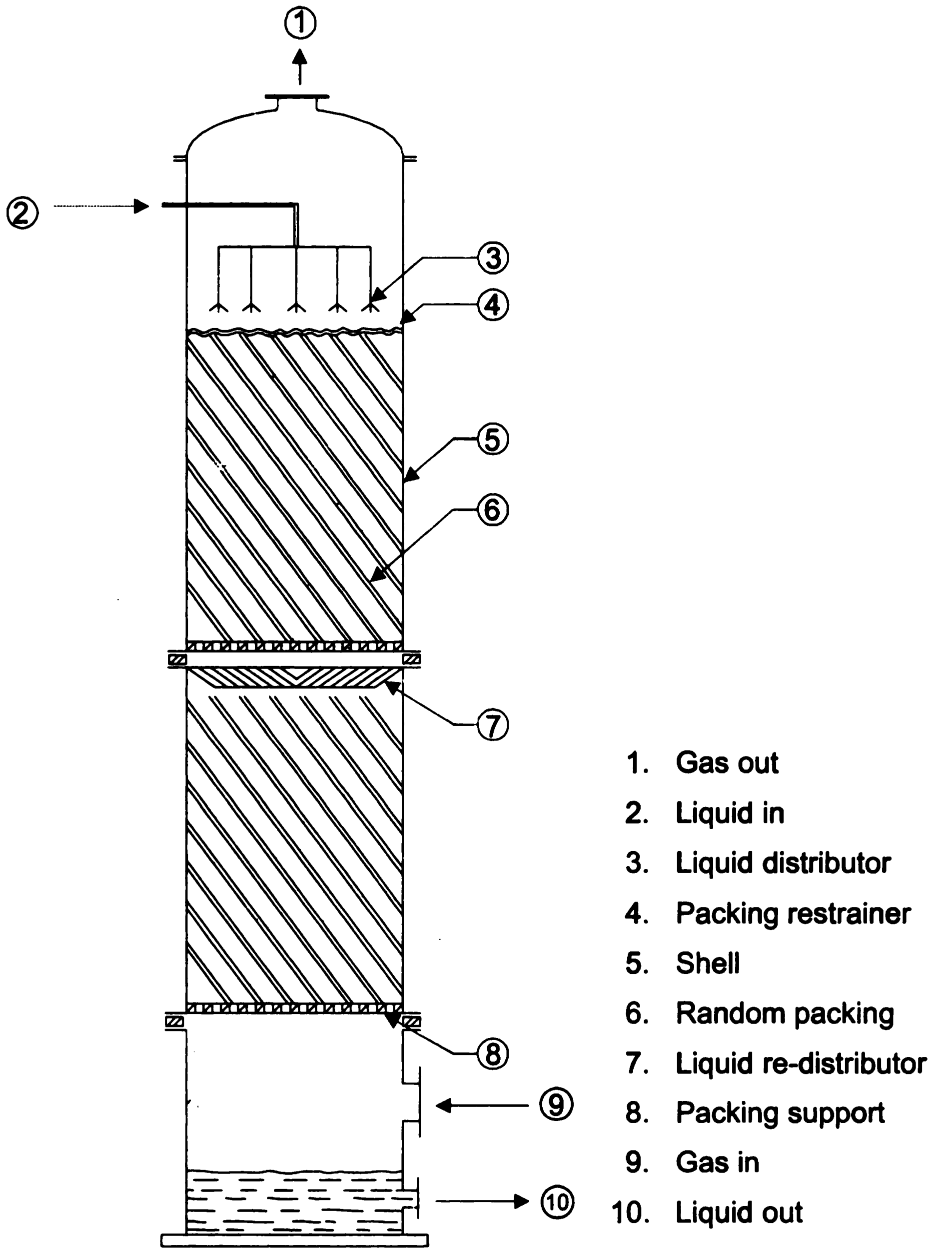


**Fig. 4.3** Packed tower.

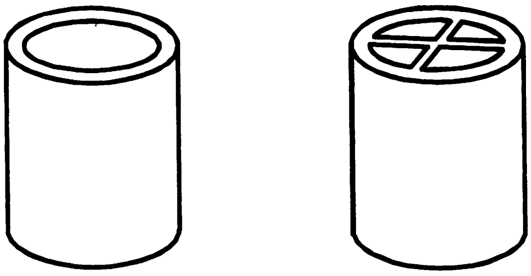
## 4.9 TYPES OF PACKINGS

There are two types of packings,

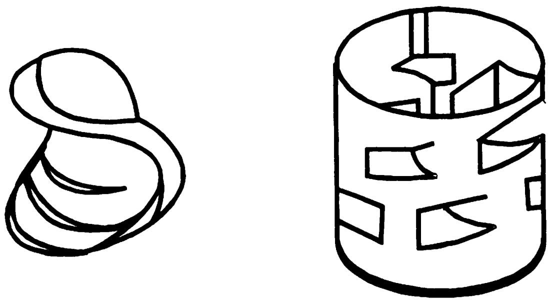
1. Random or dumped packing
2. Regular or stacked packing

### 4.9.1 Random Packing

In this type, packings are simply allowed to fall randomly. Earlier these were materials like broken stone, gravel or coke. However, due to their poor surface characteristics, they are now replaced by regular materials like Raschig rings, Berl saddles, Pall rings etc. These are made of ceramics, metals or plastics. The material of construction for these depends on the nature of fluid being handled. Ceramics are good except when alkalis and hydrofluoric acid are being used. Metals are good except in oxidising and corrosive atmospheres. Plastics deteriorate in presence of organic solvents and also at high temperatures. Advantages with thin walled metal and plastic packings over ceramics are the lightness in weight. With smaller size, random packings offer large specific surface and hence large pressure drop. However, with larger packing sizes the cost per unit volume is less. Packings in the range of 25 mm to 50 mm are used for gas rates of 0.25 $m^3/s$ and 50 mm or larger are used for gas rates of 1 $m^3/s$. During installation, the tower is filled with water and the packings are allowed to fall randomly. This prevents the disintegration of packing materials during their fall. However, when the packings are made of metals or plastics, one can drop them randomly. Some of the commonly used packings are shown in Figs. 4.4(a–d).



(a) Raschig rings (b) Partition rings

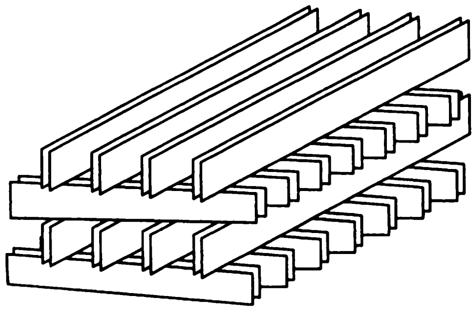


(c) Berl saddle (d) Pall ring

**Fig. 4.4** Random tower packing.

## 4.9.2 Regular Packing

In these packings there is an organised manner in which the packings are arranged in the tower. The main feature of this is the low pressure for gas and higher fluid flow rates compared to random packings. Stacked packings like stacked Raschig rings, wood grids and woven wire screens are some of the examples for regular packings. Some of the commonly used regular packings are shown in Fig. 4.5.



Wood grids

**Fig. 4.5** Regular packing.

### 4.9.3 Shell

Tower shell is made of wood, metal, stoneware, acid proof brick, glass, plastic and metals lined with glass or plastic used as material of construction depending on the corrosive nature of the liquid or gas. They are generally circular in cross-section. In most of the instances it is made of metal because of their strength and ease of operation.

### 4.9.4 Packing Supports

The packing material is normally supported in the tower by means of supports. The objective of these supports is not only that they should carry the weight of packings but also ensure a proper flow of gas and liquid with minimum restriction. They are also made of different materials like metals, ceramics and plastics.

### 4.9.5 Packing Restrainer

In order to prevent the lifting of packings, restrainers are provided at the top of the packings. Heavy screens or bars can be used for this purpose. In the case of heavy packing materials especially of ceramics, heavy bar plates resting freely on top of the packing may be used. For plastics and other light weight packings, restrainer is attached to the tower.

### 4.9.6 Liquid Distributors

The liquid distribution at the top of the tower must be uniform so that the wetting of packing is uniform. A uniformly wetted packing is essential to have effectiveness in mass transfer. With non-uniform distribution of liquid one has dry packing which is ineffective for mass transfer. A ring of perforated pipe can be used in small towers. For larger diameters it is necessary to provide a number of liquid introduction points so that distribution is uniform.

### 4.9.7 Entrainment Eliminators

At high gas velocities the gas leaving the column may carry droplets of liquid as a mist. The mist can be removed by passing the gas through a layer of mesh made of wire or polymeric materials with about 99% voids or cyclones.

### 4.9.8 Channeling

As liquid flows down over the packings as thin film, the films tend to grow thicker in some places and thinner in others and liquid collects into small rivulets and flows along some localised paths. At low liquid rates much of the packing surface may be dry or, at the most, covered by a stagnant film of liquid. This effect is known as channeling. Channeling is more severe in stacked packings than in dumped packings.

### 4.9.9 Loading

Pressure drop in a packed bed is basically due to fluid friction. As the gas flow rate is increased, the pressure drop per unit length of packing increases. Pressure drop is low when the packing is dry. With increase in liquid flow rate, pressure drop increases as it reduces the space available for gas flow. When the packing is gradually wetted with a constant flow of liquid, initially there is a linear relationship between pressure drop and gas flow rate and is parallel to that of dry packing as shown in Fig. 4.6. The linear line becomes steeper at moderate gas velocities since the gas flow retards the down flowing liquid resulting in an increase in liquid hold up. The point at which the liquid holdup starts to increase, as indicated by a change in slope of the pressure drop–gas flow rate relationship is called the loading point.

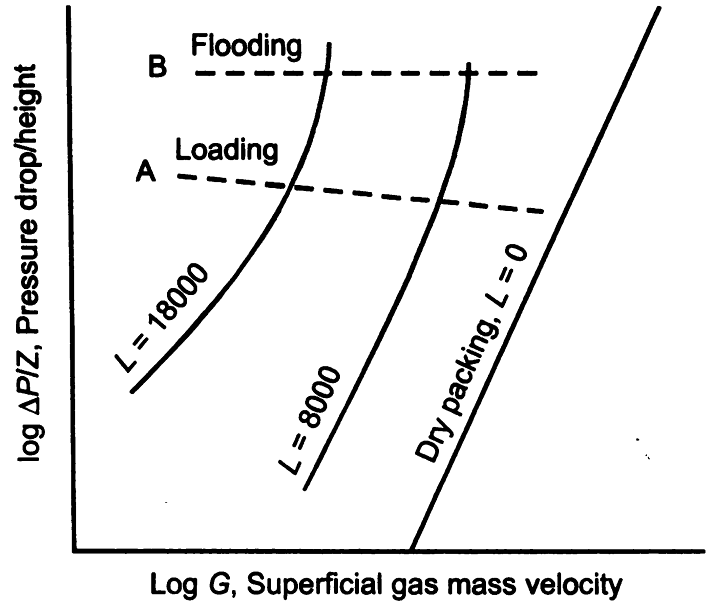


Fig. 4.6 Loading and flooding.

### 4.9.10 Flooding

With further increase in velocity of gas (beyond loading point) the pressure drop increases rapidly and pressure drop–gas flow rate relationship becomes almost vertical. At some portions of the column, the liquid becomes the continuous phase and the flooding point is said to be reached, and the accumulation of liquid is rapid and the entire column may be filled with liquid. Hence, while a bed is being operated, the gas velocity must be lesser than the flooding velocity and as flooding is approached, most or the entire packing surface is wetted, maximising the gas–liquid contact area.

As we design a column, we must choose a velocity lower than the flooding velocity. This will lead to a larger column diameter. The flooding velocity depends on the type and size of packing, liquid velocity and properties of liquid and gas. Several correlations are available in literature relating the pressure drop with flooding velocity for the design of packed columns.

## 4.10 COMPARISON OF PACKED TOWERS WITH PLATE TOWERS

| *Sl. No.* | *Criterion* | *Packed towers* | *Plate towers* |
|---|---|---|---|
| 1. | Gas pressure drop | Low–very useful in vacuum distillation | High |
| 2. | Holdup | Low and hence very useful in handling heat sensitive materials (especially in distillation) | High |
| 3. | Liquid/gas ratios | High values are preferred | Low values are preferred |
| 4. | Liquid cooling | Not so easy | Heat evolved can easily be removed. |
| 5. | Foaming systems | Can easily handle | Not so easy |
| 6. | Removal as side streams | Not so easy | More easy |
| 7. | Handling of corrosive systems | Not preferred | More preferred as trays can easily be replaced. |
| 8. | Cleaning | Not easy as plate trays | Easier |
| 9. | Thermal strain | Fragile packings tend to crush. However, metallic packings withstand the strain | Trays are more satisfactory under these circumstances. |
| 10. | Presence of solids in gas or liquid stream | Not suitable | Not suitable |
| 11. | Floor loading | Plastic packed towers are lighter than tray towers. Ceramic and metal packed towers are heavier than tray towers. (They are designed for the complete filling of tower with liquid) | Intermediate (They are designed for the complete filling of tower with liquid) |
| 12. | Cost | Not easy to predict | Not easy to predict. |

# 5

# HUMIDIFICATION

## 5.1 INTRODUCTION

Humidification operation is a classical example for an interphase transfer of mass and energy, when a gas and a pure liquid are brought into intimate contact. The term humidification is used to designate a process where the liquid is transferred to gas phase and dehumidification indicates a process where the transfer is from gas phase to liquid phase. The matter transferred between phases in both the cases is the substance which constitutes liquid phase and it either vapourises or condenses indicating either humidification or dehumidification process.

## 5.2 DEFINITIONS

The substance that is transferred (vapour) is designated by A and the main gas phase is designated by B.

### 5.2.1 Molal Absolute Humidity ($Y$)

It is defined as the moles of vapour carried by a unit mole of vapour free gas.

$$Y = \left(\frac{y_A}{y_B}\right) = \left(\frac{p_A}{p_B}\right) = \left[\frac{p_A}{(P_t - p_A)}\right] = \frac{\text{moles of A}}{\text{moles of B}} \tag{5.1}$$

where $y_A$, $y_B$ are moles of A and B respectively, $p_A$ is the partial pressure of A and $P_t$ is total pressure.

When the quantities $y_A$ and $y_B$ are expressed in mass, then it is called mass absolute humidity ($Y'$) or Grosvenor humidity.

$$Y' = Y \cdot \left(\frac{M_A}{M_B}\right) = \left[\frac{p_A}{(P_t - p_A)}\right]\left(\frac{M_A}{M_B}\right) = \frac{\text{mass of A}}{\text{mass of B}} \tag{5.2}$$

### 5.2.2 Saturated Absolute Humidity ($Y_S$)

When the vapour-gas mixture is saturated, then the partial pressure becomes equal to the vapour pressure of that substance.

$$Y_S = \frac{p_A}{p_B} = \left[\frac{p_A}{(P_t - p_A)}\right] \tag{5.3}$$

When the quantities are expressed in mass,

$$Y_S' = \left(\frac{p_A}{p_B}\right)\left(\frac{M_A}{M_B}\right) = \left[\frac{p_A}{(P_t - p_A)}\right]\left(\frac{M_A}{M_B}\right) \tag{5.4}$$

### 5.2.3 Dry Bulb Temperature (DBT)

The temperature indicated by the thermometer by ordinary immersion in the vapour–gas mixture is called dry bulb temperature.

### 5.2.4 Relative Humidity or Relative Saturation (% RH)

It is normally expressed as a percentage. If $p_A$ is the partial pressure under a given condition and $P_A$ is the vapour pressure at any dry bulb temperature (DBT) of the mixture then,

$$\%\ \text{RH} = \left(\frac{p_A}{P_A}\right) \times 100 \tag{5.5}$$

### 5.2.5 Percentage Saturation or Percentage Humidity ($H_p$)

It is defined as the percentage of humidity under given condition to the humidity under the saturated condition.

$$H_p = \left(\frac{Y}{Y_s}\right) \times 100 \tag{5.6}$$

### 5.2.6 Dew Point

This is the temperature $t_{DP}$ at which a vapour–gas mixture becomes saturated when cooled at constant total pressure out of contact with a liquid. Moment the temperature is reduced below dew point, vapour will condense as a liquid dew.

### 5.2.7 Humid Heat

The humid heat $C_S$ is the heat required to raise the temperature of unit mass of gas and its accompanying vapour by one degree at constant pressure.

$$C_S = C_A Y' + C_B \tag{5.7}$$

where $C_A$ and $C_B$ are specific heats of vapour and gas respectively.

### 5.2.8 Enthalpy

The enthalpy of a vapour–gas mixture is the sum of the enthalpies of the gas and of the vapour content. For a gas at a DBT of $t_G$, with a humidity of $Y'$, the enthalpy relative to the reference state $t_0$ is,

$$\begin{aligned} H' &= \text{Enthalpy of gas} + \text{Enthalpy of vapour component} \\ &= C_B\,(t_G - t_0) + Y'[C_A\,(t_G - t_{DP}) + \lambda_{DP} + C_{A,L}\,(t_{DP} - t_0)] \end{aligned} \tag{5.8}$$

where $\lambda_{DP}$ is latent heat of vaporisation at dew point and $C_{A,L}$ is specific heat of component A (vapour) in liquid phase.

This expression can further be simplified as low pressures are normally encountered in humidification operations. Let us consider the point P in Fig. 5.1, which actually lies on a line of constant pressure corresponding to the partial pressure of the vapour in the mixture and, for all practical purposes can be considered as lying on the line whose pressure is the saturation pressure of the vapour at the reference temperature or at $P'$.

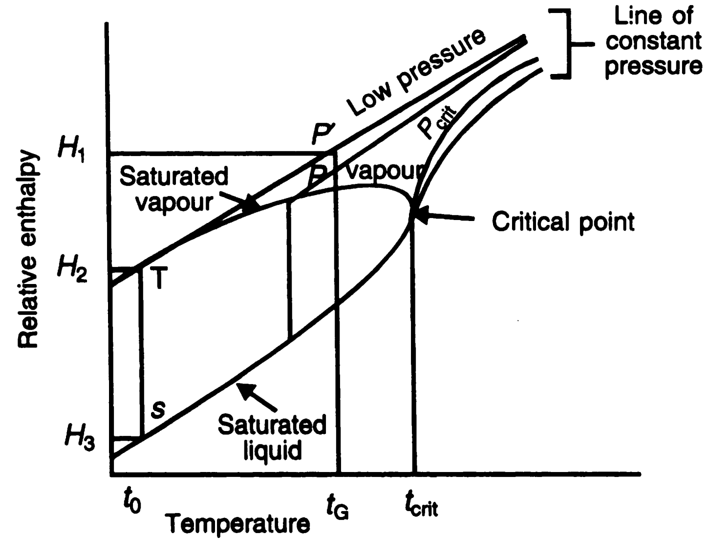


**Fig. 5.1** Typical enthalpy—temperature diagram for a pure substance.

The vapour enthalpy can then be computed by the following path P′TS and becomes enthalpy per unit mass of vapour, $C_A\ (t_G - t_0) + \lambda_0$, where $\lambda_0$ is the latent heat of vaporisation at the reference temperature. The enthalpy of the mixture, per unit mass of dry gas is then,

$$\begin{aligned} H' &= C_B\ (t_G - t_0) + Y'\ [C_A(t_G - t_0) + \lambda_0] \\ &= C_S\ (t_G - t_0) + Y'\lambda_0 \end{aligned} \qquad (5.9)$$

## 5.2.9 Humid Volume

The humid volume, $V_H$ of a vapour gas mixture is the volume of unit mass of dry gas and it is accompanying vapour at the prevailing temperature and pressure. The expression for humid volume in m³/kg of dry gas is

$$\begin{aligned} V_H &= \left[\frac{1}{M_B} + \frac{Y'}{M_A}\right] 22.41 \left[\frac{(t_G + 273)}{273}\right]\left[\frac{1.013 \times 10^5}{P_t}\right] \\ &= 8315\left[\frac{1}{M_B} + \frac{Y'}{M_A}\right]\left[\frac{t_G + 273}{P_t}\right] \end{aligned} \qquad (5.10)$$

where $P_t$ is the total pressure in N/m².

A typical psychrometric chart is shown in Fig. 5.2 from which the various properties of air–water system can be obtained. Alternatively, the equations given above can be used.

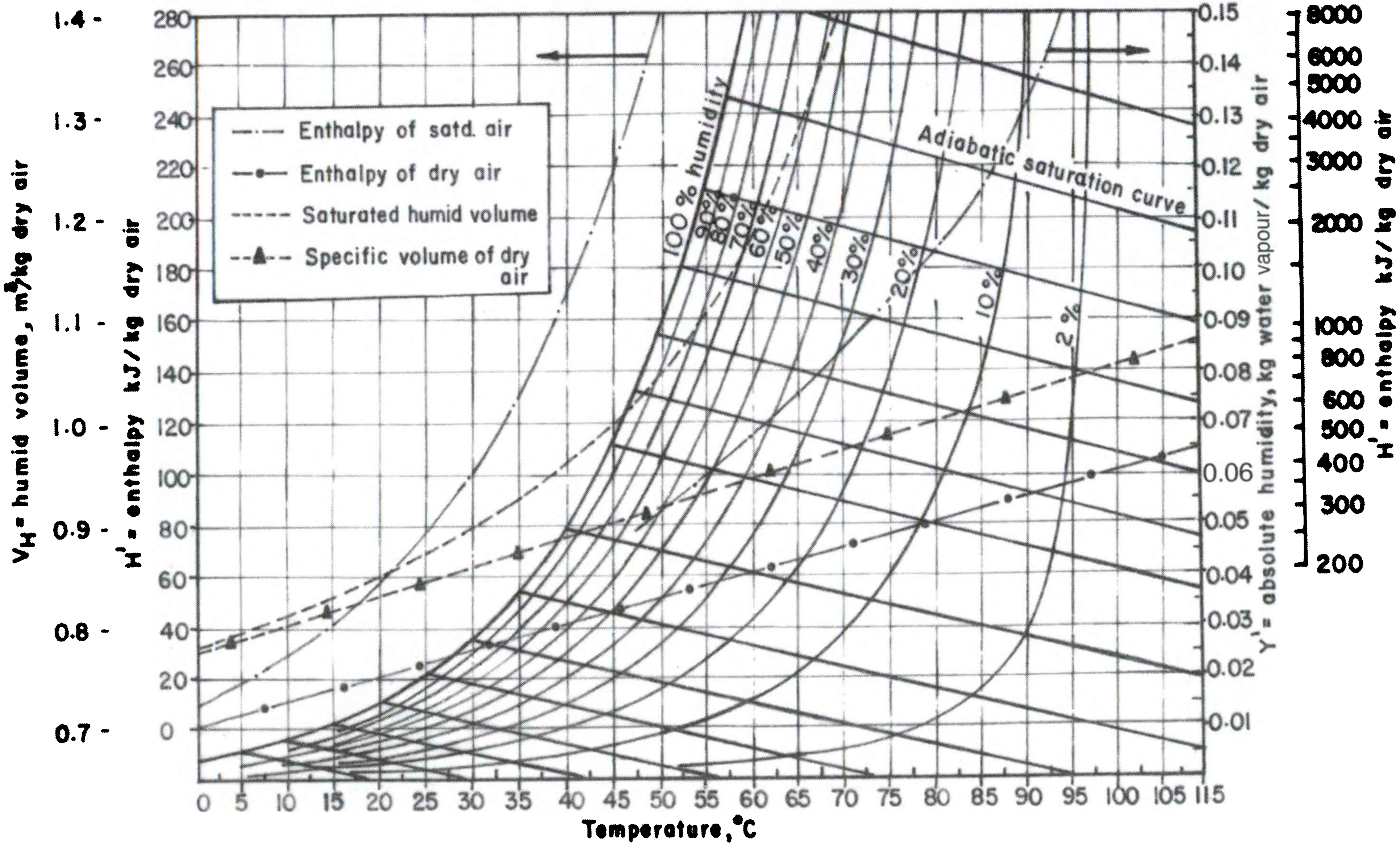


**Fig. 5.2** Psychrometric chart at 1 atm. pressure.

## 5.3 ADIABATIC SATURATION CURVES

Consider a system as shown in Fig. 5.3. A feed stream of gas is contacted with a liquid spray and as a result of diffusion and heat transfer between the gas and liquid, the gas leaves the system humidified. Assuming the operation to be adiabatic, we obtain the following.

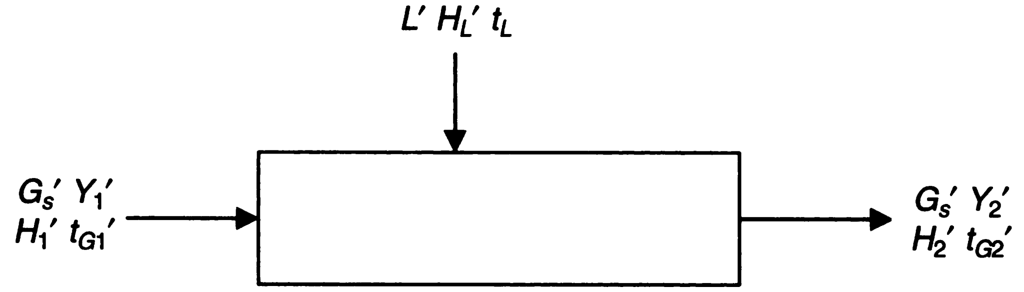


**Fig. 5.3** Adiabatic chamber.

A mass balance for vapour transferred yields,

$$L' = G_S' \, (Y_2' - Y_1') \tag{5.11}$$

Enthalpy balance yields

$$G_S' \, H_1' + L' \, H_L = G_S' \, H_2' \tag{5.12}$$

i.e.

$$H_1' + (Y_2' - Y_1') \, H_L = H_2' \tag{5.13}$$

Using the definition for enthalpy, Eq. (5.13) is modified as

$$C_{S1}(t_{G1} - t_0) + Y_1'\lambda_0 + (Y_2' - Y_1')C_{\text{A,L}}(t_L - t_0) = C_{S2}(t_{G2} - t_0) + Y_2'\lambda_0 \tag{5.14}$$

If the gas mixture leaving the system is fully saturated and hence the various quantities are denoted as $t_{\text{as}}$, $Y_{\text{as}}'$, $H_{\text{as}}$ and if the liquid enters at $t_{\text{as}}$, Eq. (5.14) becomes

$$[C_{\text{B}} \, (t_{G1} - t_0) + Y_1' \, C_{\text{A}} \, (t_{G1} - t_0)] + Y_1'\lambda_0 + [Y_{\text{as}}' - Y_1'] \, C_{\text{A,L}} \, (t_{\text{as}} - t_0)$$
$$= [C_{\text{B}} \, (t_{\text{as}} - t_0) + Y_{\text{as}}' \, C_{\text{A}} \, (t_{\text{as}} - t_0)] + Y_{\text{as}}' \, \lambda_0 \tag{5.15}$$

By subtracting $Y_1'C_{\text{A}} \, t_{\text{as}}$ from both sides and further simplifying, we get Eq. (5.15) as

$$(C_{\text{B}} + Y_1' \, C_{\text{A}}) \, (t_{G1} - t_{\text{as}}) = \text{C}_{S1} \, (t_{G1} - t_{\text{as}})$$
$$= (Y_{\text{as}}' - Y_1') \, [C_{\text{A}}(t_{\text{as}} - t_0) + \lambda_0 - C_{\text{A,L}}(t_{\text{as}} - t_0)] \tag{5.16}$$

i.e.

$$C_{S1} \, (t_{G1} - t_{\text{as}}) = (Y_{\text{as}}' - Y_1' \, (\lambda_{\text{as}}) \tag{5.17}$$

or

$$(t_{G1} - t_{\text{as}}) = (Y_{\text{as}}' - Y_1') \left(\frac{\lambda_{\text{as}}}{C_{S1}}\right) \tag{5.18}$$

Equation (5.18) is the adiabatic saturation curve on the psychromatic chart which passes through ($Y_{\text{as}}'$ $t_{\text{as}}$) and ($Y'_1$ $t_{G1}$) on the 100% curve. Since $C_{S1}$ contains the term $Y'_1$, in Eq. (5.18) represents a curve. Thus for any vapour–gas mixture, there is an adiabatic saturation temperature $t_{\text{as}}$, such that if contacted with liquid at $t_{\text{as}}$, the gas will get cooled and humidified. If the contact time is large enough, the gas will become saturated at ($Y_{\text{as}}'$ $t_{\text{as}}$) and if the contact time is insufficient, it will leave at ($Y_2'$ $t_{G2}$).

## 5.4 WET BULB TEMPERATURE (WBT)

It is the steady state temperature attained by a small amount of liquid evaporating into a large amount of unsaturated vapour–gas mixture. To measure the wet bulb temperature, a thermometer or an equivalent temperature measuring device such as a thermocouple is covered by a wick which is saturated with pure liquid and immersed in a stream of moving gas having a definite temperature and humidity. The ultimate temperature attained is called wet bulb temperature and will be less than the dry bulb temperature (if the gas is unsaturated). If the wet bulb temperature is to be measured accurately, the following precautions will have to be followed.

(i) The wick must be completely wet.
(ii) The velocity of air should be fairly large.
(iii) The make up liquid, if supplied to the bulb should be at the wet bulb temperature.

### 5.4.1 Theory of Wet Bulb Thermometry

Consider a drop of liquid immersed in a rapidly moving stream of unsaturated vapour–gas mixture shown below in Fig. 5.4. If the liquid is initially at a temperature higher than the dew point of vapour, the vapour pressure of the liquid will be higher at the drop surface than the partial pressure of vapour in the gas and hence the liquid will evaporate and diffuse into the gas. The latent heat required for evaporation is supplied at the expense of sensible heat of liquid drop due to which the temperature of the droplet will reduce. As the liquid temperature is reduced below the dry bulb temperature of the gas, heat will flow from gas to the liquid at an increasing rate due to large temperature difference. Ultimately, the rate of heat transfer from the gas to the liquid will be equal to the rate of heat required for evaporation and ultimately the temperature of the liquid will remain constant at some low value, $t_w$.

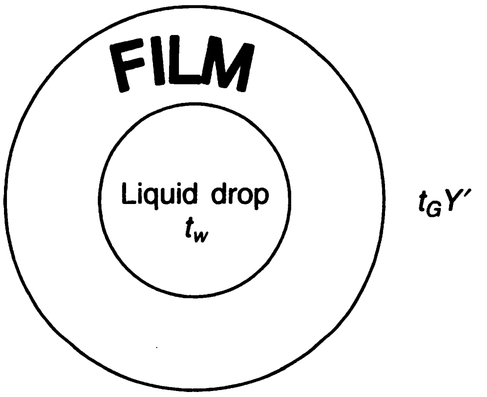


**Fig. 5.4** The wet bulb temperature.

Here both heat and mass transfer occur simultaneously. The heat lost by gas to the liquid per hour, $Q$, is given by,

$$Q = h\,A\,(t_G - t_w) \tag{5.19}$$

where $h$ is the heat transfer coefficient in kJ/hr m$^2$ K, $A$ is the area of heat transfer in m$^2$, $t_G$ is the dry bulb temperature in K and $t_w$ is the wet bulb temperature in K.

The mass of liquid evaporated per hour,

$$W = k_C A\ [C_{A1} - C_{A2})\ M_w] \tag{5.20}$$

where $k_C$ is mass transfer coefficient, m$^2$/hr, $A$ is area of mass transfer, m$^2$, $C_{A1}$ and $C_{A2}$ are concentrations of water vapour at the liquid–gas film and surrounding air respectively in kmole/m$^3$ and $M_w$ is molecular weight of water in kg/kmole. Equation (5.20) can be modified in terms of pressure and temperature as,

$$W = k_C A \frac{[(M_w)(p_{A1} - p_{A2})]}{RT} \tag{5.21}$$

where $T$ is temperature, K, $p_{A1}$ and $p_{A2}$ are partial pressure of water vapour at the interface and in the surrounding air respectively, N/m$^2$ and $R$ is gas constant J/kmole K. Humidity at the interface is given by

$$Y_1' = \left[\frac{P_w}{(P_t - P_w)}\right]\left[\frac{M_w}{M_A}\right] \tag{5.22}$$

where $P_w$ is vapour pressure of water N/m$^2$, $P_t$ is total pressure of water in N/m$^2$ and $M_A$ is molecular weight of air in kg/kmole.

Humidity in the surrounding air is given by,

$$Y_2' = \left[\frac{p_w}{(P_t - p_w)}\right]\left[\frac{M_w}{M_A}\right] \tag{5.23}$$

where $p_w$ is partial pressure of water in N/m$^2$, $P_t$ is total pressure of water in N/m$^2$ and $M_A$ is molecular weight of air in kg/kmole.
By substituting Eqs. (5.22) and (5.23) in Eq. (5.21) is modified as

$$W = k_C \cdot \frac{A\,M_w}{RT}\left[\left(\frac{M_A}{M_w}\right)(P_t - P_w)Y_1' - \left(\frac{M_A}{M_w}\right)Y_2'\ (P_t - p_w)\right] \tag{5.24}$$

$$= k_C \cdot A\left(\frac{M_A}{RT}\right)[(P_t - P_w)Y_1' - Y_2'(P_t - p_w)] \tag{5.25}$$

In humidification operations, especially when the temperatures are low, $P_w$ and $p_w$ are small in comparison to $P_t$ and hence neglected. Now the Eq. (5.25) gets simplified to

$$W = k_C \cdot A\left(\frac{M_A P_t}{RT}\right)(Y_1' - Y_2') \tag{5.26}$$

If $\lambda$ is the latent heat of vaporisation in kJ/kg, then the heat required for evaporation is given by,

$$Q_{\text{Evap}} = W\lambda = k_C A\lambda\left(\frac{M_A P_t}{RT}\right)[Y_1' - Y_2'] \tag{5.27}$$

By the principle of wet bulb thermometry, we know

Heat lost by the gas = Heat required for evaporation (5.28)

Substituting in Eq. (5.28) from Eqs. (5.19) and (5.27), we get

$$h.A\ (t_G - t_w) = k_C\, A\lambda\left(\frac{M_A P_t}{RT}\right)\ [Y_1' - Y_2'] \tag{5.29}$$

$$(t_G - t_w) = \left(\frac{k_C}{h}\right)\lambda\left(\frac{M_A P_t}{RT}\right)[Y_1' - Y_2'] = \left(\frac{k_C}{h}\right)\lambda M_A C\ [Y_1' - Y_2'] \tag{5.30}$$

By the definition of mass transfer coefficient

$$k_y = k_C \cdot C \tag{5.31}$$

$$\therefore \qquad (t_G - t_w) = \left[k_y \lambda \cdot \frac{M_A}{h}\right][Y_1' - Y_2'] \tag{5.32}$$

From Chilton–Colburn analogy, we have

$$\left(\frac{h}{\rho C_P u_0}\right)(\text{Pr})^{2/3} = \left(\frac{k_C}{u_0}\right)(\text{Sc})^{2/3} = \left(\frac{k_y}{C u_0}\right)(\text{Sc})^{2/3} \tag{5.33}$$

On simplification,

$$\left(\frac{\text{Sc}}{\text{Pr}}\right)^{2/3} = [\text{Le}]^{2/3} = \left[\frac{h}{\rho C_p k_c}\right] = \left[\frac{hC}{\rho C_p k_y}\right] \tag{5.34}$$

where Le is Lewis number. Substituting Eq. (5.34) in Eq. (5.32), gives

$$(t_G - t_w) = \left[\frac{(Y_1' - Y_2')\lambda}{(\text{Le})^{2/3} C_p}\right]\left[\frac{C M_A}{\rho}\right];\ \text{since } CM_A = \rho$$

Then,

$$(t_G - t_w) = \frac{(Y_1' - Y_2')\lambda}{(\text{Le})^{2/3} C_p} \tag{5.35}$$

The quantity $(t_G - t_w)$ is called the wet bulb depression. For the system of air–water at ordinary conditions, the humid heat $C_S$ is almost equal to the specific heat $C_p$ and the Lewis number is approximately unity.

Therefore,

$$(t_G - t_w) = \frac{(Y_1' - Y_2')\lambda}{C_S} \tag{5.36}$$

Now it can be inferred on comparing Eq. (5.18) with Eq. (5.36), both the equations are identical. The adiabatic saturation curve (line) and the wet bulb temperature (line) merge in the case of air–water system only since Lewis number is unity for that system. For the other systems, adiabatic saturation curve and wet bulb temperature lines are different.

## 5.5 GAS–LIQUID OPERATIONS

Some of the following examples are under adiabatic operations and non-adiabatic operations:

### 5.5.1 Adiabatic Operations

(i) Cooling a liquid: It occurs by transfer of sensible heat and also by evaporation. Its main application is in cooling towers where the water is cooled.

(ii) Cooling a hot gas: By providing a direct contact between hot gas and liquid, cooling of gas is effected. Fouling in heat exchangers is avoided. However, this is used only when vapours which come out from liquid are not objectionable.

(iii) Humidifying a gas: For controlling the humidity in air, this is more desirable—used in preparing air for drying.

(iv) Dehumidifying a gas: By direct contact with a cold liquid, vapours are condensed and removed. This finds application in drying and recovery of solvent vapours from gases.

### 5.5.2 Non-adiabatic Operations

(i) Evaporative cooling: A liquid or gas inside a pipe is cooled by water flowing in a film over the outside of the pipe and the latter in turn is cooled by direct contact with air.

(ii) Dehumidifying a gas: A gas–vapour mixture is brought into contact with pipes through which a refrigerant flows and the vapour condenses on the surface of pipes.

## 5.6 DESIGN OF COOLING TOWER

The operation of prime importance in industries is cooling hot water from heat exchangers, condensers and the like by direct contact with air for re-use. As the latent heat of vaporisation is so large, even a small amount of vaporisation will produce a very large cooling effect. This principle is used in the design and operation of a cooling tower. These are usually carried out in some sort of packed tower and generally a countercurrent flow of gas and liquid will be adopted. The schematic arrangement with various streams and their properties are shown in Fig. 5.5.

Consider a differential section of height $dZ$ from the bottom of the tower ① wherein the change in humidity of air is $dY'$.

$$\text{The amount of moisture transferred to air} = G_S\,[dY'] \qquad (5.37)$$

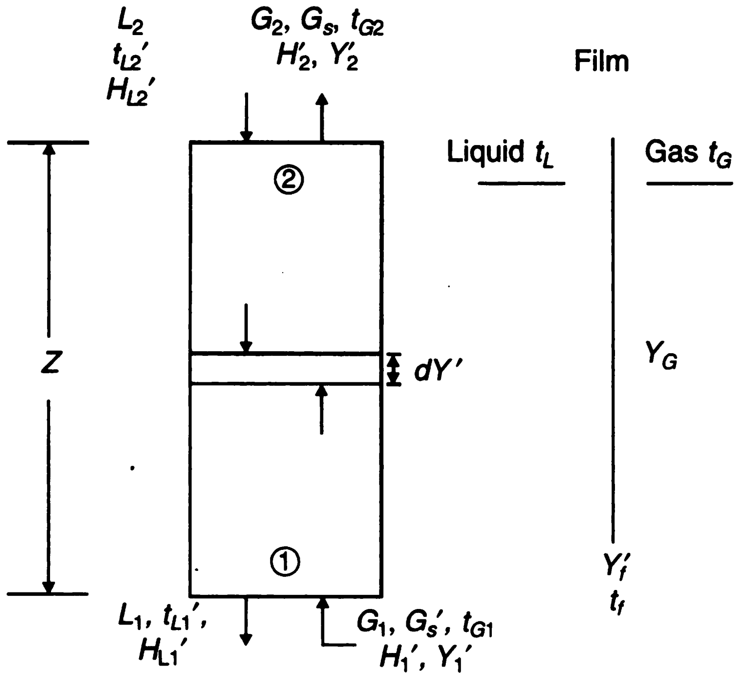


**Fig. 5.5** Flow of streams in a counter-current cooling tower.

Making an enthalpy balance for the water and air stream we get,

Heat gained by gas = Heat lost by the liquid.

$$G_S \cdot dH_G = L \cdot dH_L \tag{5.38}$$

We know,

$$dH_G = C_S \, dt_G + \lambda_0 \, dY'$$

$$dH_L = C_L \, dt_L$$

Integrating Eq. (5.38) between junctions (1) and (2), yield

$$G_S \, [H_{G2} - H_{G1}] = L[H_{L2} - H_{L1}]$$

i.e.

$$G_S \, [H_{G2} - H_{G1}] = LC_L \, [t_{L2} - t_{L1}] \tag{5.39}$$

Heat transfer rate per unit cross-sectional area of the bed for the liquid side is

$$h_L \, a \, dZ \, (t_L - t_f) = L \, C_L \, dt_L \tag{5.40}$$

Heat flux for the air is

$$h_G \, a \, dZ \, (t_f - t_G) = G_S \, C_S \, dt_G \tag{5.41}$$

Mass flux from the gas side is

$$k_y \, a \, dZ \, (Y_f' - Y_G') = Gs \, dY' \tag{5.42}$$

We know for air–water system, Lewis number is unity.

i.e.

$$\frac{h}{\rho C_p k_c} = \frac{h}{C_S k_y} = 1.0$$

i.e.

$$h_G = k_y C_S \tag{5.43}$$

Substituting Eq. (5.43) in Eq. (5.41), we get

$$k_y \, a \, C_S \, dZ \, (t_f - t_G) = G_S \, C_S \, dt_G \tag{5.44}$$

Multiplying both sides of Eq. (5.42) by $\lambda$, gives

$$k_y\, adZ\, [Y_f' - Y'_G]\, \lambda = G_S\, \lambda\, dY' \tag{5.45}$$

Adding Eq. (5.44) and (5.45) gives and rearranging, we get

$$\begin{aligned} G_S\, [C_S\, dt_G + \lambda\, dY'] &= k_y\, a\, dZ\, [C_S\, (t_f - t_G) + \lambda(Y_f' - Y_G')] \\ &= k_y\, a\, dZ\, [(C_S\, t_f + \lambda Y_f') - (C_S\, t_G + \lambda Y_G')] \end{aligned} \tag{5.46}$$

Modifying Eq. (5.46) in terms of enthalpy gives,

$$G_S\, dH_G = k_y\, a\, dZ\, [H_f - H_G] \tag{5.47}$$

Integrating Eq. (5.47) yields

$$\int_1^2 \frac{dH_G}{(H_f - H_G)} = \left(\frac{k_y a}{G_S}\right) \int_0^Z dZ = \left(\frac{k_y a}{G_S}\right) Z \tag{5.48}$$

or

$$Z = N_{tG} \cdot H_{tG} \tag{5.49}$$

where $N_{tG} = \int_1^2 \frac{dH_G}{(H_f - H_G)}$ and $H_{tG} = \left(\frac{G_S}{k_y\, a}\right)$

Equation (5.48) can be used to estimate the height of cooling tower. Already, we have deduced Eqs. (5.38) and (5.40)

$$G_S\, dH_G = L\, dH_L = L\, C_L\, dt_L \text{ (by definition of } dH_L)$$

According to Eqs. (5.40) and (5.47), $h_L\, a\, dZ\, (t_L - t_f) = L\, C_L\, dt_L$

$$G_S\, dH_G = k_y\, a\, dZ\, (H_f - H_G)$$

Combining Eqs. (5.38) and (5.40) and (5.47), we finally have

$$h_L\, a\, dZ\, (t_L - t_f) = k_y\, a\, dZ\, (H_f - H_G) \tag{5.50}$$

i.e.

$$\left[\frac{(H_f - H_G)}{(t_L - t_f)}\right] = \frac{h_L}{k_y} \text{ or } \left(\frac{(H_f - H_G)}{(t_f - t_L)}\right) = -\left(\frac{h_L}{k_y}\right) \tag{5.51}$$

Equation (5.51) gives the interfacial conditions which can be used in L.H.S. of Eq. (5.48) for estimating the height of the cooling tower. However, if the overall driving force and the bulk fluid properties are used, the film conditions are replaced by equilibrium properties and Eq. (5.49) takes the form

$$Z = N_{tOG} \cdot H_{tOG} \tag{5.52}$$

where

Number of overall gas transfer units, $N_{tOG} = \int_1^2 dH_G/(H^* - H_G)$

and

Height of overall gas transfer units, $H_{tOG} = G_S/K_ya$

***Steps involved in the use of above procedure for the design of cooling tower.***

***Step 1*:** Construct equilibrium curve
Draw the temperature—Enthalpy diagram.

***Step 2*:** Draw the operating line.
Heat lost by liquid = Heat gained by gas.

$$LC_L\,(t_{L2} - t_{L1}) = G_S\,(H_{G2} - H_{G1})$$

$$\left[\frac{(H_{G2} - H_{G1})}{(t_{L2} - t_{L1})}\right] = \left(\frac{LC_L}{G_S}\right)$$

The minimum air flow requirement is obtained by drawing a tangent to the temperature–enthalpy curve from $(t_{L1}, H_{G1})$ point.

***Step 3*:** Interfacial conditions.
Determine the interfacial conditions using Eq. (5.51)

$$\left[\frac{H_f - H_G}{t_f - t_L}\right] = -\left(\frac{h_L}{k_y}\right)$$

by drawing lines with a slope of $-(h_L/k_y)$ from operating line to equilibrium curve
or

$$\left[\frac{(H^* - H_G)}{(t - t_L)}\right] = -\left(\frac{h_L}{k_y}\right)$$

***Step 4*:** Find interfacial properties or equilibrium properties as the case may be and determine

$$(H_f - H_G) \text{ or } (H^* - H_G)$$

***Step 5*:** Graphically determine

$$\int \frac{dH_G}{(H_f - H_G)} = N_{tG}$$

or

$$\int \frac{dH_G}{(H^* - H_G)} = N_{tOG}$$

and find the height of the tower by $N_{tG} \times H_{tG}$ or $N_{tOG} \times H_{tOG}$

The difference between the temperature of liquid at the exit and the wet bulb temperature of entering air is called the wet bulb temperature approach. In the design of cooling towers, this is ordinarily specified to be from 2.5°C to 5°C.

Make up fresh water in re-circulating water system must be added to replace losses from entrainment (drift or windage), evaporation losses and blow down. Windage losses can be estimated as 0.1 to 0.3 per cent of re-circulation rate for induced draft towers. If the make up water also contains dissolved salts, a small amount of water is discarded to keep the salt concentration at a specified level.

## 5.7 RE-CIRCULATING LIQUID–GAS HUMIDIFICATION–COOLING

This is a case where the liquid enters the tower at the adiabatic saturation temperature of the entering gas mixture. This is normally achieved by re-circulating the exit liquid back to the tower. The gas not only gets cooled but also humidified in the process along the adiabatic saturation curve which passes through the entering gas conditions. Depending upon the degree of contact, the gas will approach the equilibrium conditions. A typical arrangement is shown in Fig. 5.6. Based on the temperature and humidity changes which lie in the gas phase, the mass balance is

$$GsdY' = k_y a(Y'_{as} - Y')dZ \tag{5.53}$$

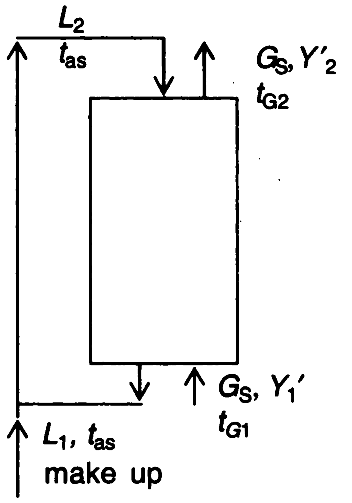


**Fig. 5.6** Schematic arrangement of re-circulation humidifier.

i.e.

$$\frac{dY'}{(Y'_{as} - Y')} = \frac{k_y a}{G_s} \int_0^Z dZ \tag{5.54}$$

and since $Y'_{as}$ is constant, integration yields,

$$\ln\left[\frac{(Y'_{as} - Y'_1)}{(Y'_{as} - Y'_2)}\right] = \left(\frac{k_y a}{G_S}\right) Z \tag{5.55}$$

i.e.

$$Z = \left(\frac{G_s}{k_y a}\right)\left\{\ln\left[\frac{Y'_{as} - Y'_1}{Y'_{as} - Y'_2}\right]\right\} \tag{5.56}$$

$$Z = (H_{tG})\,(N_{tG}) \tag{5.57}$$

where

$$H_{tG} = \frac{G}{k_y a}$$

and

$$N_{tG} = \ln\left[\frac{(Y'_{as} - Y'_1)}{(Y'_{as} - Y'_2)}\right]$$

$$\text{Let } (\Delta Y')_{av} = \frac{[(Y'_{as} - Y'_1) - (Y'_{as} - Y'_2)]}{\ln\left[\frac{(Y'_{as} - Y'_1)}{(Y'_{as} - Y'_2)}\right]} \tag{5.58}$$

$$(\Delta Y')_{av} = \frac{(Y'_2 - Y'_1)}{\ln\left[\frac{(Y'_{as} - Y'_1)}{(Y'_{as} - Y'_2)}\right]} \tag{5.59}$$

$$\therefore \qquad \ln\left[\frac{(Y'_{as} - Y'_1)}{(Y'_{as} - Y'_2)}\right] = \left[\frac{(Y'_2 - Y'_1)}{(\Delta Y')_{av}}\right] \tag{5.60}$$

Substituting Eq. (5.60) in Eq. (5.56) gives

$$Z = \left[\frac{G_S}{k_y a}\right]\left[\frac{(Y'_2 - Y'_1)}{(\Delta Y')_{av}}\right] \tag{5.61}$$

$$Z = (H_{tG})\,(N_{tG})$$

where

$$H_{tG} = \left(\frac{G_S}{k_y a}\right)$$

and

$$N_{tG} = \left[\frac{(Y'_2 - Y'_1)}{(\Delta Y')_{av}}\right]$$

Since the humidity of gas in equilibrium with liquid is $Y'_{as}$ Murphree gas-phase stage efficiency is

$$\left[\frac{(Y'_2 - Y'_1)}{(Y'_{as} - Y'_1)}\right]$$

$$\eta = \left[\frac{(Y'_2 - Y'_1)}{(Y'_{as} - Y'_1)}\right] = 1 - \frac{(Y'_{as} - Y'_2)}{(Y'_{as} - Y'_1)} \tag{5.62}$$

From Eq. (5.57)

$$\eta = 1 - \exp(-N_{tG}) \tag{5.63}$$

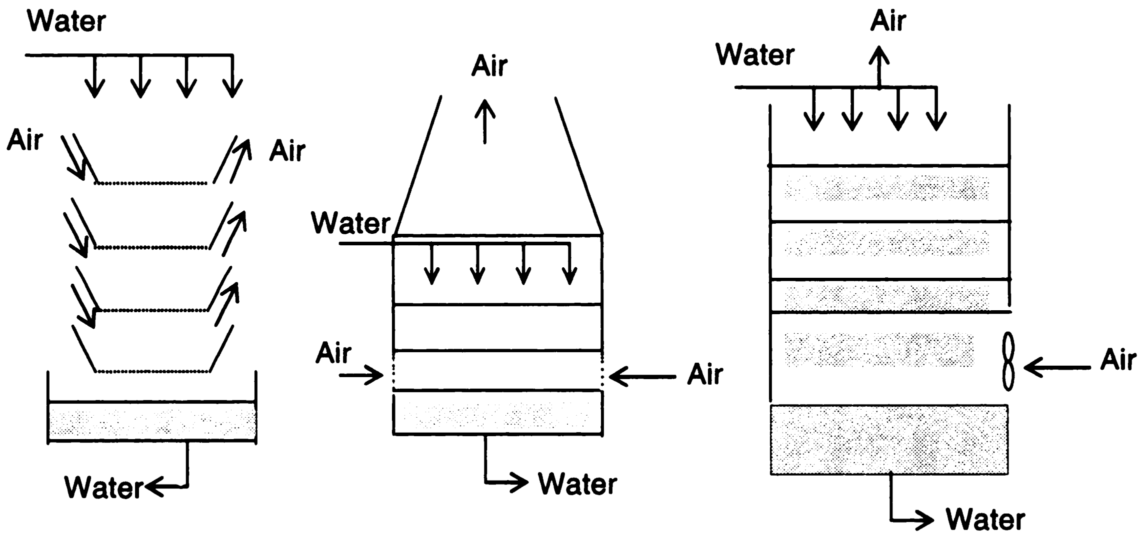
## 5.8 EQUIPMENTS

There are various types of equipments available in industries for humidification operations and they are discussed below.

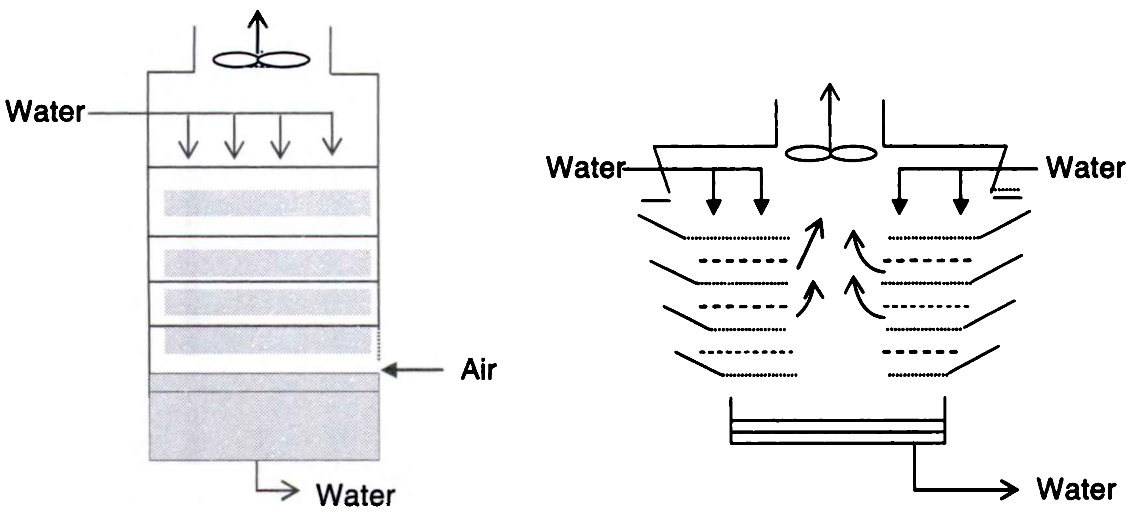
### 5.8.1 Packed Cooling Towers

As with any other operations, the cost of installation and operation will have to be minimum. As the operation involves almost a continuous contact with water, the framework and packing materials should posses good durability in such environment. Red wood is impregnated suitably with coal tar creosote. Pentachlorophenols and the like is commonly used for framework. Nowadays, towers made of plastic are also used. The internal packing, which is a staggered arrangement of horizontal slats, could be made of wood or plastic materials. Normally, more than 90% of the tower will have voids such that the pressure drop is very low. Tray towers are not used for these operations.

Typical arrangements of cooling tower is shown in Fig. 5.7.



(a) Atmospheric (b) Natural draft (c) Forced draft



(d) Countercurrent draft (e) Crosscurrent draft

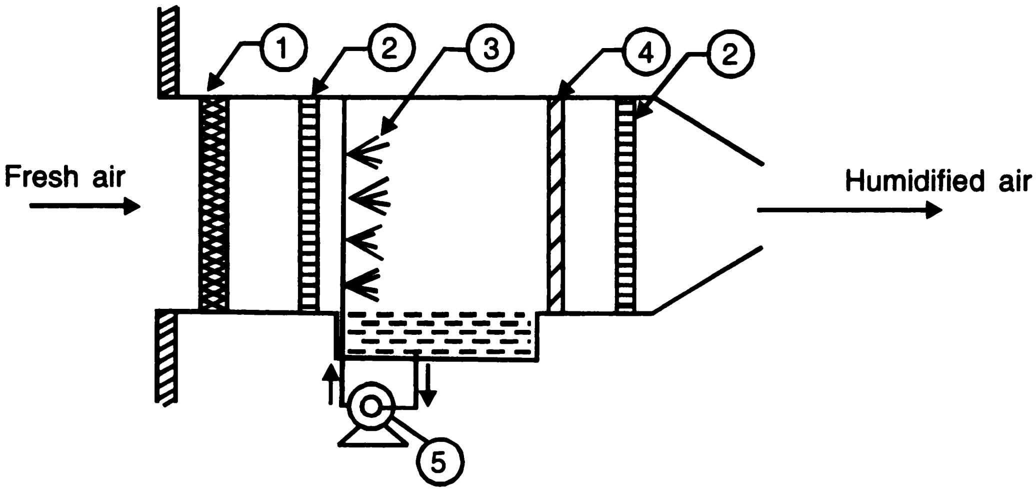
**Fig. 5.7** Cooling tower arrangement.

The types, shown in Fig. 5.7(a) and (b), are atmospheric towers depend on air movement. In natural draft towers (b) depend upon displacement of warm inside air by the cool air from outside. Chimney is needed in this type of towers. These find application in places where the humidity and air temperatures are low.

In types (c), (d) and (e) due to the provision of air by fans more uniform distribution of air can be expected. Chimneys are not needed in these arrangements. Whenever fogging is common, finned type heat exchangers can be used to evaporate the fog by heat from the hot water to be cooled.

### 5.8.2 Spray Chambers

A typical spray chamber is shown in Fig. 5.8. They are generally used for humidification and cooling operations under adiabatic conditions. They can also be used for dehumidification process. They are provided with heaters both at the inlet and outlet points of air. Preheating of air is necessary when large humidity changes are needed. Same thing can be achieved by using hot water in the spray chamber. Similarly by using water at a low temperature, dehumidification can be achieved. Operations of this nature is useful in providing conditioned air for different applications.



1. Filter 2. Heater 3. Nozzle manifold 4. Entrainment eliminators 5. Pump

**Fig. 5.8** Schematic arrangement of a spray chamber.

The process is shown in a typical psychrometric chart of Fig. 5.9.

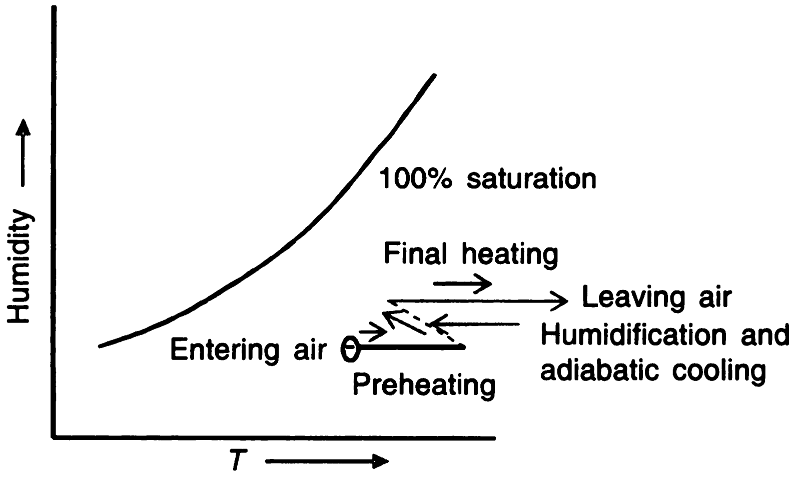


**Fig. 5.9** Process in a spray chamber.

### 5.8.3 Spray Ponds

They are also used for cooling water where close approach to the air-wet bulb temperature is not required. Examples for this are the water fountains, where the water is thrown up as a spray in air and the liquid falls back into the collection basin. Generally, these operations show high windage losses of water.

## WORKED EXAMPLES

1. An air (B) – water (A) sample has a dry bulb temperature of 50°C and a wet bulb temperature of 35°C. Estimate its properties at a total pressure of 1 atm. 1 atm = $1.0133 \times 10^5$ N/m$^2$

   Average molecular weight of air = 28.84

**Solution.**

(i) $Y'$ (Chart) = 0.03 kg water vapour/kg dry air

= 0.0483 kmol/kmol

(ii) % Humidity (Chart) = 35%

(iii) % Relative saturation = Partial pressure/Vapour pressure

Partial pressure under the given condition is given by

$$\text{Molal humidity} = \frac{p_A}{(P_t - p_A)}$$

$$0.0483 = \frac{p_A}{[1.0133 \times 10^5 - p_A]}$$

Hence, partial pressure, $p_A = 0.04672 \times 10^5$ N/m$^2$

Vapour pressure of water (steam tables) at 50°C = 92.55 mm = $0.1234 \times 10^5$ N/m$^2$

$$\therefore \quad \text{\% R.H.} = 37.86\%$$

(iv) Dew point = 31.5°C

(v) Humid heat = $C_S = C_B + C_A Y'$

= 1.005 + 1.884 (0.03) = 1.062 kJ/kg dry air °C

(vi) Enthalpy (for a reference temperature of 0°C)

(a) $H = C_S (t_G - t_0) + Y' \lambda_0$

$\lambda_0$ = 2502 kJ/kg

= 1.062 (50 – 0) + (0.03) (2502) = 128.16 kJ/kg

(b) Enthalpy of saturated air = 274 kJ/kg

Enthalpy of dry air = 50 kJ/kg

$\therefore$ Enthalpy of wet air = 50 + (274 – 50) (0.35) = 128.4 kJ/kg

(vii) Humid volume ($V_H$)

(a) $$V_H = 8315\left[\left(\frac{1}{M_B}\right)+\left(\frac{Y'}{M_A}\right)\right]\left[\frac{(t_G+273)}{P_t}\right]$$

$$= 8315\left[\left(\frac{1}{28.84}\right)+\left(\frac{Y'}{18}\right)\right]\left[\frac{325}{(1.0133\times 10^5)}\right]$$

$= 0.969$ m$^3$ mixture/kg of dry air

(b) Specific volume of saturated air = 1.055 m$^3$/kg

Specific volume of dry air = 0.91 m$^3$/kg

By interpolation $v_H$ = 0.91 + (1.055 – 0.91) (0.35)

= 0.961 m$^3$/kg of dry air **Ans.**

**2.** Air is entering into a cooling tower with characteristics as follows:
Dry bulb temperature = 25°C, Wet bulb temperature = 22°C and Pressure = 1 atm. Find (i) humidity, (ii) % humidity, (iii) % relative humidity, (iv) dew point and (v) enthalpy.

**Solution.**
From psychrometric chart,

(i) Humidity = 0.0145 kg water/kg dry air **Ans.**

(ii) % humidity = 61%. **Ans.**

(iii) $Y_S'$ (Saturation humidity) = 0.0255 kg water/kg dry air

$$Y_S' = \left[\frac{P_A}{(P_t - P_A)}\right]\times\left[\frac{18}{28.84}\right]$$

$$0.0255 = \left[\frac{P_A}{(1 - P_A)}\right]\times\left[\frac{18}{28.84}\right]$$

Vapour pressure, $P_A$ = 0.0393 atm.

$$Y' = \left[\frac{p_A}{(p_t - p_A)}\right]\times\left[\frac{18}{28.84}\right]$$

$$0.0145 = \left[\frac{p_A}{(1 - p_A)}\right]\times\left[\frac{18}{28.84}\right]$$

$p_A$ = 0.0227 atm.

$$\text{R.H.} = \left(\frac{p_A}{P_A}\right)\times 100$$

R.H. = (0.0227/0.0393) × 100 = 57.77% **Ans.**

(iv) Dew point = 19.5°C **Ans.**

(v) Humid heat, $C_S = 1005 + 1884\ Y'$

$= 1005 + 1884 \times 0.0145 = 1032.32$ J/kg °C

Enthalpy, $H = C_S\, t_G + 2502300\ Y'$

$= (1032.32 \times 25) + (2502300 \times 0.0145)$

$= 65188.25$ J/kg dry air. **Ans.**

**3.** A mixture of nitrogen–acetone vapour at 800 mm Hg and 25°C has percentage saturation of 80%. Calculate (i) absolute humidity (ii) partial pressure of acetone (iii) absolute molal humidity and (iv) volume percent of acetone. Assume vapour pressure of acetone at 25°C as 190 mm Hg.

**Solution.**

(i) $$Y_S' = \frac{P_A}{(P_t - P_A)} \times \left(\frac{58}{28}\right)$$

$$= \left[\frac{190}{(800-190)}\right] \times \left(\frac{58}{28}\right)$$

= 0.645 kg acetone/kg nitrogen

$$\%\ \text{Saturation} = \frac{Y'}{Y_S'} \times 100$$

$$80 = \left(\frac{Y'}{0.645}\right) \times 100$$

$Y' = 0.516$ kg acetone/kg $N_2$ **Ans.**

(ii) $$Y' = \frac{p_A}{(P_t - p_A)} \times \left(\frac{58}{28}\right)$$

$p_A = 159.54$ mm Hg **Ans.**

(iii) $$Y = \frac{p_A}{(P_t - p_A)} = 0.249 \text{ kmol acetone/kmol } N_2$$ **Ans.**

(iv) Volume of 0.249 k mole acetone vapour at NTP = 0.249 × 22.414 = 5.581 $m^3$

Volume of 1 kmol of $N_2$ at NTP = 22.414 $m^3$

Calculating volume of acetone and $N_2$ at 25°C, using Ideal gas law,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\text{Volume of acetone at 25°C} = \frac{(298 \times 5.581 \times 760)}{(800 \times 273)} = 5.787\ m^3$$

$$\text{Volume of } N_2 \text{ at 25°C} = \frac{(298 \times 22.414 \times 760)}{(800 \times 273)} = 23.243\ m^3$$

Hence,

Total volume of mixture = 5.787 + 23.243 = 29.03 $m^3$

Thus,

$$\% \text{ volume of acetone} = \left(\frac{5.787}{29.03}\right) \times 100 = 19.93\% \qquad \textbf{Ans.}$$

**4.** Partial pressure of water vapour in a mixture of air–water vapour at a total pressure of 106.6 kPa and a temperature of 60°C is 13.3 kPa. Express the concentration of water vapour in (i) absolute humidity (ii) mole fraction (iii) volume fraction (iv) relative humidity and (v) g water/m$^3$ mixture. Assume vapour pressure is 20.6 kPa at 60°C.

**Solution.**

$$\text{(i) } Y = \frac{p_A}{(P_t - p_A)} = \frac{13.3}{(106.6 - 13.3)}$$

$$= 0.14255 \text{ kmol water vapour/kmol dry air}$$

$$Y' = Y \times \left(\frac{18}{28.84}\right) = 0.08897 \text{ kg water vapour/kg dry air} \qquad \textbf{Ans.}$$

$$\text{(ii) Mole fraction} = \frac{p_A}{P_t} = 0.1248 \qquad \textbf{Ans.}$$

$$\text{(iii) Volume fraction} = \text{Mole fraction} = 0.1248 \qquad \textbf{Ans.}$$

$$\text{(iv) Relative humidity} = \left(\frac{p_A}{P_A}\right) \times 100 = 64.6\%$$

$$\text{Humid volume, } V_H = 8315\left(\frac{1}{M_B} + \frac{Y'}{M_A}\right)\left[\frac{(t_G + 273)}{P_t}\right]$$

$$= 8315\left(\frac{1}{28.84} + \frac{0.08897}{18}\right)\left(\frac{333}{106.6 \times 10^3}\right)$$

$$= 1.029 \text{ m}^3 \text{ mixture/kg dry air} \qquad \textbf{Ans.}$$

$$\text{(v) g water/m}^3 \text{ mixture} = \frac{Y'}{V_H} = \frac{0.08897}{1.029}$$

$$= 0.0865 \text{ kg water/m}^3 \text{ mixture}$$

$$= 86.5 \text{ g water/m}^3 \text{ mixture} \qquad \textbf{Ans.}$$

**5.** Air is available at a DBT and WBT of 30°C and 25°C respectively. Find its humidity, percentage saturation, humid volume, enthalpy and dew point.

**Solution.**

From Psychrometric chart,

$Y' = 0.0183$ kg water vapour/kg dry air

% Saturation = 67%

$$\text{Humid volume} = 8315\left(\frac{1}{M_B} + \frac{Y'}{M_A}\right)\left[\frac{(t_G + 273)}{P_t}\right]$$

$$= 0.883 \text{ m}^3 \text{ mixture/ kg dry air}$$

Humid heat, $C_s = 1005 + 1884\ Y'$

$= 1039.48$ J/kg dry air °C

Enthalpy $= C_s t_G + 2502300\ Y'$

$= 76976.49$ J/kg dry air

Dew point = 23.5°C **Ans.**

6. Air–water vapour mixture has a DBT of 55°C with humidity of 0.048 kmol water vapour/kmol dry air and 1 standard atmospheric pressure. Find absolute humidity, % humidity, humid volume, humid heat and total enthalpy.

**Solution.**

$$Y' = Y \times \left(\frac{18}{28.84}\right) = 0.03 \text{ kg water vapour/kg dry air}$$

% humidity = 25.5%

$$\text{Humid volume} = 8315\left(\frac{1}{M_B} + \frac{Y'}{M_A}\right)\left[\frac{(t_G + 273)}{P_t}\right]$$

$$= 0.978 \text{ m}^3 \text{ mixture/kg dry air}$$

Humid heat, $C_s = 1005 + 1884\ Y'$

$= 1061.52$ J/kg air °C

Enthalpy $= C_s t_G + 2502300\ Y'$

$= 133452.6$ J/kg dry air **Ans.**

7. Air at 85°C and absolute humidity of 0.03 kg water vapour/kg dry air at 1 standard atmosphere is contacted with water at an adiabatic saturation temperature and it is thereby humidified and cooled to 70% saturation. What are the final temperature and humidity of air?

**Solution.**

From psychrometric chart,

Final temperature = 46°C **Ans.**

$Y' = 0.0475$ kg water vapour/kg dry air. **Ans.**

**8.** Air at a temperature of 30°C and a pressure of 100 kPa has a relative humidity of 80%.

(i) Calculate the molal humidity of air.

(ii) Calculate the molal humidity of this air if its temperature is reduced to 15°C and its pressure increased to 200 kPa, condensing out some of the water.

(iii) Calculate the weight of water condensed from 100 $m^3$ of the original wet air in cooling to 15°C and compressing to 200 kPa.

(iv) Calculate the final volume of the wet air of part (iii).

**Solution.**

*Data:* Vapour pressure of water at 30°C = 4.24 kPa
Vapour pressure of water at 15°C = 1.70 kPa

(i) Saturated molal humidity of air, $Y'_S = \dfrac{P_A}{(P_t - P_A)}$

$$= 4.24/(100 - 4.24)$$

$$= 0.04428 \text{ kmol/kmol dry air.}$$

$$\text{Relative humidity} = \frac{p_A}{P_A}$$

$$0.8 = \frac{p_A}{4.24}$$

Hence,

$$p_A = 3.392 \text{ kPa}$$

$$\text{Molal humidity} = \frac{p_A}{(P_t - p_A)} = \frac{3.392}{(100 - 3.392)}$$

$$= 0.0351 \text{ kmol/kmol dry air.}$$

$$= 0.0351 \times 18/28.84 = 0.0219 \text{ kg/kg dry air}$$ **Ans**

(ii) Under these conditions the air will be saturated at 15°C as some water is condensed.

Saturated molal humidity of air, $Y_s = \dfrac{P_A}{(P_t - P_A)}$

$$= 1.7/(200 - 1.7)$$

$$= 0.00857 \text{ k mol/kmol dry air}$$

$$Y'_s = 0.00857 \times 18/28.84$$

$$= 0.00535 \text{ kg/kg dry air}$$ **Ans.**

(iii) Humid volume: $V_{H,\text{Original}} = 8315\left(\dfrac{1}{M_B} + \dfrac{Y'}{M_A}\right)\left[\dfrac{(t_G + 273)}{P_t}\right]$

$$= 8315\left(\frac{1}{28.84}+\frac{0.0219}{18}\right)\left[\frac{(30+273)}{100000}\right]$$

$$= 0.9042 \text{ m}^3\text{/kg dry air}$$

100 m$^3$ of original mixture contains, $\frac{100}{0.9042}$ = 110.6 kg dry air

Therefore, water present in original air = 110.6 × 0.0219 = 2.422 kg

Water present finally = 110.6 × 0.00535 = 0.5918 kg

Water condensed from 100 m$^3$ of original mixture = 2.422 – 0.5918
= 1.830 kg **Ans.**

(iv) $$V_{H,\text{final}} = 8315\left[\frac{1}{28.84}+\frac{0.00535}{18}\right]\left[\frac{(15+273)}{200000}\right]$$

$$= 0.4187 \text{ m}^3\text{/kg dry air}$$

Final volume of mixture = 110.6 × 0.4187 = 46.329 m$^3$ **Ans.**

**9.** An air–water vapour sample has a dry bulb temperature of 55°C and an absolute humidity 0.030 kg water/kg dry air at 1 standard atm pressure. Using humidity chart, if vapour pressure of water at 55°C is 118 mm Hg, calculate the relative humidity, the humid volume in m$^3$/kg dry air, enthalpy in J/kg dry air and the heat required if 100 m$^3$ of this air is heated to 110°C.

**Solution.**

DBT = 55°C and Humidity = 0.030 kg water/kg dry air

$$Y = \frac{Y'}{(18/28.84)} = \left[\frac{0.03}{18}\right]\Big/\left(\frac{1}{28.84}\right)$$

$$= 0.04807 \text{ kmol of water vapour/kmol of dry air}$$

$$0.04807 = \left[\frac{p_A}{P_t - p_A}\right] = \left[\frac{p_A}{760 - p_A}\right]$$

$$p_A = 34.86 \text{ mm Hg}$$

(i) Relative humidity $= \frac{p_A}{P_A} = \frac{34.86}{118} = 29.5\%$

$$\text{Saturated humidity} = \left[\frac{118}{760-118}\right]$$

$$= 0.184 \text{ kmol of water vapour/kmol of dry air}$$

$$\% \text{ Humidity} = \left[\frac{\text{Humidity at given condition}}{\text{Humidity at saturated condition}}\right] \times 100$$

$$= \left[\frac{0.04807}{0.184}\right] \times 100 = 0.261 \qquad \textbf{Ans.}$$

(ii) $V_H = 8315\left(\frac{1}{M_B}+\frac{Y'}{M_A}\right)\left[\frac{(t_G+273)}{P_t}\right]$

$$= 8315\left[\frac{1}{28.84}+\frac{0.03}{18}\right]\left[\frac{55+273}{101300}\right] = 0.978 \text{ m}^3\text{/kg dry air}$$

$V_H = V_{H,\text{Dry air}} + (V_{H,\text{Sat. Air}} - V_{H,\text{Dry air}}) \times \%$ saturation (from chart)

$= 0.93 + (1.1 - 0.93)\ 0.261 = 0.974$ m$^3$/kg dry air **Ans.**

(iii) Humid heat, $C_S = 1005 + 1884\ Y'$

$= 1005 + 1884 \times 0.03$

$= 1061.52$ J/kg dry air

Enthalpy, $H = C_S\ (t_G - t_0) + Y'\lambda_0$

$= 1061.52\ (55 - 0) + 0.03 \times 2502300$

$= 133452$ J/kg dry air $= 133.452$ kJ/kg dry air

*From chart*: $H = H_{\text{dry}} + (H_{\text{sat}} - H_{\text{dry}}) \times 0.261$

$= 56 + (350 - 56) \times 0.261$

$= 132.7$ kJ/kg dry air **Ans.**

(iv) Heat needed if volume of air = 100 m$^3$

$$\text{Mass of dry air} = \frac{\text{Volume}}{\text{Humid volume}}$$

$= 100/0.978 = 102.25$ kg dry air

Enthalpy, $H_{\text{final}} = C_S\ (t_G - t_0) + Y'\lambda_0$

$= 1061.52\ (110 - 0) + 0.03 \times 2502300$

$= 191836$ J/kg dry air

$=191.836$ kJ/kg dry air

Heat added $= (H_{\text{final}} - H_{\text{initial}}) \times$ Mass of dry air

$= (191.836 - 133.452) \times 102.25 = 5969.76$ kJ **Ans.**

**10.** A plant requires 2,000 kg/min of cooling water to flow through its distillation equipment condensers. The water will leave the condensers at 50°C. It is planned to design a countercurrent cooling tower in order to cool this water to 30°C from 50°C for reuse, by contact with air. Air is available at 30°C dry bulb temperature and 24°C wet bulb temperature. 30% excess air will be used and the make up water will enter at 15°C. For the packing to be used, the value of the mass transfer coefficient is expected to be 2500 kg/(h)(m$^3$)($\Delta Y'$), provided the minimum liquid rate and gas rates are 12,000 and 10,000 kg/(h)(m$^2$) respectively. Determine the diameter of the cooling tower and makeup water to be used.

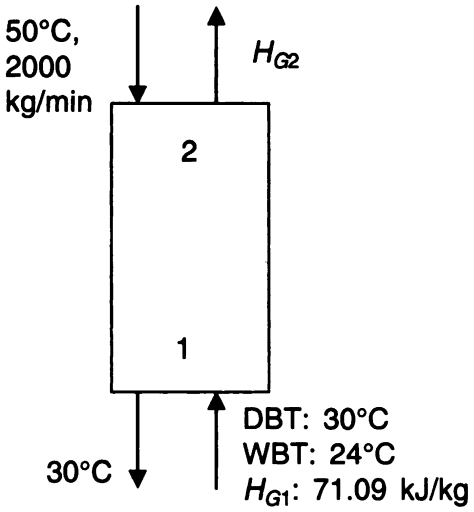


**Fig. 5.10(a)** Example 10.

**Solution.**

Flow rate of water to be cooled, $L$ = 2000 kg/min.

Inlet temperature of water = 50°C

Outlet temperature of water = 30°C

Humidity of incoming air (DBT 30°C and WBT 24°C) = 0.016 kg/kg

Specific heat of water = 4.18 kJ/kg °C

Let us now compute the (Temperature – Enthalpy) data:

Temperature 20°C, Saturated humidity = 0.016 kg/kg (from chart)

$$\begin{aligned}\text{Enthalpy} &= C_{p,\text{air}}\,(t_G - t_0) + [C_{p,\text{w.v}}.(t_G - t_0) + \lambda_0]\,Y' \\ &= 1.005 \times 20 + [1.884 \times 0.016 \times 20] + 2502 \times 0.016 \\ &= 60.735 \text{ kJ/kg}\end{aligned}$$

In the same manner the enthalpy for other temperatures are also estimated and have been given below:

| Temp, °C | 20 | 30 | 40 | 50 | 55 |
|---|---|---|---|---|---|
| Enthalpy, kJ/kg | 60.735 | 101.79 | 166.49 | 278.72 | 354.92 |

Alternatively, these values can be obtained from the psychrometric chart also corresponding to enthalpy at saturated conditions.

Enthalpy of incoming air,

$$\begin{aligned}H_{G1} &= 1.005 \times 30 + [1.884 \times 0.016 \times 30] + 2502 \times 0.016 \\ &= 71.09 \text{ kJ/kg}\end{aligned}$$

Now draw the temperature vs. enthalpy curve from the above data.

Locate ($t_{L1}$, $H_{G1}$), i.e., (30, 71.09) the operating condition at the bottom of the tower and draw the tangent to the curve.

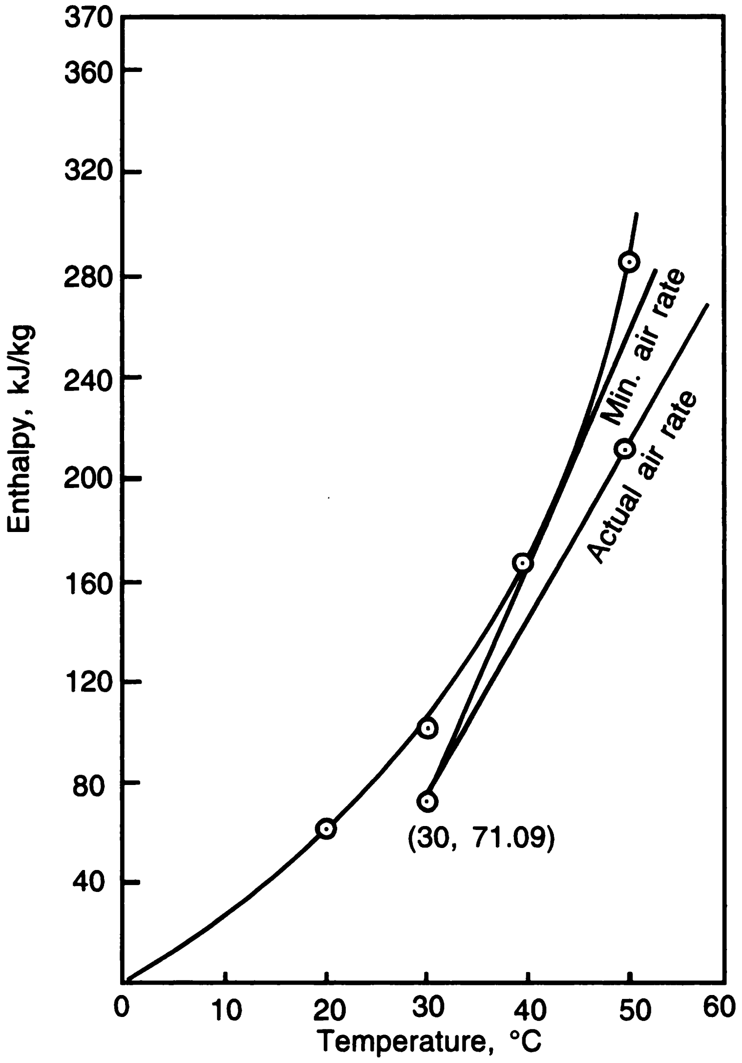


Fig. 5.10(b) Example 10, temperature–enthalpy plot.

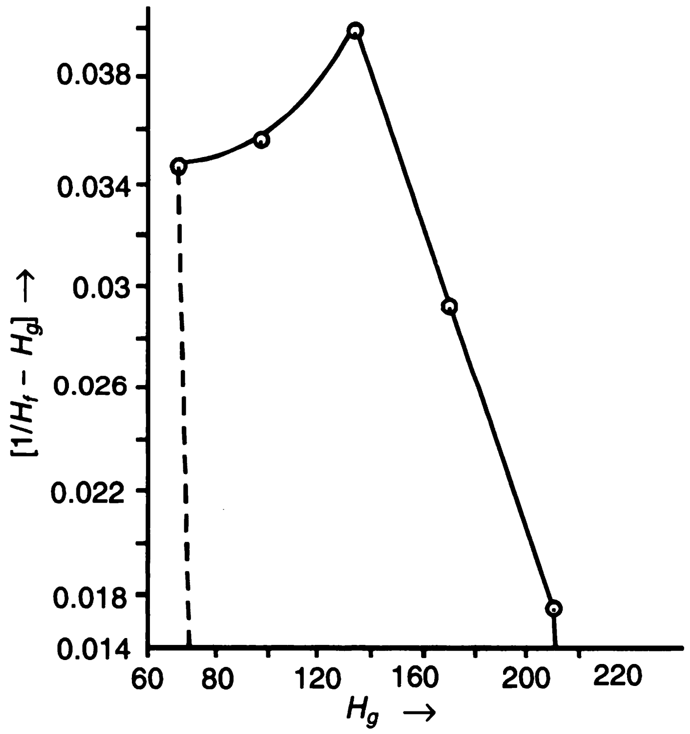


Fig. 5.10(c) Example 10, $[1/(H_f - H_G)]$ vs $H_G$.

$$\text{Slope of the tangent} = \left(\frac{LC_L}{G_S}\right)_{\min} = \frac{(H_{G2} - H_{G1})}{(t_{L2} - t_{L1})}$$

$H_{G2}$ (from graph) = 253 kJ/kg

$$\left(\frac{LC_L}{G_S}\right)_{\min} = \frac{(253 - 71.09)}{(50 - 30)} = 9.1$$

Therefore,

$$G_{S\min} = 2000 \times 60 \times 4.18/9.1 = 55120.88 \text{ kg/h.}$$

$$G_{\text{actual}} = 1.3 \times G_{\min} = 1.3 \times 55120.88 \text{ kg/hr} = 71657.14 \text{ kg/h.}$$

$$\text{Slope of the operating line} = \left(\frac{LC_L}{G_S}\right)_{\text{act}} = \frac{2000 \times 60 \times 4.18}{71657.14} = 7$$

i.e

$$\frac{(H_{G2} - H_{G1})}{(t_{L2} - t_{L1})} = 7$$

Therefore,

$$H_{G2}\text{, act} = [7 \times 20] + 71.09 = 211.09 \text{ kJ/kg}$$

The minimum gas rate is to be 10000 kg/h · $m^2$.
Therefore, the maximum area of tower (based on gas) is given by

$$\frac{\text{Actual gas rate}}{\text{Minimum gas rate}} = \frac{71657.14}{10000} = 7.1657 \text{ m}^2$$

The minimum liquid rate is to be = 12000 kg/h $m^2$. Therefore, the maximum area of tower (based on liquid) is given by,

$$\frac{\text{Actual liquid rate}}{\text{Minimum liquid rate}} = \frac{2000 \times 60}{12000} = 10 \text{ m}^2.$$

The minimum area of the tower, among the ones estimated (from gas and liquid flow rates) will be chosen, as this will alone meet the expected minimum flow rates of gas and that of liquid to ensure the mass transfer coefficient of 2500 kg/(h)($m^3$)($\Delta y'$). Hence, the area of tower is 7.1657 $m^2$. Therefore, the diameter is

$$\left[\frac{7.1657 \times 4}{3.14}\right]^{0.5} = 3.02 \text{ m}$$

The line joining points (30, 71.09) and (50, 211.09) is the operating line.

Let us assume that the resistances to mass transfer lies basically in gas phase. Hence, the interfacial conditions and the equilibrium conditions are one and the same. From the vertical lines drawn between the operating line and the equilibrium curve we get the conditions of gas and that of equilibrium conditions. The values are tabulated as follows:

| *Temperature*, °C | *Enthalpy*, kJ/kg | | $1/[H^* - H_G]$, kg/kJ |
|---|---|---|---|
| | $H^*$ | $H_G$ | |
| 30 | 101.79 | 71.09 | 0.0326 |
| 35 | 133.00 | 103.00 | 0.0333 |
| 40 | 166.49 | 140.00 | 0.0378 |
| 45 | 210.00 | 173.00 | 0.0270 |
| 50 | 278.72 | 211.09 | 0.0148 |

$$N_{OG} = \int \frac{dH_G}{[H^* - H_G]}$$

The above integral is evaluated graphically by plotting $1/[H^* - H_G]$ against $H_G$ from the above table

Area under the curve = $N_{OG} = 4.26$

The gas flow rate is 10000 kg/(h)(m$^2$) based on an area of tower of 7.1657 m$^2$

$$H_{OG} = \frac{G_s}{K_y a} = \frac{10000}{2500} = 4 \text{ m}$$

Height of the tower is

$= H_{OG} \times N_{OG} = 4 \times 4.26 = 17.04$ m

Makeup water (*M*) is based on the evaporation loss (*E*), blow down loss (*B*) and windage loss (*W*)

$$\therefore \quad M = E + B + W$$

Windage loss = 0.2% of circulation rate

$= 0.002 \times 2000 = 4$ kg/min = 240 kg/h

Blow down loss = Neglected

Evaporation loss is calculated assuming that the outlet air leaves fully saturated (based on the enthalpy of leaving gas 211.09 kJ/kg) and is equal to 0.064 kg/kg

Evaporation loss = $71657.14 \times (0.064 - 0.016) = 3439.5$ kg/h

Total makeup water = $240 + 3439.5 = 3679.5$ kg/h. **Ans.**

**11.** A cooling tower is used to cool 1,00,000 kg/h of water from 30°C to 17°C with air entering at 8°C and a humidity of 0.004 kg/kg. Air leaves the tower at 19°C at fully saturated condition. The cross-sectional area of the tower is 14.4 m$^2$. Calculate air velocity in kg/h · m$^2$ and quantity of makeup water needed.

**Solution.**

Humidity of incoming air = 0.004 kg/kg dry air

Humidity of leaving air = 0.015 kg/kg dry air

Enthalpy of incoming air = 18.11 kJ/kg dry air

Enthalpy of leaving air = 57.16 kJ/kg dry air

Let *W* be the water evaporated

$$W = m_{\text{dry air}} (0.015 - 0.004)$$

$$= 0.011\, m_{\text{dry air}} \qquad (1)$$

Making an energy balance,

Total heat in = Total heat out

Heat in entering water + Heat in entering air = Heat in leaving water + Heat in leaving air

$$(100000)(4.18)(30-0) + (m_{\text{dry air}})(H_{\text{in air}}) = (100000 - W)(4.18)(17-0) + (m_{\text{dry air}})(H_{\text{out air}})$$

Substituting for $W$ from Eq. (1), we get

$$30 \times 10^5 + 18.11 m_{\text{dry air}} = (100000 - 0.011 m_{\text{dry air}})(4.18)17 + 57.16\, m_{\text{dry air}}$$

$$(30 \times 10^5 - 17 \times 10^5) = m_{\text{dry air}} [(57.16 - 18.11 - (17 \times 0.011 \times 4.18)]$$

$$m_{\text{dry air}} = 141{,}997.3 \text{ kg/h}$$

Air velocity = 9860.9 kg/h · m$^2$ **Ans.**

Makeup water = 141997.3 × 0.011 = 1561.97 kg/h **Ans.**

**12.** A horizontal spray chamber with recirculated water is used for adiabatic humidification and cooling of air. The chamber has a cross-section of 2 m$^2$ with air rate of 3.5 m$^3$/s at dry bulb temperature of 65°C and absolute humidity of 0.017 kg water/kg dry air, the air is cooled and humidified to dry bulb temperature of 34°C and leaves at 90% saturation. For the system the volumetric mass transfer coefficient may be taken as 1.12 kg/m$^3$ s (mole fraction). The density of the air is 1.113 kg/m$^3$. Determine the length of the chamber for the requirements.

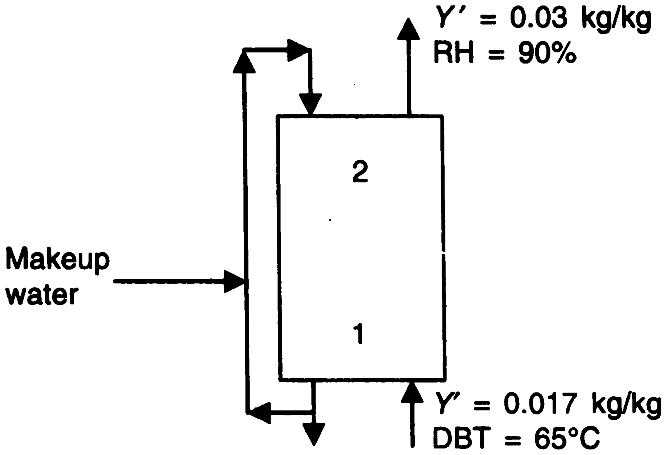


**Fig. 5.11** Example 12.

**Solution.**

Cross-sectional area of chamber 2 m$^2$

Air flow rate: 3.5 m$^3$/s

Humidity of incoming air: 0.017 kg/kg

$$\text{Mass flow rate of air} = 3.5 \times 1.113 = 3.8955 \text{ kg/s}$$

$$G'_S = 3.8955/1.017 = 3.83 \text{ kg/s}$$

$$\text{Humidity of leaving air} = 0.03 \text{ kg/kg}$$

$$Y'_{\text{as}} = 0.032 \text{ kg/kg}$$

For recirculation humidifier

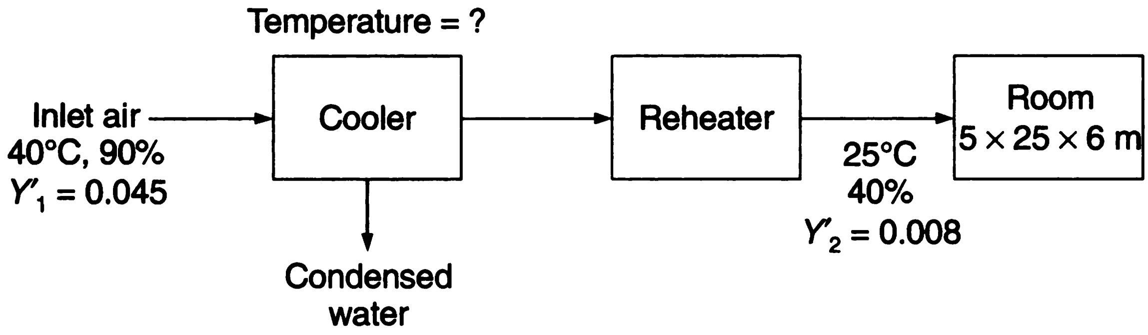
$$\ln\left[\frac{(Y'_{as} - Y'_1)}{(Y'_{as} - Y'_2)}\right] = \frac{K_{ya}Z}{G_S}$$

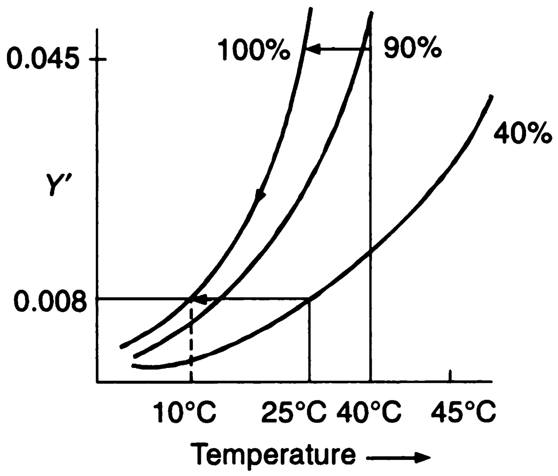
$$\ln\left[\frac{(0.032-0.017)}{(0.032-0.030)}\right] = 1.12 \times \frac{Z}{3.83}$$

$$Z = 6.89 \text{ m}$$

13. Atmospheric air at 40°C with 90% saturation is cooled and separated out the condensed water. Then it is reheated in a heat exchanger for conditioning at 25°C with 40% saturation using steam at 1 atm pressure. This conditioned air is supplied to a conference room of size 5 × 25 × 6 m without any facility for recirculation. Determine the temperature at which it is cooled and the volume of outside air at entry conditions.

**Solution.**





Temperature at cooler = 10°C.

Volume of the conditioned,

$$\text{air} = 5 \times 25 \times 6$$
$$= 750 \text{ m}^3.$$

Humid volume of entering air

$$V_{H1} = \left[\frac{1}{28.84} + \left(\frac{1}{18} \times 0.045\right)\right] 22.414 \left[\frac{273+40}{273}\right]$$

$$= 0.9553 \text{ m}^3\text{/kg dry air.}$$

Humid volume of conditioned air

$$V_{H_2} = \left[\frac{1}{28.84} + \left(\frac{1}{18} \times 0.008\right)\right] 22.414 \left[\frac{273+25}{273}\right]$$

$$= 0.8592 \text{ m}^3\text{/kg dry air.}$$

$$1 \text{ kg dry air} = 0.8592 \text{ m}^3$$

$$\therefore\ 750 \text{ m}^2 \text{ occupies} = \frac{750}{0.8592}$$

$$= 872.875 \text{ kg dry air}$$

$$1 \text{ kg dry air} = 0.9553 \text{ m}^3$$

$$\text{then } 872.875 \text{ kg dry air} = 0.9553 \times 872.875$$

$$= 833.86 \text{ m}^3$$

Volume of entering air = 833.86 m$^3$.

## EXERCISES

1. An air water system has DBT of 65°C with a humidity of 0.042 kg water vapour/kg dry air. What is the WBT and % saturation?

   (**Ans:** 41°C and 20%)

2. A horizontal spray chamber with recirculated water is used for adiabatic humidification and cooling of air. The chamber has a cross-section of 3 m$^2$ with an air rate of 5 m$^3$/s at a dry bulb temperature of 60°C and absolute humidity of 0.017 kg water/kg dry air, the air is cooled and humidified to a dry bulb temperature of 34°C and leaves at 80% RH. For the system volumetric mass transfer coefficient may be taken as 1.12 kg/m$^3 \cdot$ s ($\Delta Y'$). The density of the air is 1.113 kg/m$^3$. Determine the length of the chamber for the requirements.

   (**Ans:** 4.59 m)

3. A drier is used to remove 100 kg of water per hour from the material being dried. The available air has a humidity of 0.010 kg per kg of bone dry air and a temperature of 23.9°C and is heated to 68.3°C before entering the drier. The air leaving the drier has a wet bulb temperature of 37.8°C and a dry bulb temperature of 54.4°C. Calculate the (i) consumption rate of wet air, (ii) humid volume of air before and after preheating, (iii) wet bulb temperatures of air before and after preheating, and (iv) dew point of the air leaving the drier.

   (**Ans:** (i) 3.672 kg/h, (ii) 0.8586 m$^3$/kg dry air and 0.987 m$^3$/kg dry air, (iii) 300 K and 303 K and (iv) 308.5 K)

4. A spray chamber of 2 metres length and 2.5 m$^2$ area has been used for adiabatic humidification and cooling of air. Water is sprayed into the chamber through nozzles and the coefficient of heat transfer is found to be

1360 kcal/h m$^3$°C. Air is passed at a rate of 3,000 cubic metres per minute at 70°C, containing water vapour 0.014 kg per every kg dry air.

(i) What exit temperature and humidity can be expected for the air?
(ii) What is the amount of make–up water?

(**Ans:** (i) 32°C and 0.032 kg/kg, assuming fully saturated
(ii) 54.108 kg/min)

5. A tray drier contains 10 trays in a tier on racks at 10 cm apart. Each tray is 3.5 cm deep and 90 cm wide and there are 16 m$^2$ of drying surface. It is desired that the material on trays is dried by blowing a part of recycled air with fresh air, drawn through a heater kept in the drier itself. Atmospheric air enters at 26.6°C having a humidity content of 0.01. Further, it is desired that the air entering trays have a dry bulb temperature of 93.3°C and humidity 0.05 kg water/kg dry air. The air velocity at the entrance of trays is to be 3.3 m/s. The material looses water at a constant rate of 30 kg of water per hour. Determine:

(i) Percentage recirculation of air.
(ii) Heat load.

(**Ans:** (i) 63.5%, (ii) 82697.2 kJ/h)

6. Calculate the cross-sectional area and depth of packing required in wooden slats packed water-cooling tower. The tower is required to cool 37,735 kg/h of water initially at 54.4°C to 32.2°C, by counter current contact with air at atmospheric pressure having a dry bulb temperature of 25°C and a wet bulb temperature of 21.0°C. The air rate will be 30% more than the minimum air flow rate, and the superficial velocity will be 6,970 kg/h (air). $H_{tOG}$ = 2.65 metres (enthalpy based).

*Data:*

| *Temperature*, °C | *Enthalpy*, kcal/kg |
|---|---|
| 4.4 | 8.47 |
| 10 | 11.30 |
| 15.5 | 14.72 |
| 21 | 18.96 |
| 26.07 | 24.3 |
| 32.2 | 31.1 |
| 37.7 | 39.9 |
| 43.3 | 51.36 |
| 48.8 | 66.5 |
| 54.4 | 86.7 |
| 57.2 | 99.52 |

(**Ans:** Area = 2.2623 m$^2$, Height = 10.878 m)

7. The air supply for a drier has dry bulb temperature of 26°C and a wet bulb temperature of 17°C. It is heated to 85°C by heating coils and introduced into the drier. In the drier, it cools along the adiabatic cooling line and leaves the drier fully saturated.

(i) What is its humidity initially and after heating?
(ii) What is the dew point of the initial air?

(iii) How much water will be evaporated per 100 $m^3$ of entering air?
(iv) How much heat is needed to heat 100 $m^3$ air to 85°C?
(v) At what temperature does the air leave the drier?

(**Ans:** (i) 0.01 kg/kg and 0.033 kg/kg, (ii) 285 K, (iii) 2.659 kg, (iv) 6984 2 kJ, and (v) 306.5 K)

**8.** 350 $m^3$/min of air at 70°C and 1 atmosphere pressure having a wet bulb temperature of 30°C is to be adiabatically humidified and cooled in a chamber using recirculated water. The chamber is 1.5 m. wide, 1.5 m high and 2.0 m long. The coefficient of heat transfer has been estimated to be 1200 kcal/(h)($m^3$)(°C). The specific volume and specific heat of the entering air are 0.85 $m^3$/kg and 0.248 kcal/(kg)(°C) respectively. Determine the (i) temperature, (ii) humidity of the exit air, and (iii) Estimate the number of transfer units.

(**Ans:** (i) 46.6°C, (ii) 0.02 kg/kg, (iii) 0.754)

**9.** Fresh air at 25°C in which partial pressure of water vapour is 15 mm Hg is blown at a rate of 215 $m^3$/h first through a preheater and then adiabatically saturated in a spray chamber to 100% saturation and again reheated. This reheated air has a humidity of 0.024 kg water vapour/kg dry air. It is assumed that the fresh air and the air leaving the reheaters have the same percentage humidity. Determine (i) the temperature of air after preheater, spray chamber and reheater and (ii) heat requirements for preheating and reheating.

(**Ans:** (i) 53.5°C, 28°C and 32.5°C (ii) 7287.5 kJ and 1174.65 kJ)

**10.** Air is to be cooled and dehumidified by counter-current contact with water in a packed tower. The tower is to be designed for the following conditions, DBT and WBT are 28°C and 25°C respectively. Flow rate of inlet air 700 kg/h of dry air. Inlet and outlet temperatures of water are 10°C and 18°C respectively. For the entering air estimate (i) humidity (ii) %R.H. (iii) dew point and (iv) enthalpy.

(**Ans:** (i) 0.019 kg/kg, (ii) 88.08%, (iii) 23.5°C and (iv) 79.22 kJ/kg dry air)

**11.** Air is available at a DBT of 30°C and a WBT of 25°C respectively. Determine (i) humidity, (ii) percentage saturation, (iii) humid volume, (iv) enthalpy, and (v) dew point.

(**Ans:** (i) 0.013 kg/kg, (ii) 59%, (iii) 0.874 $m^3$/kg dry air, (iv) 63.4 kJ/kg, and (v) 16°C)

**12.** Air–water sample mixture has a DBT of 50°C and a humidity of 0.03 kg water vapour/kg dry air. If the pressure is 1 atmosphere, find (i) % humidity (ii) humid volume (iii) dew point and (iv) enthalpy.

(**Ans:** (i) 36%, (ii) 138 $m^3$/kg dry air, (iii) 31.75°C, and (iv) 42 kJ/kg)

**13.** 1.5 $m^3$/s of air is required for a specific operation at 65°C and 20% humidity. This is prepared from air available at a DBT of 27°C and a WBT of 18°C by direct spray of water into the air stream followed by passage over steam heated finned tube. Estimate the water and heat needed per second.

(**Ans:** 0.0496 kg/s and 128.07 kJ/kg dry air)

**14.** Air at a DBT of 40°C and a WBT of 30°C is to be dried by first cooling to 16°C to condense water vapour and then reheating to 25°C. Calculate (i) the initial humidity and % humidity and (ii) the final humidity and % humidity of air.

(**Ans:** (i) 0.023 kg/kg, 48% (ii) 0.012 kg/kg, 50%)

**15.** Air at a temperature of 20°C and a pressure of 760 mm Hg has a relative humidity of 80%.

(i) Calculate the humidity of air.

(ii) Calculate the molal humidity of this air if its temperature is reduced to 10°C and its pressure increased to 1900 mm Hg, condensing out some of the water.

*Data*: VP of water at 20°C = 17.5 mm Hg
VP of water at 10°C = 9 mm Hg

(**Ans:** (i) 0.01171 kg/kg, (ii) $4.759 \times 10^{-3}$ kmol/kmol)

**16.** Air at a DBT of 35°C and WBT of 30°C and at 1 atm is passed into an evaporator. The DBT and WBT of air at the outlet of evaporator are 45°C and 38°C respectively. Determine (i) humidity and relative humidity (ii) percent saturation of air at the exit of evaporator and (iii) weight of water evaporated.

(**Ans:** (i) 0.0255 kg/kg, 70.7% (ii) 65% and (iii) 0.0155 kg)

# 6

# DRYING

## 6.1 INTRODUCTION

Drying refers to the removal of relatively small amounts of moisture from a substance which is generally a solid. However, in some specific cases, it includes the removal of moisture from liquids and gases as well. Drying is generally a final step in the production process and the product from the dryer is often sent for final packaging.

## 6.2 DEFINITIONS OF MOISTURE AND OTHER TERMS ON DRYING

When an insoluble solid is dried, the moisture is lost to the surrounding air and the solid attains an equilibrium moisture content depending on the relative saturation of air. The different moisture contents exhibited by a substance when exposed to air of different saturation levels is shown in Fig. 6.1.

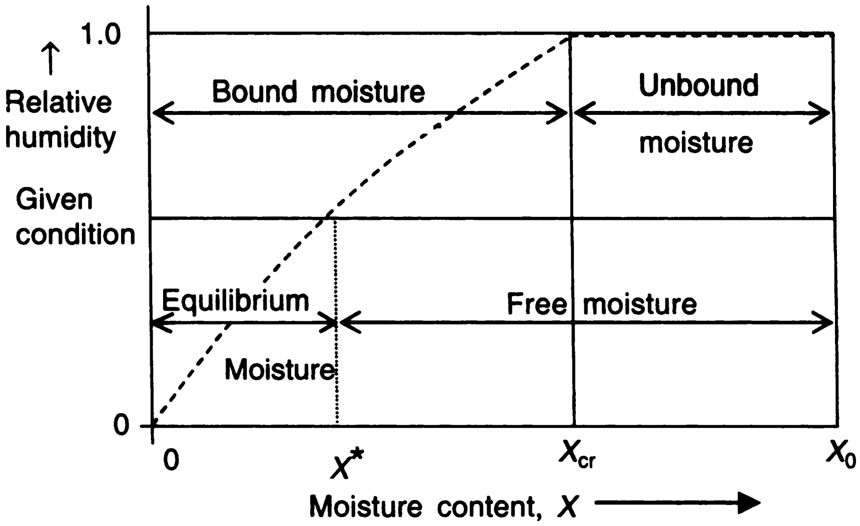


**Fig. 6.1** Moisture contents at different humidity conditions.

### 6.2.1 Moisture Content (Wet Basis), X

This is defined as the weight of moisture per unit weight of wet substance.

### 6.2.2 Moisture Content (Dry Basis), X

This is defined as the weight of moisture per unit weight of bone dry substance.

### 6.2.3 Equilibrium Moisture, X*

This is the moisture content of a substance when it is at equilibrium with a given partial pressure of the vapour. It is the limiting moisture content to which a given material can be dried under specific conditions of air temperature and humidity.

### 6.2.4 Bound Moisture

This refers to the moisture contained by a substance which exerts an equilibrium vapour pressure less than that of the pure liquid at the same temperature. Liquid may become bound by retention in small capillaries, by solution in cell or fibre walls or by adsorption on solid surface.

### 6.2.5 Unbound Moisture

This refers to the moisture contained by a substance which exerts an equilibrium vapour pressure equal to that of the pure liquid at the same temperature.

### 6.2.6 Free Moisture (X–X*)

This is the moisture contained by a substance in excess of equilibrium moisture. Only free moisture can be removed with air of given temperature and humidity. It may include both bound and unbound moisture.

### 6.2.7 Critical Moisture Content

It is the moisture content when the constant rate drying period ends and falling rate drying period starts.

### 6.2.8 Fibre–Saturation Point

It is the moisture content of cellular materials (e.g. wood, fibre) at which the cell walls are completely saturated while the cavities are liquid free. It may be defined as the equilibrium moisture content, as the humidity of the surrounding atmosphere approaches saturation.

### 6.2.9 Constant Rate Drying Period

Constant rate drying period is that drying period during which the rate of water removed per unit area of drying surface is constant.

### 6.2.10 Falling Rate Drying Period

It is a drying period during which the instantaneous drying rate continually decreases. Some of the substances show a linear behaviour and some show non-linear behaviour during falling rate drying period. In certain instances we observe both. The linear one is due to unsaturated surface drying wherein one sees certain dry spots on the drying surface and the non-linear one is observed when the moisture movement is controlled by diffusion mechanism.

### 6.2.11 Funicular State

It is a condition that occurs in drying a porous body when capillary suction results in air being sucked into the pores. This generally indicates the first falling rate drying period. The drying rate varies linearly with free moisture content in this period.

### 6.2.12 Pendular State

As the water is progressively removed from the solid, the fraction of the pore volume that is occupied by air increases. When the fraction reaches a certain limit, there is insufficient water left to maintain continuous film across the pores, the interfacial tension in the capillaries breaks, and the pores filled with air, which now becomes continuous phase. The left out water is relegated to small isolated pores and interstices of the pores. This state is called the pendular state and it generally refers to the second falling rate drying period. During this period, the variation of drying rate with free moisture content is non-linear.

## 6.3 HYSTERESIS

Many substances exhibit different equilibrium moisture relationships during the adsorption and desorption of moisture as shown in Fig. 6.2. This phenomenon of following different paths is known as hysteresis in drying.

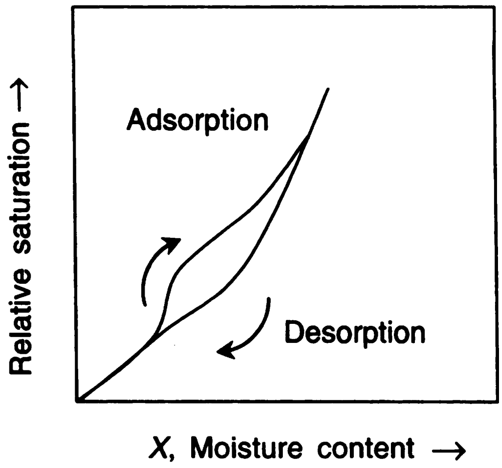


**Fig. 6.2** Hysteresis in drying.

## 6.4 DRYING OF SOLUBLE SOLIDS

Soluble solids show insignificant equilibrium moisture content when exposed to gases whose partial pressure of vapour is less than that of the saturated solution of the solid. A typical trend is shown in Fig. 6.3.

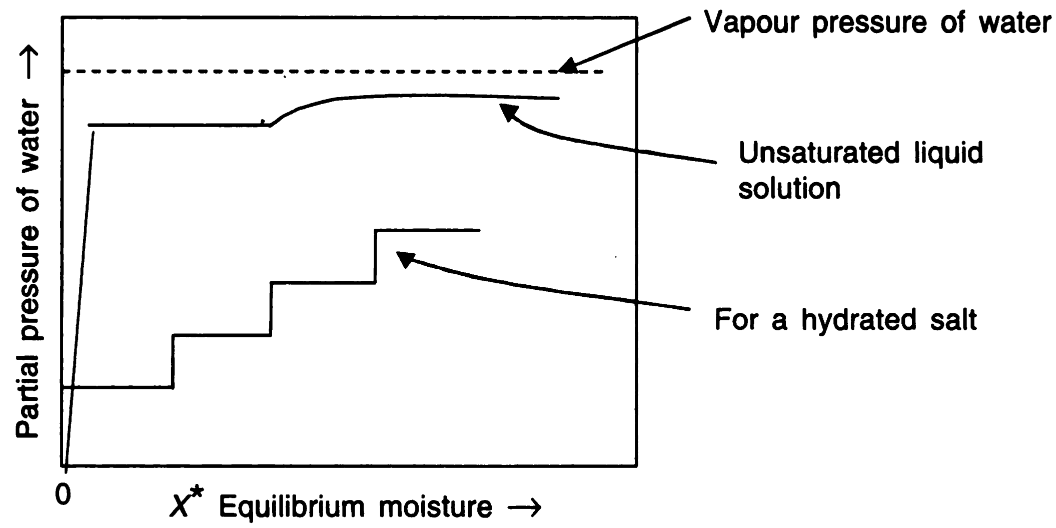


**Fig. 6.3** Equilibrium moisture content of soluble solids.

## 6.5 CLASSIFICATION OF DRYING OPERATIONS

Drying operations can be broadly classified as,

(i) Batch drying
(ii) Continuous drying

### 6.5.1 Batch Drying

Here the material to be dried is fed to a drier and exposed to drying media under unsteady state conditions.

#### 6.5.1.1 Drying test

In order to determine the drying schedule and also the size of drying equipments, it is necessary to know the time required for drying a substance. The rate of drying is determined by suspending a substance in a chamber in a stream of air and measuring its weight periodically. The operation is carried out under constant drying condition by maintaining the same temperature, humidity and air flow rate. A typical drying curve is shown in Fig. 6.4. This is drawn by estimating weight itself or determining moisture content on dry basis and plotting against time.

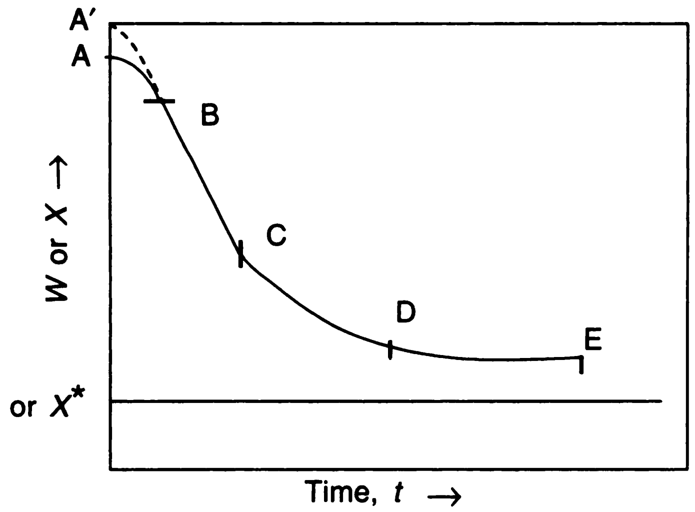


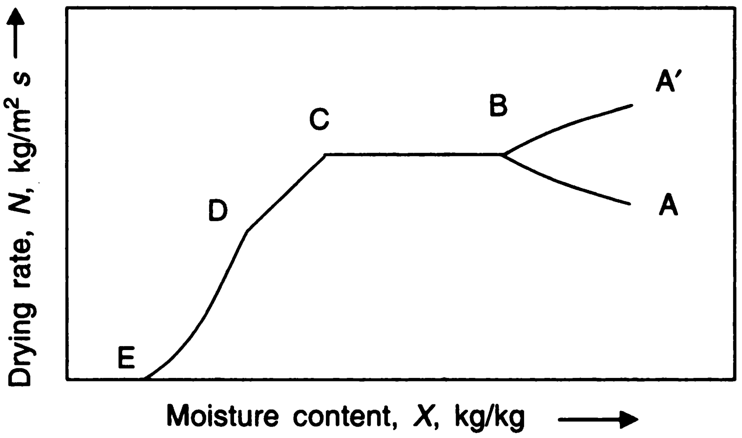
**Fig. 6.4** Drying curve.

From the drying curve, the rate of drying, $N$ is calculated as,

$$N = \left(-\frac{L_S}{A}\right)\left(\frac{dX}{dt}\right) = -\frac{1}{A}\cdot\frac{dw}{dt} \tag{6.1}$$

where $L_S$ is the mass of bone dry solid; $A$ is the drying surface, from which drying takes place.

However, in the case of through circulation drying, A is defined as the cross-sectional area of the bed, perpendicular to the direction of air flow. When the drying rate is plotted against moisture content (on dry basis), the rate curve is obtained and a typical rate curve is shown in Fig. 6.5.



ED is internal movement of moisture controls (second falling rate period)
DC is unsaturated surface drying (first falling rate)
CB is constant rate drying period
A'B and AB are unsteady drying (initial stage)

**Fig. 6.5** Rate curve.

When the solid to be dried is fully wet, the surface will be covered with a thin film of liquid and will have unbound moisture. If the air is unsaturated with a humidity of $Y$ and if the gas at the liquid surface is $Y_S$ (saturated humidity), the rate of drying at constant rate period is expressed as,

$N_C = k_y(Y_S - Y)$, where $k_y$ is the mass transfer coefficient.

### 6.5.1.2 Time of drying

From Eq. (6.1) we have,

$$N = \left(\frac{-L_S}{A}\right)\left(\frac{dX}{dt}\right)$$

Rearranging and integrating Eq. (6.1) to determine the time needed to dry the material from $X_1$ to $X_2$, we get,

$$t = \int_0^t dt = \left(\frac{-L_S}{A}\right)\int_{X_1}^{X_2}\frac{dx}{N} \tag{6.2}$$

**(a) The constant rate period:** The drying period is said to be constant rate period when both $X_1$ and $X_2$ are greater than critical moisture content $X_c$. Under such conditions, drying rate remains constant and $N = N_C$.

The Eq. (6.2) can now be rearranged as $\int_0^t dt = \left(\frac{-L_S}{AN_C}\right) \int_{X_1}^{X_2} dX$ and on integration this yields,

$$t = L_S \frac{(X_1 - X_2)}{AN_C} \tag{6.3}$$

**(b) The falling rate period:** If $X_1$ and $X_2$ are both less than $X_C$, the drying rate $N$ decreases with decrease in moisture content. Equation (6.2) can be integrated graphically by plotting $(1/N)$ in y-axis against moisture content $X$ in x-axis or by using a numerical technique. However, when $N$ varies linearly with $X$ in the region CE, the drying rate can be expressed mathematically as,

$$N = aX + b \tag{6.4}$$

where $a$ is the slope of the line and $b$ is a constant. The Eq. (6.2) can be integrated between the limits

$$t = 0,\ x = X_1$$

$$t = t,\ x = X_2 \text{ and we get}$$

$$t = \int_0^t dt = \frac{L_S}{A} \int_{X_2}^{X_1} \frac{dX}{(aX + b)} = \left(\frac{L_S}{aA}\right) \ln \left[\frac{(aX_1 + b)}{(aX_2 + b)}\right] \tag{6.5}$$

However, $N_1 = a\,X_1 + b$, $N_2 = a\,X_2 + b$ and $a = \left[\frac{(N_1 - N_2)}{(X_1 - X_2)}\right]$

Substituting these in Eq. (6.5) gives

$$t = \left[\frac{L_S(X_1 - X_2)}{A(N_1 - N_2)}\right] \ln \left[\frac{N_1}{N_2}\right] = \frac{L_S(X_1 - X_2)}{A \cdot N_m} \tag{6.6}$$

where $N_m$ is the logarithmic mean rate of drying.
In a specific case of drying from $X_c$ to $X^*$

$$N_C = aX_C + b \tag{6.7}$$

$$N^* = 0 = aX^* + b \tag{6.8}$$

Subtracting Eq. (6.8) from Eq. (6.7) gives,

$$N_C = a\ (X_C - X^*) \tag{6.9}$$

Also, subtracting Eq. (6.8) from Eq. (6.4), we get

$$N = a\ [X - X^*] \tag{6.10}$$

Eliminating $a$ in Eq. (6.10), using Eq. (6.9)

$$N = N_C \frac{(X - X^*)}{(X_C - X^*)} \tag{6.11}$$

Replacing $N_1$ and $N_2$ in Eq. (6.6) in terms of $N_C$ and $N^*$ and also $X_1$ and $X_2$ in terms of $X_C$ and $X^*$, we get,

$$t = \left[\frac{L_S(X_C - X^*)}{AN_C}\right] \ln \left\{\frac{[N_C(X_1 - X^*)(X_C - X^*)]}{[(X_C - X^*)\,N_C(X_2 - X^*)]}\right\}$$

$$t = \left[\frac{L_S(X_C - X^*)}{AN_C}\right] \ln \left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right] \tag{6.12}$$

## 6.6 PARAMETERS AFFECTING DRYING RATE DURING CONSTANT RATE DRYING PERIOD

Gas velocity, gas temperature, gas humidity and thickness of the drying solid are the parameters which affect drying rate.

### 6.6.1 Effect of Gas Velocity (G)

When radiation and conduction effects are present, the effect of gas rate will be less significant. However, when they are negligible, then drying rate $N_C$ is proportional to $G^{0.71}$ for parallel flow of gas and to $G^{0.37}$ for perpendicular flow of gas.

### 6.6.2 Effect of Gas Temperature

Increased air temperature, $T_G$ increases the driving force, $(T_G - T_S)$ for heat transfer and hence $N_C$ is directly proportional to $(T_G - T_S)$. $T_S$ is the surface temperature of drying solid and is assumed to be at the WBT condition during constant rate drying.

### 6.6.3 Effect of Gas Humidity

As the humidity of air decreases, the driving force $(Y_S - Y)$ available for mass transfer increase, and hence $N_C$ is proportional to $(Y_S - Y)$. $Y_S$ is the saturation humidity of air corresponding to $T_S$.

### 6.6.4 Effect of Thickness of Drying Solid

When heat transfer occurs through the solid, $N_C$ increases with decrease in solid thickness. However, if drying occurs from all surfaces, $N_C$ is independent of thickness.

## 6.7 MOISTURE MOVEMENT IN SOLIDS

When drying takes place, moisture moves from the inner core to the external surface and evaporates. The nature of movement influences the drying during the falling rate period and the following theories have been proposed to explain the moisture movement in solids.

### 6.7.1 Liquid Diffusion

Due to concentration gradients between the higher concentration in depths of the solid and the low concentration at the surface, moisture movement takes place. This type of phenomenon is exhibited by substances like soap, glue, gelatin, textiles and paper.

During constant rate period, the rate of moisture movement from inner core and the rate of removal of moisture from the surface balance each other. However, after sometime, dry spots appear on the surface resulting in unsaturated surface drying and then the moisture movement from the solid takes place which is entirely controlled by the diffusion rates within the solid. Whenever, the constant drying rates are very high, the drying substance may exhibit only diffusion controlled falling rate drying.

### 6.7.2 Capillary Movement

In some of the porous solids, moisture moves through the capillaries in them which is quite similar to the burning of lamp with wick. These capillaries extend from water reservoir to the drying surface. As the drying process is initiated, the moisture starts moving by capillarity to the drying surface and maintains a uniformly wetted surface, which corresponds to constant rate drying period. Subsequently air replaces the water and the wetted area at the surface also decreases leading to unsaturated surface drying. After sometime, when the sub-surface water also dries up, the liquid surface recedes into capillaries and water evaporates from there setting in second falling rate period. This phenomenon is exhibited by clays, paints and pigments.

### 6.7.3 Vapour Diffusion

When one surface of a wet solid is heated and the other surface allows the drying to take place, the moisture gets vapourised from the hot surface and diffuses outward as a vapour from the other surface.

### 6.7.4 Pressure Diffusion

When bound moisture is removed from a colloidal non-porous solid, it tends to shrink when the substance is dried very rapidly. The moisture present on the surface is removed very quickly and the moisture movement from the inner core to the outer surface will not be equal to the rate of removal of moisture from the surface. During this process, an impervious membrane forms and prevents the movement of moisture under such circumstances from the inner core to the surface. The outer surface will be fully dry whereas the inner core will be wet under such conditions. This phenomenon is called **case hardening**.

However, under certain circumstances, the shrinkage of outside layers of solid may also squeeze out moisture to the surface.

## 6.8 SOME MORE ASPECTS ON FALLING RATE DRYING

The moisture movement during the falling rate period is governed by either unsaturated surface drying or internal diffusion controlling mechanism.

### 6.8.1 Unsaturated Surface Drying

In this phase, the rate of drying will vary linearly with moisture content. The moisture removal mechanism is same as that in the constant rate period and the general effects of temperature, humidity, gas flow rate and thickness of the solid are the same as for constant rate drying.

### 6.8.2 Internal Diffusion Controlling

In this period of drying, the moisture movement is controlled by the pores in the drying substance. The drying rate decreases with decrease in moisture content.

## 6.9 THROUGH CIRCULATION DRYING

When a gas passes through a bed of solids, the drying zone varies as shown in Fig. 6.6. At the point, where the gas enters, maximum drying occurs and a zone of drying of bound moisture forms. In this zone there is a gradual rise in temperature. This zone is followed by a zone of drying unbound moisture. The temperature in this zone remains constant and the particles are at their wet bulb temperature. The zone of drying unbound moisture is followed by a zone of initial moisture concentration where the solids also remain at their initial temperature. The gas leaves the system fully saturated.

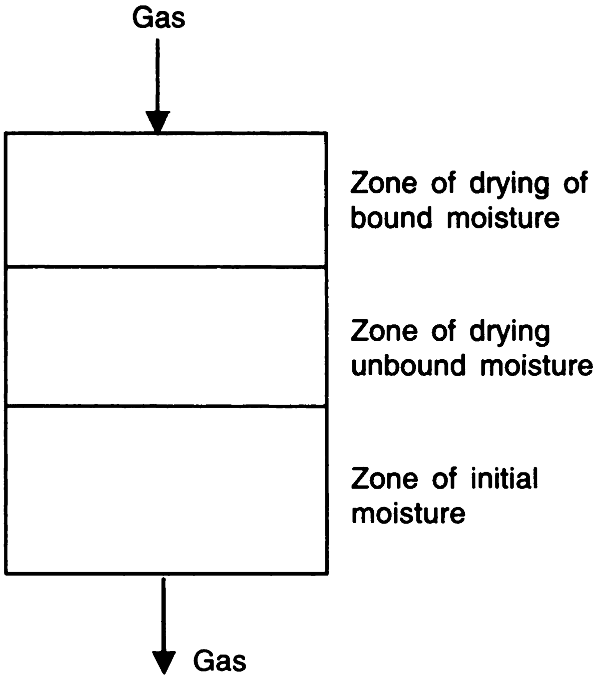


**Fig. 6.6** Through circulation drying.

### 6.9.1 The Rate of Drying of Unbound Moisture

Let us consider that a gas of humidity $Y_1$ enters the bed at a moisture free flow rate of $G_S$, kg/m$^2$hr. The maximum drying rate $N_{max}$ will occur if the gas leaving the bed is saturated at adiabatic saturation temperature and hence at the humidity $Y_{as}$,

$$N_{max} = G_S(Y_{as} - Y_1) \tag{6.13}$$

However, if $Y_2$ is the humidity of leaving air and $N$ is the drying rate, then

$$N = G_S(Y_2 - Y_1) \tag{6.14}$$

For a differential section of the bed, where the change in humidity for incoming air of humidity $Y_1$ is $dY$ and leaves at a humidity of "Y", the rate of drying $dN$ is given by

$$dN = G_S\, dY = k_y\, dS\, (Y_{as} - Y) \tag{6.15}$$

where $S$ is the interfacial surface per unit area of bed cross-section. If $a$ is the interfacial area per unit volume of bed and if $Z_S$ is the bed thickness, then

$$dS = a \cdot dZs \tag{6.16}$$

Substituting Eq. (6.16) in Eq. (6.15) and integrating it yields,

$$\int_{Y_1}^{Y_2}\left(\frac{dY}{Y_{as} - Y}\right) = \int_0^{Zs}\left(\frac{k_y a dZ_S}{G_S}\right) \tag{6.17}$$

$$\ln\left[\frac{(Y_{as} - Y_1)}{(Y_{as} - Y_2)}\right] = N_{tG} = \left(\frac{k_y a Z_S}{G_S}\right) \tag{6.18}$$

where $N_{tG}$ is the number of gas phase transfer units in the bed. From Eqs. (6.13), (6.14) and (6.15), we get

$$\frac{N}{N_{max}} = \left[\frac{(Y_2 - Y_1)}{(Y_{as} - Y_1)}\right] = 1 - \left[\frac{(Y_{as} - Y_2)}{(Y_{as} - Y_1)}\right]$$

$$= 1 - \exp^{-N_{tG}} = 1 - \exp^{-\left(\frac{k_y a Z_s}{Gs}\right)} \tag{6.19}$$

Using Eq. (6.19) we can predict drying rate when $k_y$ a is known.

### 6.9.2 Drying of Bound Moisture

In order to analyse the drying rate in this regime, it is preferable to make experimental investigation as both particle and gas characteristics play a significant role.

## 6.10 CONTINUOUS DIRECT HEAT DRIER

It is the one in which there is a continuous flow of solid and gas through the drier. The drying takes place as the solid moves through the drier.

### 6.10.1 Material and Energy Balance

Consider a continuous drier with gas flowing countercurrent to the flow of solids which is dried from a moisture content of $X_1$ to $X_2$. The flow rate of dry solids is $L_S$. The gas enters at a moisture free flow rate of $G_S$ and at a temperature of $t_{G_2}$ and a humidity of $Y_2$. It leaves at a temperature of $t_{G_1}$ and at a humidity of $Y_1$.

A schematic diagram shown in Fig. 6.7 indicates the flow of various streams and their conditions.

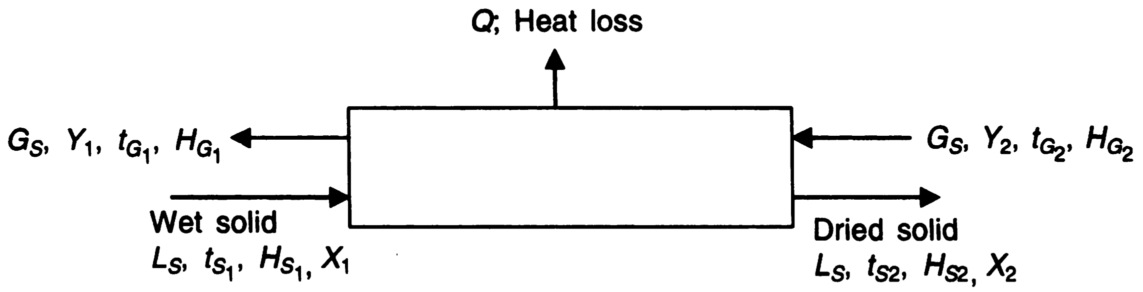


**Fig. 6.7** Schematic diagram of a continuous direct heat drier.

Balance for moisture gives,

$$L_S X_1 + G_S Y_2 = L_S X_2 + G_S Y_1 \tag{6.20}$$

i.e.

$$L_S (X_1 - X_2) = G_S (Y_1 - Y_2) \tag{6.21}$$

Enthalpy of the wet solids, $H_S$ is given by

$$H_S = C_S (t_S - t_0) + X(C_A)\,(t_S - t_0) + \Delta H \tag{6.22}$$

where $C_S$ is specific heat of solids, $C_A$ is the specific heat of moisture, $X$ is the moisture content and $\Delta H$ is the integral heat of wetting.
An energy balance gives,

Heat entering = Heat leaving + Heat loss

$$L_S H_{S_1} + G_S H_{G_2} = L_S H_{S_2} + G_S H_{G_1} + Q \tag{6.23}$$

For an adiabatic operation, where there is no heat loss, $Q = 0$. Sometimes the solids are conveyed through drier in some supports in which case the heat carried in and out by these materials should also be accounted.

### 6.10.2 Rate of Drying for Continuous Direct Heat Driers

Direct heat driers fall under two categories, namely, whether the high temperature or low temperature prevails in the drier.

#### 6.10.2.1 Drying at high temperature

Figure 6.8 shown below indicates the temperature profile of both solid and gas.

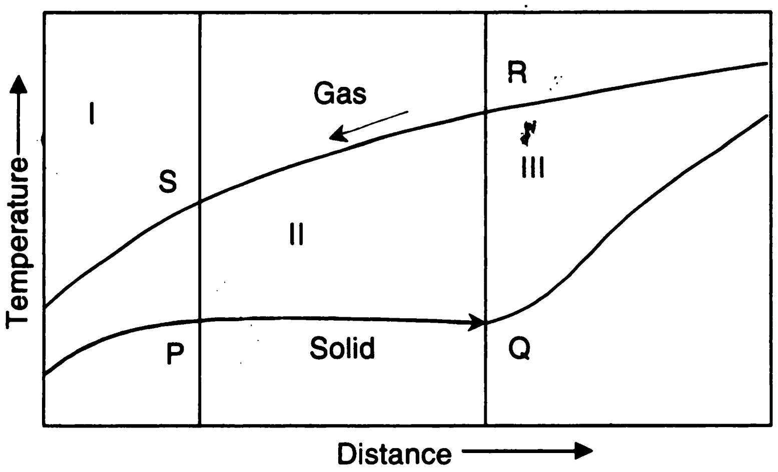


**Fig. 6.8** Temperature profile of gas and solid in a continuous drier.

For analysis, let us divide the drier into three zones. The zone I is a preheating zone, where the solid is heated and very little drying takes place. In zone II, the temperature of solid remains fairly constant and the surface and unbound moisture are evaporated. The point Q corresponds to the critical moisture content. In zone III, unsaturated surface drying and removal of bound moisture takes place. There is a sharp increase in the temperature of the solid. The zone II represents a major portion of the drier and it will be interesting to see how the gas temperature and humidity varies in this section.

The point R (see Fig. 6.9) denotes the condition of air entering zone II. When the drying takes place adiabatically without any heat loss, the variation of gas temperature and humidity is along the adiabatic saturation line $RS_1$. The solid temperature could be A (corresponding to point Q) or A′ (corresponding to point P).

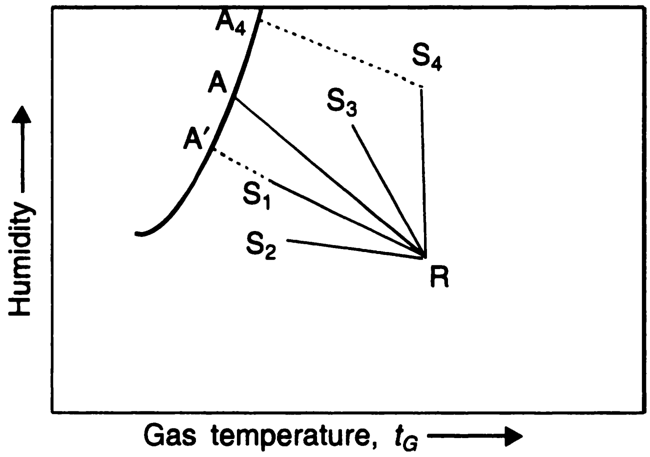


**Fig. 6.9** Temperature–humidity variation in a continuous drier.

However, if heat losses are there, then the path could be $RS_2$. In case the heat is added within the drier then the path would be $RS_3$. The path will be $RS_4$ if the gas temperature is kept constant and the solid temperature is likely to be $A_4$.

Now, let us make an energy balance. The heat lost $q_G$ by the gas is transferred partially to the solid $q$ and partially lost as $Q$. For a drier of differential length, $dz$

$$dq_G = dq + dQ \tag{6.24}$$

i.e.

$$dq = dq_G - dQ = U \cdot ds\,(t_G - t_S) \tag{6.25}$$

$$= U \cdot adz\,(t_G - t_S) \tag{6.26}$$

where $S$ is the interfacial surface/cross-sectional area, $a$ is the interfacial surface/drier volume, $U$ is the overall heat transfer coefficient, $t_G$ is the gas temperature and $t_S$ is the solid temperature.

$$dq = G_S \cdot C_S \cdot dt_G \tag{6.27}$$

where $G_S$ is the dry gas flow rate, $C_S$ is the humid heat of gas and $dt_G$ is drop in gas temperature.

i.e.
$$dq = G_S \cdot C_S \cdot dt_G = Ua\ (t_G - t_S)\ dz \tag{6.28}$$

$$dN_{toG} = \frac{dt_G}{(t_G - t_S)} = \left(\frac{Ua}{G_S C_S}\right) \tag{6.29}$$

i.e.
$$N_{toG} = \frac{\Delta t_G}{\Delta t_m} \tag{6.30}$$

i.e.
$$H_{toG} = \frac{G_S C_S}{Ua} \tag{6.31}$$

$\Delta t_m$ is logarithm of average temperature difference from gas to solid.

### 6.10.2.2 Drying at low temperatures

Since the drying takes place at low temperature the preheating of solid is not a major factor. The preheating zone merges with zone II (Refer Fig. 6.10). In zone II unbound and surface moisture are removed and the moisture content of leaving solid reaches critical moisture content, $X_C$ as in drying at high temperatures. The unsaturated surface drying and evaporation of bound moisture occurs in zone III. The humidity of incoming gas increases from $Y_2$ to $Y_C$ as it leaves zone III.

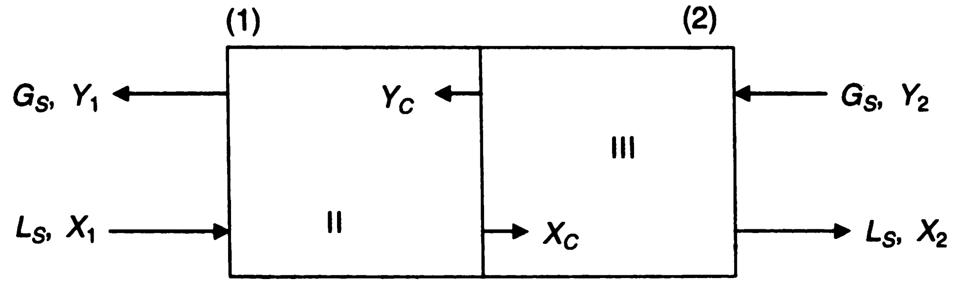


**Fig. 6.10** Continuous countercurrent drier of low temperature operation.

The retention time can be calculated using Eq. (6.1)

$$t = t_{\text{II}} + t_{\text{III}} \tag{6.32}$$

$$= \frac{L_S}{A}\left[\int_{X_C}^{X_1} \frac{dX}{N} + \int_{X_2}^{X_C} \frac{dX}{N}\right] \tag{6.33}$$

where $(A/L_S)$ is the specific exposed drying surface.

In zone II, $X > X_C$, the rate of drying $N = N_C$ is given by Eq. (6.2)

$$t_{\text{II}} = \frac{L_S}{A} \int_{X_C}^{X_1} \frac{dX}{[k_y(Ys - Y)]} \tag{6.34}$$

Also a material balance yields,

$$G_S \cdot dY = L_S \cdot dX \tag{6.35}$$

Substituting for $dX$ in Eq. (6.34) from Eq. (6.35), we get

$$t_{\mathrm{III}} = \frac{L_S}{A} \cdot \frac{G_S}{L_S} \int_{Y_C}^{Y_1} \frac{dY}{[k_y(Y_s - Y)]} \tag{6.36}$$

Assuming $Y_S$ to be constant (which will correspond to saturation humidity at the wet bulb temperature of incoming air and there is no heat loss).

$$t_{\mathrm{II}} = \left(\frac{L_S}{A}\right)\left(\frac{G_S}{L_S}\right)\left(\frac{1}{k_y}\right) \cdot \ln\left[\frac{(Y_S - Y_C)}{(Y_S - Y_1)}\right] \tag{6.37}$$

In zone III, $X < X_C$ we have from Eq. (6.11)

$$N = \frac{[N_C(X - X^*)]}{(X_C - X^*)}$$

Applying Eq. (6.2) to constant rate period, we get

$$N_C = k_y\,(Y_S - Y) \tag{6.38}$$

Substituting for $N_C$ in Eq. (6.11) from Eq. (6.38), we get,

$$N = \frac{k_y(Y_S - Y)(X - X^*)}{(X_C - X^*)} \tag{6.39}$$

Substituting in Eq. (6.33) from Eq. (6.39), we get

$$t_{\mathrm{III}} = \frac{L_S(X_C - X^*)}{A \cdot k_y} \int_{X_2}^{X_C} \frac{dX}{(Y_S - Y)(X - X^*)} \tag{6.40}$$

This cannot be integrated directly as $Y$ also varies with $X$.
A simple material balance for moisture across any section yields

$$G_S[Y - Y_2] = L_S\,[X - X_2]$$

$$Y = Y_2 + (X - X_2)\left(\frac{S_S}{G_S}\right) \tag{6.41}$$

The humidity $Y_S$ can be determined from the humidity $Y_1$ of inlet gas. $X^*$ can be determined experimentally for the given humidity $Y_2$. By substituting for $Y$ in Eq. (6.40) from Eq. (6.41) we can estimate $t_{\mathrm{III}}$.
However, for the case where $X^* = 0$, we shall analyse Eq. (6.40).
By differentiating Eq. (6.41), we get,

$$G_S \cdot dY = L_S \cdot dX \tag{6.42}$$

From Eq. (6.40) using Eq. (6.42), we get

$$t_{\text{III}} = \left(\frac{L_S}{A}\right)\left(\frac{X_C}{k_y}\right)\left(\frac{G_S}{L_S}\right)\int_{Y_2}^{Y_C} \frac{dY}{(Y_S - Y)\left[(Y - Y_2)\left(\frac{G_S}{L_S}\right) + X_2\right]}$$

$$= \left(\frac{L_S}{A}\right)\left(\frac{X_C}{k_y}\right)\left(\frac{G_S}{L_S}\right)\frac{1}{\left[(Y_S - Y_2)\frac{G_S}{L_S} + X_2\right]}\ln\left[\frac{X_C\,(Y_S - Y_2)}{X_2(Y_S - Y_C)}\right] \tag{6.43}$$

The above Eq. (6.43) cannot be applied when the internal diffusion controls the drying process. Since we have assumed that in the falling rate period, drying rate varies linearly with free moisture content. Whenever the drying is controlled by internal diffusion, $t_{\text{III}}$ can be determined experimentally. In the case of parallel flow driers,

$$t_{\text{II}} = \left(\frac{L_S}{A}\right)\left(\frac{G_S}{L_S}\right)\left(\frac{1}{k_y}\right)\int_{Y_1}^{Y_C} \frac{dY}{(Ys - Y)}$$

$$= \left(\frac{G_S}{L_S}\right)\left(\frac{L_S}{A}\right)\left(\frac{1}{k_y}\right)\ln\left[\frac{(Y_S - Y_1)}{(Y_S - Y_C)}\right] \tag{6.44}$$

$$t_{\text{III}} = L_S\frac{(X_C - X^*)}{A\,.\,ky}\int_{X_2}^{X_C} \frac{dX}{[(Y_S - Y)(X - X^*)]}$$

$$= \left(\frac{G_S}{L_S}\right)\left(\frac{L_S}{A}\right)\left(\frac{X_C}{k_y}\right)\frac{1}{\left[(Y_S - Y_C)\frac{G_S}{L_S} - X_2\right]}\ln\left[\frac{X_C(Y_S - Y_2)}{X_2(Y_S - Y_C)}\right] \tag{6.45}$$

## 6.11 DRYING EQUIPMENTS

Drying equipments are classified on different basis, as mentioned below.

### 6.11.1 Based on Contact between Drying Substance and Drying Material

(a) Direct contact: In these dryers, there is a direct contact between hot gas and the drying substance. For example, rotary dryer, spray dryer, etc.

(b) Indirect contact: There is no direct contact between gas and drying substance. For example, drum dryer, mechanically agitated dryer, etc.

### 6.11.2 Based on the Type of Operation

(a) Batch dryer: Tray dryer, freeze dryer, etc.

(b) Continuous dryer: rotary dryer, mechanically agitated dryer and tunnel dryer.

### 6.11.3 Based on the Nature of Substance being Dried

(a) Materials in sheets or masses carried on conveyors or trays,
  (i) Batch dryers: Atmospheric tray and vacuum tray.
  (ii) Continuous dryers: Tunnel.

(b) Materials which are granular or loose,
  (i) Rotary dryers: rotary and roto–Louvre
  (ii) Turbo dryers
  (iii) Conveyors
  (iv) Filter–dryer combinations

(all these dryers operate on continuous basis)

(c) Materials in continuous sheets,
  (i) Cylinder dryer
  (ii) Festoon dryers

(d) Materials in the form of pastes or sludges or caking crystals,
  (i) Atmospheric agitator dryers (mechanically agitated)
  (ii) Vacuum dryers

(e) Materials in the form of solution,
  (i) Drum dryers: Atmospheric and vacuum
  (ii) Spray dryers

(f) Special dryers,
  (i) Freeze dryers
  (ii) Infrared dryers
  (iii) Dielectric dryers

Some of the dryers used in industries have been described as follows.

### 6.11.4 Atmospheric Compartment Dryers

This dryer consists of a rectangular chamber with insulating walls and a door. The chamber is partitioned to enable the loading of materials in these compartments. In some of the designs, trucks or cars may run into the dryer with provisions for closing doors. These dryers also have provisions for heating the air inside and also circulating it over the trays/shelves. Dampers are also provided to regulate the flow of air.

### 6.11.5 Vacuum Compartment Dryer

It consists of a rectangular chamber with a number of shelves. These shelves are hollow and during operation they are filled with steam or hot water. Steam enters through a steam inlet manifold and there is a provision to remove the condensate and also non condensables. The material to be dried is loaded in trays and placed on shelves. The door is closed after placing the material to be dried and vacuum is created in the chamber by means of a vacuum pump. The water released from the substance is condensed in a condenser placed between the dryer and the vacuum pump. If the vapour from the drying substance has a value, these dryers enable us in the recovery of these vapours.

These dryers are used for materials that cannot withstand high temperatures, as in the case of atmospheric compartmental dryer, such as pharmaceuticals. They are also suitable for systems where in the contact between air or other oxidizing gases are to be avoided.

### 6.11.6 Tunnel Dryers

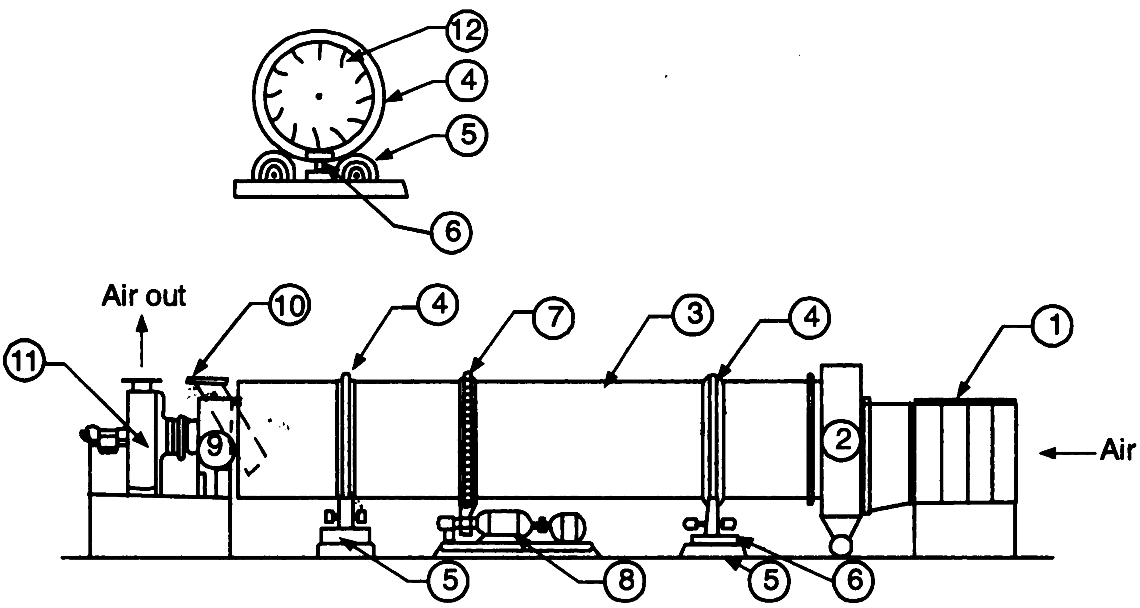
The compartmental dryers operate on batch basis. The dryer is in the form of a long tunnel. However, if a continuous operation is desired the material to be dried can be loaded in trucks or cars and sent through the tunnel. The air flow could be either co-current or countercurrent at right angles to the path of travel of trucks. Heaters are also provided in the dryer in different sections so that the air may be sent through the trucks, taken to a re-heater and sent back again to the trucks in the same section.

These are used for drying bricks, ceramic products and other material which have to be dried rather slowly but in large quantities.

### 6.11.7 Rotary Dryers

It consists of a cylindrical shell slightly inclined to the horizontal and mounted so that it can be rotated. The feed material enters at the elevated end of the dryer and due to the rotation of the shell, the material slowly moves towards the lower end and finally leaves the dryer. Inside the shell, flights are present which help in lifting the solids and showering them over air stream. Heat can be supplied from outside to the shell of the dryer or through the hot air which flows either co-currently or countercurrently. The rotating shell carries forged tires which ride on rolls. Thrust rolls prevent the endwise travel of the shell. The shell is driven by a gear arrangement which is driven by a motor. The shell rotates at a peripheral speed of 20 to 25 m/min.

This dryer is used for drying granular or crystalline substances which has to be handled in bulk and cannot be used for sticky material. A typical drier is shown in Fig. 6.11.



1. Air Heater 2. Stationary hood 3. Dryer shell 4. Tires 5. Supporting rolls 6. Thrust rolls
7. Drive gear 8. Motor and speed reducer 9. Air discharge hood 10. Feed chute
11. Discharge 12. Flights

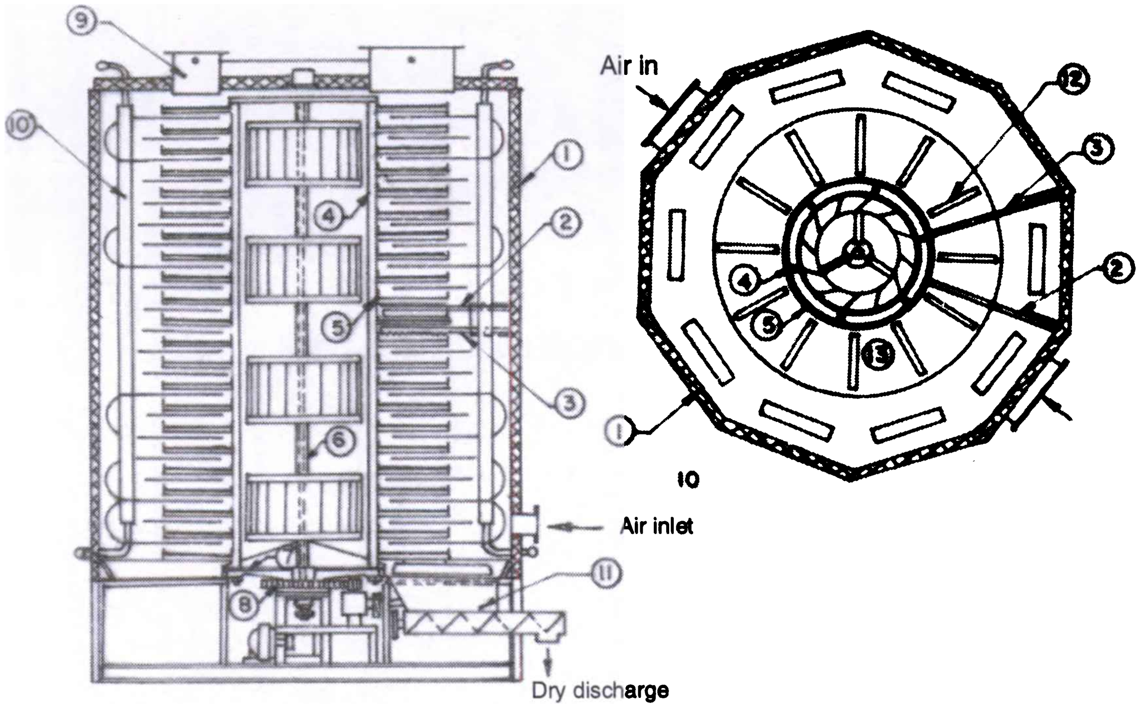
**Fig. 6.11** Rotary dryer.

### 6.11.8 Roto-Louvre Dryer

This is a modified form of rotary dryer described in Section 6.11.7. The dryer consists of an outer cylindrical shell and a tapered inner shell made of number of overlapping plates. The space between the shell and the overlapping plates is divided into longitudinal channels by ribs. These channels are open at the large end to receive hot air and are closed at the smaller end. The louvres are inclined against the flow of rotation so that they do not lift the solid but merely serve to keep the solid from dropping down the channels. The air from these channels flow through the bed of solids. As the material entering the dryer has a higher moisture content, the tapering provides a thinner bed to start with and hence a lower resistance for gas/air flow. As the air flows through a bed of solids, the air comes more nearly in equilibrium with the material and drying rate is faster and hence a shorter size of the shell. Further, as the material is not lifted and showered down, the tendency for degradation of material is less. Here, the material rolls along the bottom of inner surface, and reaches the product outlet region.

### 6.11.9 Turbo Dryer

This dryer, as shown in Fig. 6.12, is of vertical orientation with a cylindrical or polygonal shell. At the bottom we have a base plate which is driven by a gear coupled to a motor assembly. From this base plate vertical rods rise to the top, which are suitably connected to a guide bearing. Around these rods are cylindrical bands of sheet metal to which are attached wedge shaped trays. The whole assembly rotates as a single unit. Feed enters through the feed opening, fills the trays and as they rotate they pass under a fixed leveling scraper. After one revolution they pass under another scraper that scrapes the charge on the tray through the transfer slots to fall to the tray below. Every tray is provided with a



1. Casing 2. Scraper 3. Leveler blade 4. Rods 5. Metal Bands 6. Fan shaft 7. Base casting
8. Drive gears 9. Feed opening 10. Re-heaters 11. Chute for dried product 12. Transfer slots 13. Trays

**Fig. 6.12** Turbo dryer.

leveler scraper and a scraper to push the solids to next lower tray. The dried material leaving the last tray is finally scraped into a hopper and discharged out of the dryer by a screw conveyor. Air enters the drier at the bottom of the shell. With the help of fans attached to the central shaft, air is circulated over the material on trays. As it picks up moisture, its temperature drops and the saturation level goes up. In order to maintain a fairly steady temperature, the circulating air is continuously reheated with the finned tube heaters. The humidified air finally leaves the top of the dryer. The major advantage of this dryer is, it occupies a lesser floor space and low power consumption compared to rotary dryer and Roto–Louvre dryers.

### 6.11.10 Conveyor Dryers

It is a type of tunnel dryer in which the granular solids to be dried is loaded in a conveyor belt of wire screen of a suitable mesh. The belt screen retains the solid but permits the flow of air through the conveyor and also the solid. In some cases the flow of air could be downwards also.

### 6.11.11 Filter Dryer Combination

When the material to be dried is in suspension in a liquid, a rotary continuous filter is used. It consists of a drum which is fed at the top surface of the drum. After the filtration, hot air is blown through the filtered material, so that filtration and drying take place at the same time.

## 6.11.12 Cylinder Dryers

A typical cylinder drier is shown in Fig. 6.13. This consists of steam heated rolls with facilities to remove the condensate. The rolls are driven by motors. The material to be dried in the form of continuous sheets of paper or textiles passes over these rolls and gets spooled to the final product spool. The speed of the train of rolls must suitably be adjusted, through it is complicated, to compensate the shrinkage of drying material. A typical dryer assembly in a paper manufacturing unit may have 50 to 75 rolls.

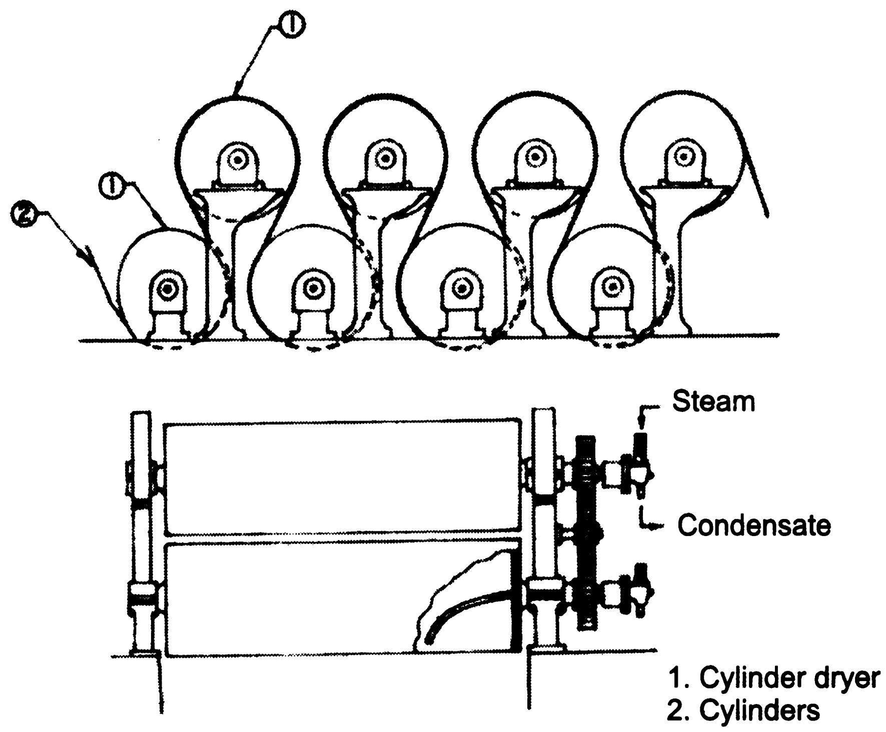


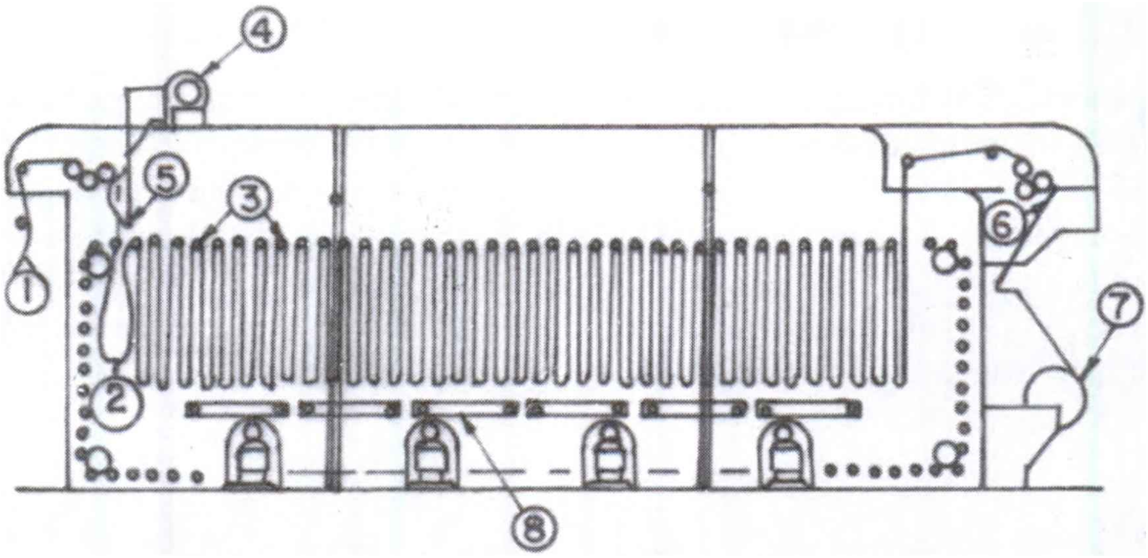
**Fig. 6.13** Cylinder dryer.

## 6.11.13 Festoon Dryers

In this dryer, as shown in Fig. 6.14, wet sheet of material passes over a series of rolls and drops down to form a series of loops. A continuous chain conveyor carrying cross bars is so timed with respect to the speed of the sheet that the loops drop down to a certain predetermined length pass as the next roll comes along to catch the next loop. The proper loop formation is ensured by the loop blower. The material as it leaves the dryer is fully dried and rolls on to the product roll. Hot air needed for drying is obtained by passing air through a bank of finned tube heaters. The moist air leaves through air discharge port.

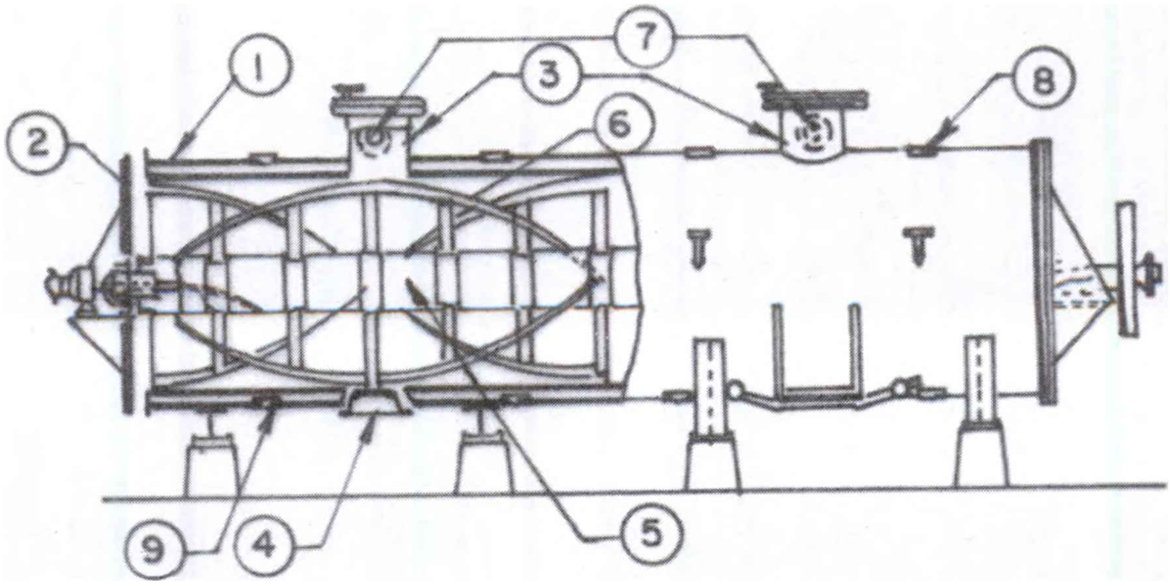
## 6.11.14 Mechanically Agitated Dryer

A mechanically agitated dryer, as shown in Fig. 6.15, consists of a horizontal jacketed cylindrical shell. Inside is a central shaft carrying agitator blades



1. Entering sheet 2. Festoons 3. Cross bars 4. Loop blower
5. Air nozzle 6. Exit sheet 7. Product roll 8. Heaters.

**Fig. 6.14** Festoon dryer.



1. Jacketed shell 2. Heads 3. Feed inlet 4. Discharge door 5. Agitator shaft
6. Agitator blades 7. Vapour outlets 8. Steam inlets 9. Condensate outlets

**Fig. 6.15** Mechanically agitated dryer.

arranged in a way, that one set of blades moves the material in one direction and the other set in the opposite direction. Heat needed for drying is sent through the jacket. The central shaft is rotated by a motor. The feed enters through the feed inlet point and leaves from the product outlet point.

## 6.11.15 Drum Dryer

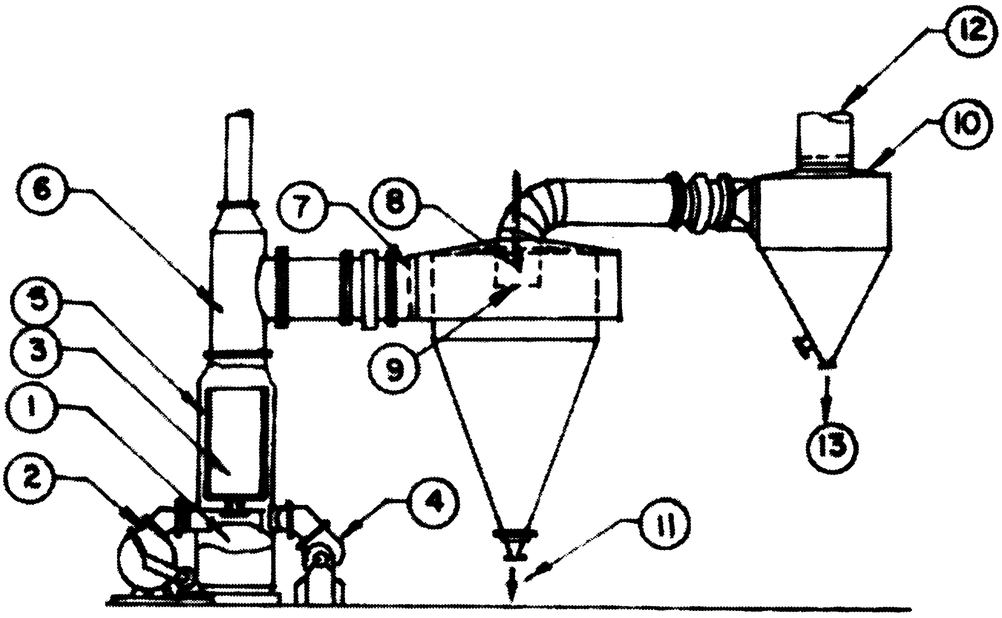
A typical double roll drum dryer consists of two large steam heated cast iron rolls with a smooth external surface. The rolls rotate toward each other and the liquid to be dried is fed directly into the V-shaped space between the rolls. In small units generally there is no special feeding device whereas in larger dryers swinging pipe or a traveling discharge pipe is used to keep the feed uniform. Thickness of the material deposited on the rolls is determined by the space between them. Doctor knife placed near the top of the rolls on the outside, removes the dried product which is taken away with a conveyor. In very small capacity driers, only one roll will be present which will be dipping, in a feed trough. The doctor knife at the lower part of the drum removes the dried solid.

### 6.11.16 Vacuum Drum Dryers

When the substance being dried is heat sensitive, the drying can be accomplished under vacuum. A typical vacuum drum dryer consists of a single drum made of cast iron. The drum is heated with steam with facility to remove the condensate. The pool of feed liquid is maintained at the bottom of the unit and is pumped up to the spreader trough which is very close to the bottom section of the drum. The dried product is removed by a doctor knife and taken out subsequently by a conveyor. The entire unit is maintained under vacuum.

### 6.11.17 Spray Dryers

Spray dryers are extensively used for drying solutions, pharmaceuticals, detergent products, fruit juices and milk. Many designs are available. In a typical design shown in Fig. 6.16, the liquid to be dried is atomised and introduced into the large drying chamber with a conical bottom. The droplets are dispersed into a stream of hot air. The particles of liquid evaporate rapidly and dry before they can be carried to the sides of the chamber and the bulk of the dried powder which results, falls to the bottom of the chamber from where they are removed by a stream of air to the dust collector. For atomisation, either spray nozzles or rapidly rotating disks are used. Nozzles



1. Burner 2. Primary air blower 3. Combustion chamber 4. Secondary air blower
5. Secondary air passage 6. Hot air pipe 7. Hot air value 8. Spray nozzle
9. Air discharge pipe 10. Dust collector 11. Main product discharge
12. Air discharge 13. Dust discharge

**Fig. 6.16** Spray dryer.

are relatively inflexible in their operating characteristics and do not permit even moderate variation in liquid flow rates with out large changes in droplet size. They are also subjected to rapid erosion and wear. Rotating disks are about 30 cm in diameter and rotate at speeds of 3000 to 12000 rpm. They also easily handle variations in liquid flow rates.

### 6.11.18 Freeze Drying

Substances which cannot be heated even to moderate temperatures, are frozen by exposure to very cold air and placed in a vacuum chamber, where the moisture sublimes and is pumped off by steam jet ejectors or mechanical vacuum pumps. This is used for drying fish, vegetables like peas, vitamins and other heat sensitive materials.

### 6.10.19 Infrared Drying

It has been used in the drying of paint films on objects such as automobile bodies. The radiation is usually supplied by infrared lamps and the material to be dried travels in a tunnel lined with banks of such lamps. This process is suitable only for the drying of thin films on the surface of the material to be dried and never for cases where the water or solvent to be removed is deep inside the solid. It is a very expensive drying operation.

### 6.10.20 Dielectric Drying

In this operation the object to be dried is passed through a very high frequency (2 to 100 × $10^6$ cycles) electrostatic field. This generates heat uniformly throughout the object. Its only important field of application is in polymerising the resin that forms the bond between layers of plywood which is a rare drying operation. However, some people will disagree in calling it as a very expensive drying operation.

## WORKED EXAMPLES

1. Air containing 0.005 kg of water vapour per kg of dry air is preheated to 52°C in a dryer and passed to the lower shelves. It leaves these shelves at 60% relative humidity and is reheated to 52°C and passed over another set of shelves, again leaving at 60% relative humidity. This is again repeated for the third and fourth sets of shelves, after which the air leaves the dryer. On the assumption that the material in each shelf has reached the wet bulb temperature and heat loss is negligible, estimate: (i) the temperature of the material on each tray; (ii) the amount of water removed, in kg/hr, if 300 $m^3$/min of moist air leaves the dryer.

   **Solution.**

   (i) Air leaves the pre-heater of the dryer at 325 K
   Humidity of incoming air = 0.005 kg water/kg dry air

It enters the first shelf. So, the wet bulb temperature = 25°C

Moisture is removed along wet bulb temperature line till 60% R.H. is reached. This gives the exit condition of air from first shelf.

From the chart, Humidity of air leaving first shelf = 0.016 kg water/kg dry air. Dry bulb temperature of exit air is at 27°C and is at a humidity of 0.016 kg water/kg dry air. This air is again heated to 52°C dry bulb temperature in second heater. So, air leaves heater at 52°C and at a humidity of 0.016 kg water/kg dry air. When it leaves the second shelf, the corresponding dry bulb temperature is 34°C and the humidity is 0.023 kg water/kg dry air. This air enters the third shelf after preheating to 52°C. Similarly for third shelf, exit air has a humidity of 0.028 kg water/kg dry air and has a dry bulb temperature of 39°C. The air leaving the fourth shelf has a humidity of 0.032 kg water/kg dry air and a dry bulb temperature of 42°C. (The figure is only indicative and does not correspond to actual one.)

The solid temperatures correspond to WBT and they are 23°C, 27°C, 32°C and 34°C respectively. **Ans.**

(ii) Final moist air conditions: ($Y'$) = 0.032 kg water/kg dry air

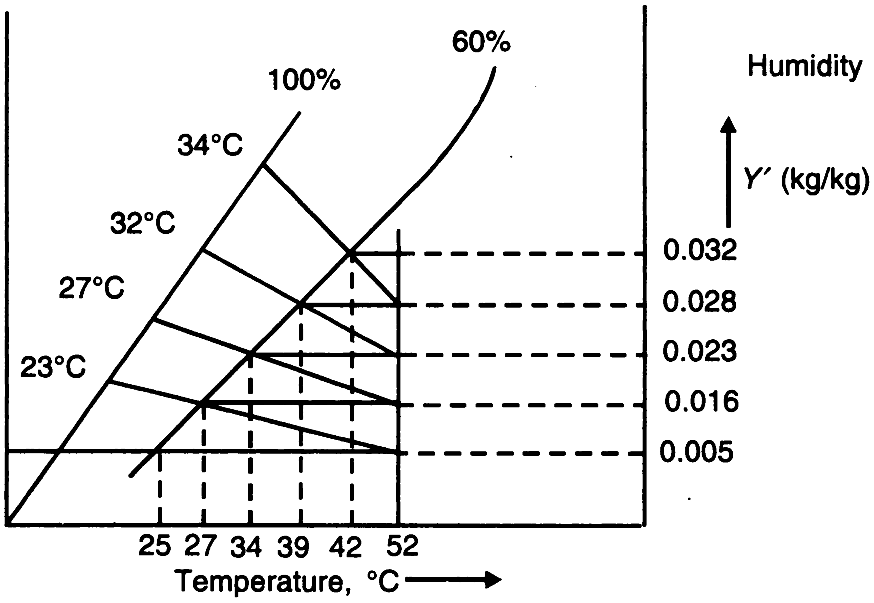


**Fig. 6.17** Example 1 Humidity vs temperature.

Dry bulb temperature = 42°C

$$V_H = 8315\left(\frac{1}{M_{\text{air}}}\right)+\left(\frac{Y'}{M_{\text{water}}}\right)\left[\frac{(t_G+273)}{P_t}\right]$$

$$V_H = 8315\left(\frac{1}{28.84}\right)+\left(\frac{0.032}{18}\right)\left[\frac{(42+273)}{1.013\times10^5}\right]$$

$V_H$ = 0.945 m$^3$/kg dry air.

$$\text{Amount of dry air leaving/hr} = \frac{(300\times60)}{0.945} = 1.905\times10^4 \text{ kg}$$

Water removed/hr = 1.905 × $10^4$ (0.032 – 0.005) = 514.35 kg/hr. **Ans.**

2. A batch of the solid, for which the following table of data applies, is to be dried from 25 to 6 percent moisture under conditions identical to those for which the data were tabulated. The initial weight of the wet solid is 350 kg, and the drying surface is 1 $m^2/8$ kg dry weight. Determine the time for drying.

| $X \times 100, \frac{\text{kg moisture}}{\text{kg dry solid}}$ | $N \times 100, \frac{\text{kg moisture evaporated}}{\text{hr} \cdot \text{m}^2}$ |
|---|---|
| 35 | 30 |
| 25 | 30 |
| 20 | 30 |
| 18 | 26.6 |
| 16 | 23.9 |
| 14 | 20.8 |
| 12 | 18 |
| 10 | 15 |
| 9 | 9.7 |
| 8 | 7 |
| 7 | 4.3 |
| 6.4 | 2.5 |

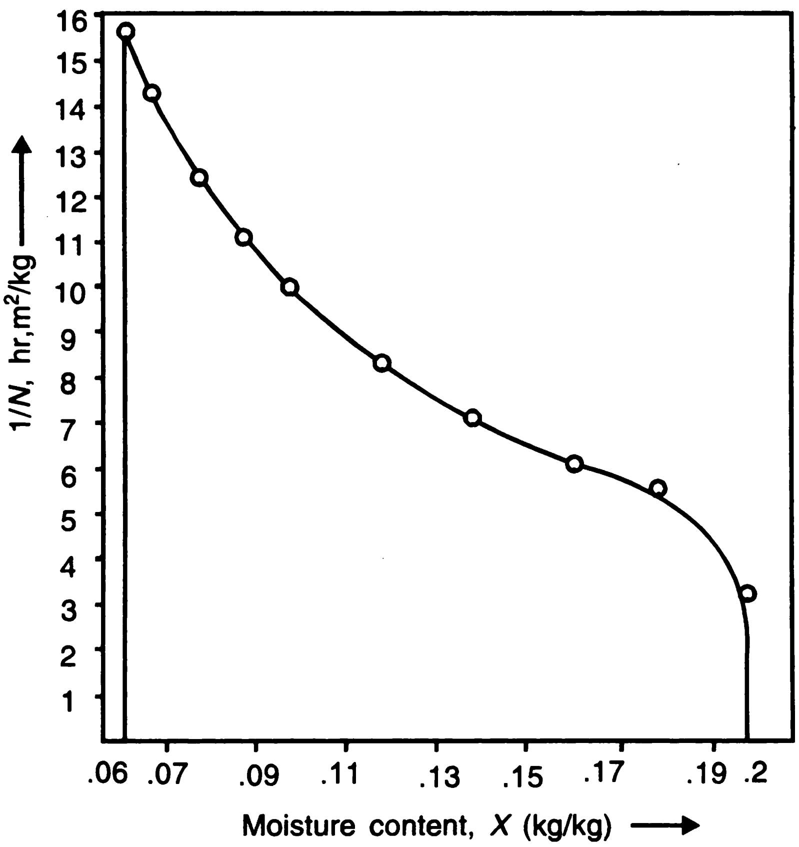


Fig. 6.18 Example 2 1/$N$ vs $X$ for falling rate period.

**Solution.**

$$X_1 = \frac{0.25}{(1 - 0.25)} = 0.333, \qquad X_2 = \frac{0.06}{(1-0.06)} = 0.0638,$$

Initial weight of wet solid = 350 kg

Initial moisture content = 0.333 kg moisture/kg dry solid

So, total moisture present in wet solid (initially) = 350 × 0.25 = 87.5 kg moisture

Weight of dry solid, $L_S = 262.5$ kg

$$A = \frac{262.5}{8} = 32.8125 \text{ m}^2, \quad \text{or} \quad \frac{L_S}{A} = 8 \text{ kg/m}^2$$

$$X_{Cr} = 0.20, \qquad N_C = 0.3 \text{ kg/m}^2\text{hr}$$

So for constant rate period, drying time is

$$t_I = \frac{L_S}{AN_C}[X_1 - X_{Cr}] = \left[\frac{262.5}{(32.8125 \times 0.3)}\right][0.333 - 0.2] = 3.55 \text{ hr.}$$

For falling rate period, we are finding drying time graphically,

| $X$ | 0.2 | 0.180 | 0.16 | 0.14 | 0.120 | 0.100 | 0.090 | 0.080 | 0.07 | 0.064 |
|---|---|---|---|---|---|---|---|---|---|---|
| $1/N$ | 3.33 | 5.56 | 6.25 | 7.14 | 8.32 | 10.00 | 11.11 | 12.5 | 14.29 | 15.625 |

Area = 1.116,

$$\therefore \quad \text{Time} = \text{Area under the curve} \times \frac{L_S}{A} = 1.116 \times \frac{L_S}{A} = 1.116 \times 8 = 8.928 \text{ hr.}$$

$\therefore$ Total time = 8.928 + 3.55 = 12.478 hr. **Ans.**

3. A wet slab of material weighing 5 kg originally contains 50 percent moisture on wet basis. The slab is 1 m × 0.6 m × 7.5 cm thick. The equilibrium moisture is 5 per cent on wet basis. When in contact with air, the drying rate is given in the table below. Drying takes place from one face only.
   (i) Plot the drying rate curve and find the critical moisture content.

| Wet slab wt, kg | 5.0 | 4.0 | 3.6 | 3.5 | 3.4 | 3.06 | 2.85 |
|---|---|---|---|---|---|---|---|
| Drying rate, kg/(hr)(m$^2$) | 5.0 | 5.0 | 4.5 | 4.0 | 3.5 | 2.00 | 1.00 |
| $X$, Dry basis | 1.00 | 0.6 | 0.44 | 0.4 | 0.36 | 0.224 | 0.14 |

   (ii) How long will it take to dry the wet slab to 15 percent moisture on wet basis?

**Solution.** Weight of wet solid = 5 kg

Moisture content = 0.50 moisture/kg wet solid

$$= \frac{0.5}{[(0.5 \text{ moisture}) + (0.5 \text{ dry solid})]} = \frac{0.5}{(1 - 0.5)}$$

$\therefore X_1 = 1$ = moisture/dry solid

For 5 kg wet solid, moisture = 5 × 0.5 = 2.5 kg
Weight of dry solid = 5 – 2.5 = 2.5 kg

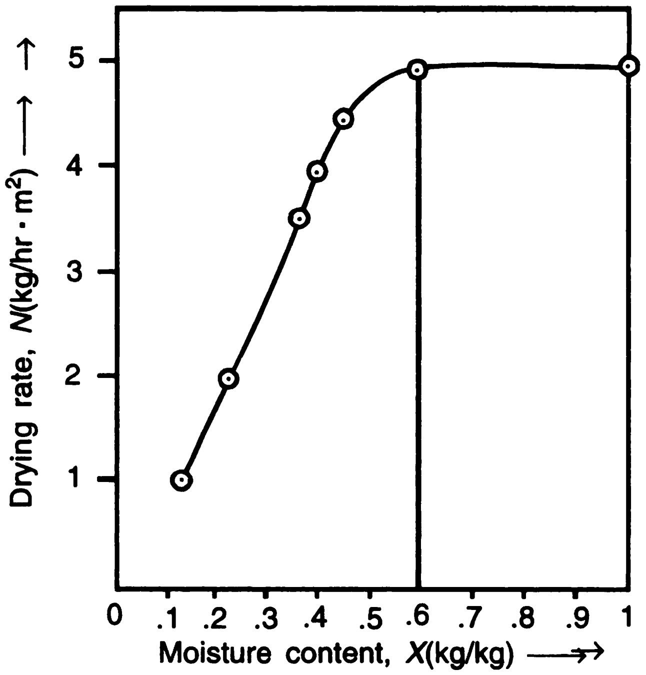


**Fig. 6.19(a)** Example 3 Drying rate curve.

$$x^* = 0.05, \qquad X^* = \frac{0.05}{(1-0.05)} = 0.0526$$

$$\text{Moisture content in dry basis} = \frac{\text{weight of wet solid} - \text{weight of dry solid}}{\text{weight of dry solid}}$$

**Ans.**

(i) $X_{Cr} = 0.6$ kg moisture/kg dry solid.

(ii) From $X = 0.6$ to 0.44 the falling rate curve is non-linear and from $X = 0.44$ to 0.14, falling rate period is linear.

$$X_2 = \frac{0.15}{(1 - 0.15)} = 0.1765.$$

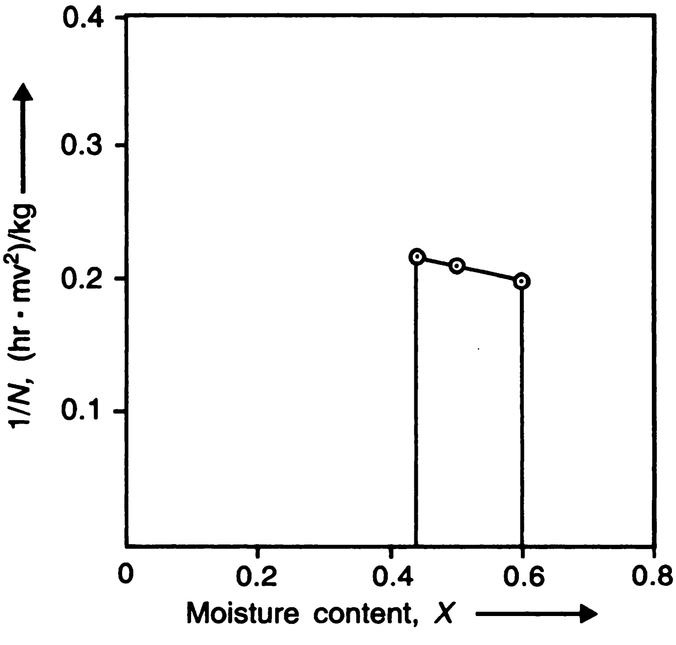


**Fig. 6.19(b)** Example 3 1/N vs X.

So, we can find time for drying from 0.6 to 0.44 graphically and then for $X = 0.44$ to 0.1765, we can go in for analytical solution as the '$N$' vs '$X$' relation is linear.

Time taken for constant rate drying period (From $X = 1$ to $X = 0.6$)

$$t_I = \left[\frac{L_S}{AN_C}\right] [X_1 - X_{Cr}] = \left[\frac{2.5}{(5 \times 0.6)}\right] [1 - 0.6] = 0.333 \text{ hr}$$

(from $X = 0.44$ to 0.1765)

$$t_{II} = \frac{L_S}{AN_C} \times \left\{(X_{Cr} - X^*) \ln \left[\frac{(X_1 - X^*)}{(X_2' - X^*)}\right]\right\}$$

$$\left(X^* = \frac{0.05}{(1 - 0.05)} = 0.0526\right)$$

$$t_{II} = \left[\frac{2.5}{(5 \times 0.6)}\right] \times \left\{(0.6 - 0.0526) \times \ln \left[\frac{(0.44 - 0.0526)}{(0.176 - 0.0526)}\right]\right\}$$

$= 0.522$ hr **Ans.**

From graph, $t_{III}$ (From $X = 0.6$ to $X = 0.44$) $= \dfrac{(0.0336 \times 2.5)}{0.6} = 0.14$ hr.

Total time $= t_I + t_{II} + t_{III} = 0.333 + 0.522 + 0.14 = 0.995$ hr or 59.58 min. **Ans.**

**4.** Data on drying rate curve of a particular solid is given below. The weight of the dry material in the solid is 48.0 kg/m$^2$. Calculate the time required to dry the material from 25% to 8% moisture (dry basis).

*Data:*

| $X$ | 0.30 | 0.20 | 0.18 | 0.15 | 0.14 | 0.11 | 0.07 | 0.05 |
|---|---|---|---|---|---|---|---|---|
| $N$ | 1.22 | 1.22 | 1.14 | 0.90 | 0.80 | 0.56 | 0.22 | 0.05 |

where $X$ is the moisture content in kg water/kg dry solid and $N$ is the drying rate in kg/(hr) (m$^2$).

**Solution.**

$N_C = 1.22$ kg/m$^2$hr, $X_{Cr} = 0.2$, $X_1 = 0.25$, $X_2 = 0.08$, $L_S/A = 48$ kg/m$^2$

Time taken for constant rate drying period,

$$t_I = \left[\frac{L_S}{AN_C}\right] [X_1 - X_{Cr}]$$

$$t_I = 48 \frac{[0.25 - 0.2]}{1.22} = 1.967 \text{ hr.}$$

| $X$ | 0.18 | 0.15 | 0.14 | 0.11 | 0.07 | 0.05 |
|---|---|---|---|---|---|---|
| $1/N$ | 0.8772 | 1.111 | 1.25 | 1.7857 | 4.545 | 20 |

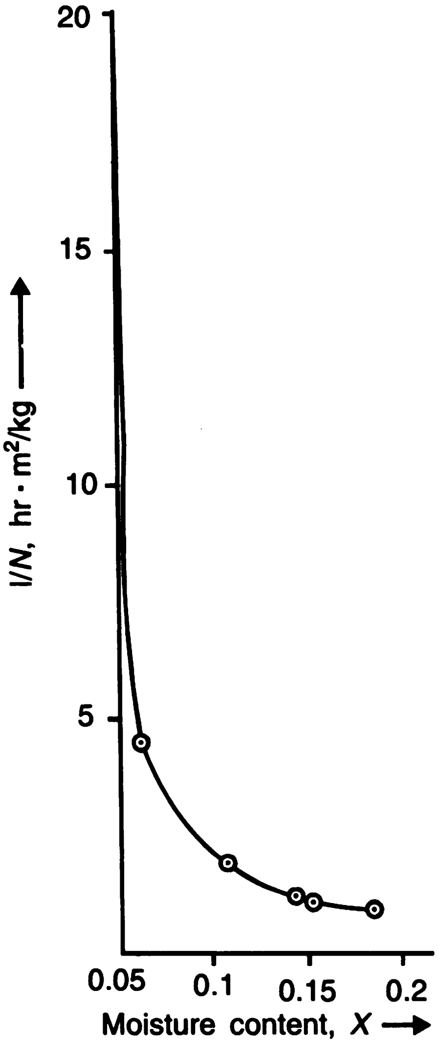


**Fig. 6.20** Example 4 $1/N$ vs $X$ for falling rate.

$$\text{Area under the curve} = 14 \times 0.025 \times 1 = 0.35$$

$$t_{\text{II}} = 0.35 \times 48 = 16.8 \text{ hr}$$

$$\text{Total time taken} = t_{\text{I}} + t_{\text{II}} = 1.967 + 16.8 = 18.767 \text{ hr}$$ **Ans.**

5. In a drying experiment, a tray drier, containing a single tray of 1 m$^2$ area is used, to dry crystalline solids. The following data have been collected:

| *Sl. No* | *Time* (hr) | *Weight of wet material*, kg |
|---|---|---|
| 1 | 0.0 | 5.314 |
| 2 | 0.4 | 5.238 |
| 3 | 0.8 | 5.162 |
| 4 | 1.0 | 5.124 |
| 5 | 1.4 | 5.048 |
| 6 | 1.8 | 4.972 |
| 7 | 2.2 | 4.895 |
| 8 | 2.6 | 4.819 |
| 9 | 3.0 | 4.743 |
| 10 | 3.4 | 4.667 |
| 11 | 4.2 | 4.524 |
| 12 | 4.6 | 4.468 |
| 13 | 5.0 | 4.426 |
| 14 | 6.0 | 4.340 |
| 15 | Infinity | 4.120 |

| *Sl. No.* | *Time*, hr | *Weight of wet material*, kg | *Moisture content dry basis* | *Drying rate*, kg/hr · m$^2$ |
|---|---|---|---|---|
| 1 | 0 | 5.314 | 0.29 | — |
| 2 | 0.4 | 5.238 | 0.271 | 0.19 |
| 3 | 0.8 | 5.162 | 0.253 | 0.19 |
| 4 | 1.0 | 5.124 | 0.244 | 0.19 |
| 5 | 1.4 | 5.048 | 0.225 | 0.19 |
| 6 | 1.8 | 4.972 | 0.206 | 0.19 |
| 7 | 2.2 | 4.895 | 0.188 | 0.1925 |
| 8 | 2.6 | 4.819 | 0.169 | 0.19 |
| 9 | 3.0 | 4.743 | 0.151 | 0.19 |
| 10 | 3.4 | 4.667 | 0.133 | 0.19 |
| 11 | 4.2 | 4.524 | 0.098 | 0.179 |
| 12 | 4.6 | 4.468 | 0.084 | 0.14 |
| 13 | 5 | 4.426 | 0.074 | 0.105 |
| 14 | 6 | 4.340 | 0.053 | 0.086 |
| 15 | Infinity | 4.120 | 0.0 | — |

(i) Calculate and plot drying rates. Find the critical moisture content.

(ii) If dry air is available at 40°C with an absolute humidity of 0.01 kg/kg dry air and the drier is maintained at 90°C, calculate the amount of air required in first 2 hours. Assume the air is heated up to 90°C and the dry air leaves the drier at 90°C with 5% saturation.

(iii) Test the consistency of the falling rate period. (Choose critical moisture content and any one point in falling rate period.)

**Solution.**

From the above data after getting the rate curve it is clear that $X_{Cr} = 0.11$. The loss in weight is due to moisture evaporated. After two hours the weight is

$$(4.972 + 4.895)/2 = 4.934 \text{ kg}$$

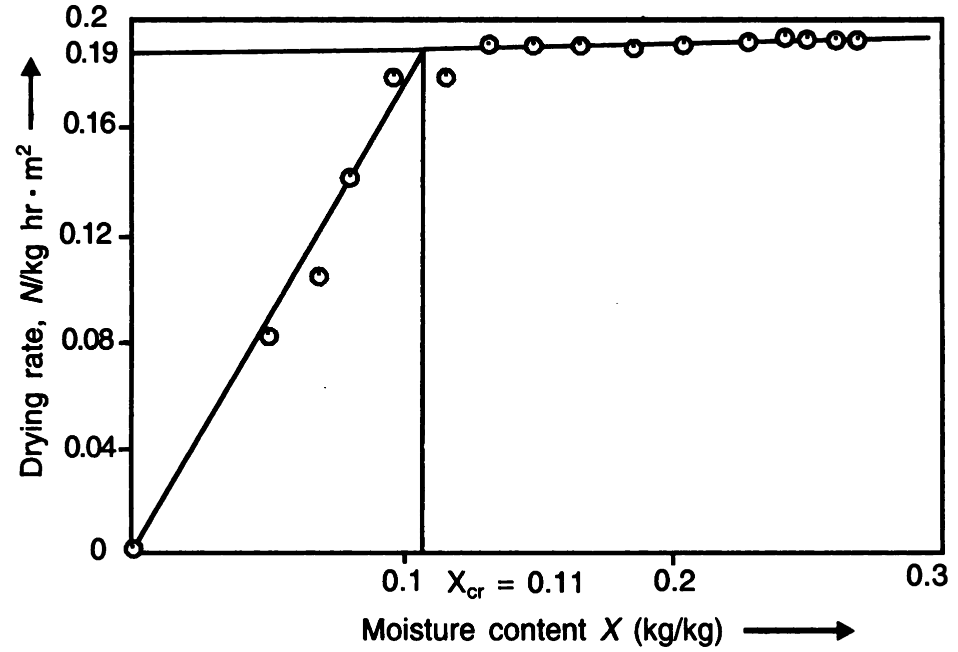


**Fig. 6.21** Example 5 Drying rate curve.

The water evaporated in 2 hours is

$$5.314 - 4.934 = 0.38 \text{ kg}$$

Humidity of incoming air = 0.01 kg/kg

Humidity of leaving air = 0.03 kg/kg (for 90°C with 5% saturation)

Water carried away by air = $G_s(Y_{\text{out}} - Y_{\text{in}})$

$$0.38 = G_s\,(0.03 - 0.01)$$

Therefore,

$$G_s \text{ is } = \frac{0.38}{0.02} = 19 \text{ kg of air for 2 hours.}$$

$$t_{\text{falling}} = \frac{L_S}{AN_C} \times \left\{(X_{Cr} - X^*) \times \ln\left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]\right\}$$

Let us choose readings (11) and (13) to check the consistency

$$= \left[\frac{4.12}{(1) \times (0.19)}\right] \times (0.11 - 0) \times \ln\left[\frac{0.098}{0.074}\right] = 0.67 \text{ hours}$$

Here, $X^*$ is taken as 0. Actual time is 0.8 hour. **Ans.**

**6.** A woolen cloth is dried in a hot air dryer from an initial moisture content of 100% to a final moisture content of 10%. If the critical moisture content is 55% and the equilibrium moisture content is 6% (at dryer condition), calculate the saving in drying time if the material is dried to 16% instead of 10%. All other drying conditions remain the same.
All moisture contents are on the dry basis.

$X_1$ = 100%, $X_{\text{Cr}}$ = 55%, $X_2$ = 10%, $X^*$ = 6% all are on dry basis.

**Solution.**

$X_2'$, = 16%

$$t_{\text{T1}} = \frac{L_S}{AN_C}[X_1 - X_{\text{Cr}}] + \left\{\frac{L_S}{AN_C}(X_{\text{Cr}} - X^*) \ln\left[\frac{(X_{\text{Cr}} - X^*)}{(X_2 - X^*)}\right]\right\}$$

$$t_{\text{T}} = \frac{L_S}{AN_C}[X_1 - X_{\text{Cr}}] + \left\{\frac{L_S}{AN_C}(X_{\text{Cr}} - X^*) \ln\left[\frac{(X_{\text{Cr}} - X^*)}{(X_2 - X^*)}\right]\right\} \qquad \ldots (1)$$

$$t_{\text{T}}' = \frac{L_S}{AN_C}[X_1 - X_{\text{Cr}}] + \left\{\frac{L_S}{AN_C}(X_{\text{Cr}} - X^*) \ln\left[\frac{(X_{\text{Cr}} - X^*)}{(X_2' - X^*)}\right]\right\} \qquad \ldots (2)$$

i.e. (1)/(2) is,

$$\frac{t_T}{t'_T}=\frac{[X_1-X_{Cr}]+\left\{(X_{Cr}-X^*)\ln\left[\dfrac{X_{Cr}-X^*}{X_2-X^*}\right]\right\}}{[X_1-X_{Cr}]+\left\{(X_{Cr}-X^*)\ln\left[\dfrac{X_{Cr}-X^*}{X'_2-X^*}\right]\right\}}$$

$$\frac{t_T}{t_{T'}}=\frac{[1-0.55]+\left\{(0.55-0.06)\ln\left[\dfrac{(0.55-0.06)}{(0.1-0.06)}\right]\right\}}{[1-0.55]+\left\{(0.55-0.06)\ln\left[\dfrac{(0.55-0.06)}{(0.16-0.06)}\right]\right\}}$$

$$\frac{t_T}{t'_T}=\frac{1.678}{1.2287}$$

$$\frac{t_T}{t'_T}=1.3657$$

$$\frac{t'_T}{t_T}=0.7322$$

$$t'_T=0.7322\ t_T$$

The reduction in drying time is,

$$\frac{(t_T-t'_T)}{t'_T}=0.2678$$

Thus, the time reduces by 26.78%. **Ans.**

7. A filter cake is dried for 5 hours from an initial moisture content of 30% to 10% (wet basis). Calculate the time required to dry the filter cake from 30% to 6% (wet basis).

Equilibrium moisture content = 4% on dry basis

Critical moisture content = 14% on dry basis

Assume that the rate of drying in the falling rate period is directly proportional to the free moisture content.

**Solution.**

$$x_1=0.3,\quad x_f=0.10,\quad X_1=\frac{0.3}{(1-0.3)}=0.4286,\quad X_{Cr}=0.14$$

$$X_f=X_2=\frac{0.1}{(1-0.1)}=0.111,\quad X^*=0.04,$$

Then,
$$X_2'=\frac{0.06}{(1-0.06)}=0.064$$

$$t_T=\left[\frac{L_S}{AN_C}\right][(X_1-X_{Cr})+\left\{(X_{Cr}-X^*)\ \ln\left[\frac{(X_{Cr}-X^*)}{(X_2-X^*)}\right]\right\}$$

$$5 = \left[\frac{L_S}{AN_C}\right](0.4286 - 0.14) + \left\{(0.14 - 0.04)\ln\left[\frac{(0.14 - 0.04)}{(0.111 - 0.04)}\right]\right\}$$

Thus, $L_S/AN_C = 15.487$ hrs

$$t_T' = \frac{L_S}{AN_C}\left[(X_i - X_{Cr}) + \left\{(X_{Cr} - X^*)\ln\left[\frac{(X_{Cr} - X^*)}{(X_2' - X^*)}\right]\right\}\right]$$

$$t_T' = 15.487\left[(0.4286 - 0.14) + \left\{(0.14 - 0.04)\ln\left[\frac{(0.14 - 0.04)}{(0.064 - 0.04)}\right]\right\}\right]$$

$t_T' = 6.68$ hrs **Ans.**

**8.** 1000 kg dry weight of non-porous solid is dried under constant drying conditions with an air velocity of 0.75 m/s, so that the surface area of drying is 55 m$^2$. The critical moisture content of the material may be taken as 0.125 kg water/kg dry solids?

(i) If the initial rate of drying is 0.3 g/m$^2 \cdot$ s. How long will it take to dry the material from 0.15 to 0.025 kg water/kg dry solid?

(ii) If the air velocity were increased to 4.0 m/s, what would be the anticipated saving in time if surface evaporation is controlling.

**Solution.**

$L_S = 1000$ kg, Air velocity = 0.75 m/s, $A = 55$ m$^2$, $X_{Cr} = 0.125$, $X_1 = 0.15$, $X_2 = 0.025$. Assume $X^* = 0$, $N_C = 0.3$ g/m$^2 \cdot$ s or $0.3 \times 10^{-3}$ kg/m$^2$ s

$$t_T = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1 - X_{Cr}) + \left\{(X_{Cr} - X^*)\ln\left[\frac{(X_{Cr} - X^*)}{(X_2 - X^*)}\right]\right\}\right]$$

$$t_T = \left[\frac{1000}{(55 \times 0.3 \times 10^{-3})}\right] \times \left[(0.15 - 0.125) + \left\{(0.125 - 0) \ln\left[\frac{(0.125 - 0)}{(0.025 - 0)}\right]\right\}\right]$$

$= 13708$ s

$t_T = 3.8077$ hr

(ii) Assuming only surface evaporation, and assuming air moves parallel to surface

$\because$ $N_C \propto G^{0.71}$ and $G = V \times \rho$

i.e. $N_C \propto V^{0.71}$

$$\therefore \quad \frac{N_{C1}}{N_{C2}} = \frac{(V_1)^{0.71}}{(V_2)^{0.71}}$$

$$\frac{(0.3 \times 10^{-3})}{N_{C2}} = \frac{(0.75)^{0.71}}{(4)^{0.71}}$$

$N_{C2} = 0.985 \times 10^{-3}$ kg/m² s.

$$t_T = \left[\frac{1000}{(55 \times 0.985 \times 10^{-3}}\right] \times$$

$$\left[(0.15 - 0.125) + \left\{(0.125 - 0) \ln \left[\frac{(0.125-0)}{(0.025-0)}\right]\right\}\right] \times \frac{1}{3600}$$

$= 1.1597$ hrs.

So, time saved = 3.8077 – 1.1597 = 2.648 hr **Ans.**

**9.** A plant wishes to dry a certain type of fibreboard. To determine drying characteristics, a sample of 0.3 × 0.3 m size with edges sealed was suspended from a balance and exposed to a current of hot dry air. Initial moisture content was 75%. The sheet lost weight at the rate of $1 \times 10^{-4}$ kg/s until the moisture content fell to 60%. It was established that the equilibrium moisture content was 10%. The dry mass of the sample was 0.90 kg. All moisture contents were on wet basis. Determine the time for drying the sheets from 75% to 20% moisture under the same drying conditions.

**Solution.**

$x_1 = 0.75$, $x^* = 0.1$, $x_{Cr} = 0.6$ $L_S = 0.90$ kg,

$A = (0.3 \times 0.3) \times 2$ (both upper and lower surfaces are exposed) $= 0.18$ m².

$A \times N_C = 10^{-4}$ kg/s,

$x_1 = 0.75$, $x_2 = 0.2$, $X_{Cr} = \frac{0.6}{0.4} = 1.5$,

$X_1 = 3$, $X_2 = 0.25$, $X^* = \frac{0.1}{0.9} = 0.111$

$$t_T = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1 - X_{Cr}) + \left\{(X_{Cr} - X^*) \ln \left[\frac{(X_{Cr} - X^*)}{(X_2 - X^*)}\right]\right\}\right]$$

$$t_T = \frac{0.90}{10^{-4}} \times \left[(3 - 1.5) + \left\{(1.5 - 0.111) \ln \left[\frac{(1.5 - 0.111)}{(0.25 - 0.111)}\right]\right\}\right]$$

$= 11.74$ hr **Ans.**

**10.** A commercial drier needed 7 hours to dry a moist material from 33% moisture content to 9% on bone dry basis. The critical and equilibrium moisture content were 16% and 5% on bone dry basis respectively. Determine the time needed to dry the material from a moisture content of 37% to 7% on bone dry basis if the drying conditions remain unchanged.

**Solution.**

$X_1 = 0.33$, $X^* = 0.05$, $X_{Cr} = 0.16$, $X_2 = 0.09$, $X_1' = 0.37$,

$X_2' = 0.07$, $t_T = 7$ hrs

$$t_T = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1 - X_{Cr}) + \left\{(X_{Cr} - X^*) \ln \left[\frac{(X_{Cr} - X^*)}{(X_2 - X^*)}\right]\right\}\right]$$

$$7 = \frac{L_S}{AN_C} \times \left[(0.33-0.16) + \left\{(0.16-0.05)\ln\left[\frac{(0.16-0.05)}{(0.09-0.05)}\right]\right\}\right]$$

$$\frac{L_S}{AN_C} = 24.8866 \text{ h}$$

Now, $X_1 = 0.37$, $X_2 = 0.07$

$$t_T = 24.8866 \times \left[(0.37-0.16) + \left\{(0.16-0.05)\ln\left[\frac{(0.16-0.05)}{(0.07-0.05)}\right]\right\}\right]$$

$= 9.893$ h **Ans.**

11. A slab of paper pulp 1.5 m × 1.5 m × 5 mm, is to be dried under constant drying conditions from 65% to 30% moisture (wet basis) and the critical moisture is 1.67 kg free water/kg dry pulp. The drying rate at the critical point has been estimated to be 1.40 kg/(m$^2$)(hr). The dry weight of each slab is 2.5 kg. Assuming drying to take place from the two large faces only, calculate the drying time to be provided.

**Solution.**

$x_1 = 0.65$, $L_S = 2.5$ kg,

$A = (1.5 \times 1.5) \times 2$ (drying takes place from both the larger surface only) $= 4.5$ m$^2$.

$N_C = 1.4$ kg/m$^2$hr, $x_2 = 0.3$, $X_{Cr} = 1.67$,

$$X_1 = \frac{0.65}{(1-0.65)} = 1.857,\quad X_2 = \frac{0.3}{(1-0.3)} = 0.4286,$$

Assuming $X^* = 0$

$$t_T = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1 - X_{Cr}) + \left\{(X_{Cr} - X^*)\ln\left[\frac{(X_{Cr}-X^*)}{(X_2-X^*)}\right]\right\}\right]$$

$$t_T = \left(\frac{2.5}{4.5 \times 1.4}\right) \times [1.857-1.67] + \left\{(1.67-0)\ln\left[\frac{(1.67-0)}{(0.4286-0)}\right]\right\}$$

$t_T = 0.976$ h **Ans.**

12. A slab of paper pulp 1.5 m × 1.5 m × 5 mm, thick is to be dried under constant drying conditions from 15% to 8.5% moisture (dry basis). The equilibrium moisture is 2.5% (dry basis) and the critical moisture is 0.46 kg free water kg dry pulp. The drying rate at the critical point has been estimated to be 1.40 kg/(m$^2$)(hr). Density of dry pulp is 0.22 gm/cc. Assuming drying to take place from the two large faces only, calculate the drying time to be provided.

**Solution.**

$X^* = 0.025$, $N_C = 1.4$ kg/m$^2$hr, $X_{Cr} = 0.46$, $X_1 = 0.15$, $X_2 = 0.085$,
Density of dry pulp = 0.22 g/cc, $A = (1.5 \times 1.5) \times 2 = 4.5$ m$^2$,

Volume of material = $1.5 \times 1.5 \times 0.5 = 1.125 \times 10^{-2}$ m$^3$,

$\therefore \quad L_S = 1.125 \times 10^{-2} \times 0.22 \times 10^3 = 2.475$ kg

$$t_T = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1 - X_{Cr}) + \left\{(X_{Cr} - X^*) \ln \left[\frac{(X_{Cr} - X^*)}{(X_2 - X^*)}\right]\right\}\right]$$

But here initial moisture is less than the $X_{Cr}$. So there is no constant rate drying period and only falling rate period is observed.

$$\therefore \quad t_T = \frac{L_S}{AN_C}[X_{Cr} - X^*] \ln \left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right].$$

$$t_T = \left\{\left[\frac{2.475}{4.5 \times 1.4}\right] \times [0.46 - 0.025] \ln \left[\frac{(0.15 - 0.025)}{(0.085 - 0.025)}\right]\right\}$$

$= 0.125$ hr or $7.526$ min **Ans.**

**13.** Under constant drying conditions, a filter cake takes 5 hours to reduce its moisture content from 30% to 10% on wet basis. The critical moisture is 14% and the equilibrium moisture 4%, both on dry basis. Assuming the rate of drying in the falling rate period to be directly proportional to the free moisture content, estimate the time required to dry the cake from 30% to 6% moisture on wet basis.

**Solution.**

$x_1 = 0.3, \quad x_2 = 0.1,$

$X_1 = 0.3/0.7 = 0.4286, \quad X^* = 0.04, \quad X_{Cr} = 0.14,$

$X_2 = 0.1/0.9 = 0.111, \quad t_T = 5$ hrs,

$x_1' = 0.3$ and hence, $X_1' = 0.4286$. $x_2' = 0.06$ and $X_2' = 0.0638$,

$$5 = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1 - X_{Cr}) + \left\{(X_{Cr} - X^*) \ln \left[\frac{(X_{Cr} - X^*)}{(X_2 - X^*)}\right]\right\}\right] \quad \ldots (1)$$

$$t_T' = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1' - X_{Cr}) + \left\{(X_{Cr} - X^*) \ln \left[\frac{(X_{Cr} - X^*)}{(X_2' - X^*)}\right]\right\}\right] \quad \ldots (2)$$

Dividing (1)/(2)

$$\frac{5}{t_T'} = \frac{[0.4286 - 0.14] + \left\{0.14 - 0.04) \ln \left[\frac{(0.14 - 0.04)}{(0.111 - 0.04)}\right]\right\}}{[0.4286 - 0.14] + \left\{(0.14 - 0.04) \ln \left[\frac{(0.14 - 0.04)}{(0.0638 - 0.04)}\right]\right\}}$$

$$\frac{5}{t_T'} = 0.3228/0.4321$$

Hence,

$t_T' = 6.69$ h. **Ans.**

**14.** Sheet material, measuring 1 m$^2$ and 5 cm thick, is to be dried from 45% to 5% moisture under constant drying conditions. The dry density of the material is 450 kg/m$^3$ and its equilibrium moisture content is 2%. The available drying surface is 1 m$^2$. Experiments showed that the rate of drying was constant at 4.8 kg/(hr)(m$^2$) between moisture contents of 45% and 20% and thereafter the rate decreased linearly. Calculate the total time required to dry the material from 45% to 5%. All moisture contents are on wet basis.

**Solution.**

$A = 1\text{ m}^2$, 5 cm thick, $x_i = 0.45$, $x^* = 0.02$, $x_{Cr} = 0.2$, $x_2 = 0.05$,

$$X^* = \frac{0.02}{(1 - 0.02)} = 0.02041, \quad N_C = 4.8\text{ kg/m}^2\text{hr}, \quad X_{Cr} = 0.25,$$

$$X_1 = \frac{0.45}{(1 - 0.45)} = 0.818, \quad X_2 = 0.0526.$$

Density of dry pulp = 450 kg/m$^3$

Volume of material = $1 \times 5 \times 10^{-2} = 0.05\text{ m}^3$

$L_S = 450 \times 0.05 = 22.5$ kg

$$t_T = \left(\frac{L_S}{AN_C}\right) \times \left[(X_1 - X_{Cr}) + \left\{(X_{Cr} - X^*) \ln\left[\frac{(X_{Cr} - X^*)}{(X_2 - X^*)}\right]\right\}\right]$$

$$t_T = \left[\frac{22.5}{(1 \times 4.8)}\right] \times \left[(0.818 - 0.25) + \left\{(0.25 - 0.02041) \ln\left[\frac{(0.25 - 0.02041)}{(0.0526 - 0.02041)}\right]\right\}\right]$$

$= 4.78$ hr **Ans.**

15. Wet solids containing 120 kg/hour of dry stuff are dried continuously in a specially designed drier, cross-circulated with 2,000 kg per hour of dry air under the following conditions:

Ambient air temperature = 20°C
Exhaust air temperature = 70°C
Evaporation of water = 150 kg/hr
Outlet solids moisture content = 0.25 kg/hr
Inlet solids temperature = 15°C
Outlet solids temperature = 65°C
Power demand = 5 kW
Heat loss = 18 kW

Estimate heater load per unit mass of dry air and fraction of this heat used in evaporation of moisture.

Data:

Mean specific heat of dry air = 1 kJ kg$^{-1}$ K$^{-1}$
Enthalpy of saturated water vapour = 2,626 kJ per kg
Mean specific heat of dry materials = 1.25 kJ kg$^{-1}$ K$^{-1}$
Mean specific heat of moisture = 4.18 kJ kg$^{-1}$ K$^{-1}$.

**Solution.**

Total quantity of solid 120 kg dry stuff

Air used is 2000 kg/hr dry air

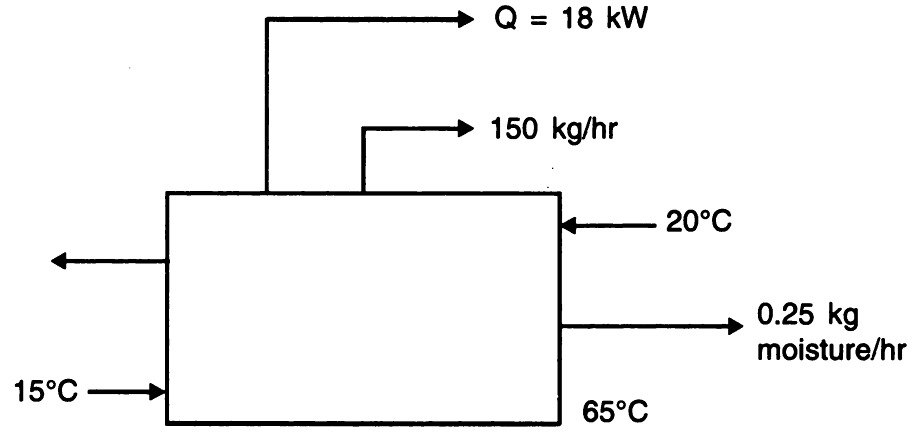


**Fig. 6.22** Example 15.

Basis: 1 hour

Heat required for heating 150 kg water from 15°C to 65°C

$$150 \times 4.18 \times (65 - 15) = 31350 \text{ kJ}$$

Heat required for 150 kg water evaporation

$$150 \times 2626 = 393900 \text{ kJ}$$

Heat required for heating air from 20°C to 70°C

$$2000 \times 1 \times (70 - 20) = 100000 \text{ kJ}$$

Heat required for heating moisture in solid from 15°C to 65°C

$$0.25 \times 4.18 \times (65 - 15) = 52.25 \text{ kJ}$$

Heat required for heating dry solid from 15°C to 65°C

$$120 \times 1.25 \times (65 - 15) = 7500 \text{ kJ}$$

Heat lost = 18 × 3600 = 64800 kJ

So, total heat required/hr

393900 + 100000 + 52.25 + 7500 + 64800 = 566252.25 kJ/hr. = 157.3 kW

i.e. 166 kW of heat is needed for 2000 kg/hr of dry air.

$$\text{Heat required/mass of dry air} = \frac{157.3}{2000} = 0.0787 \text{ kW}$$

(i) Heat needed for evaporation = [393900 + 31350]/3600 = 118.13 kW **Ans.**

(ii) Fraction of this heat needed for evapouration = $\frac{118.13}{157.3} = 0.751$ or 75.1%. **Ans.**

**16.** A drum drier is being designed for drying of a product from an initial total moisture content of 12% to final moisture content of 4%. An overall heat transfer coefficient (*U*) 1700 W/m$^2$C is being estimated for the product. An average temperature difference between the roller surface and the product of 85°C will be used for design purpose. Determine the surface area of the roller required to provide a production rate of 20 kg product per hour.

**Solution.**

Initial moisture content = 12%

Final moisture content = 4%

Production rate = 20 kg final product/hour

4 kg moisture is present in 100 kg product

In 20 kg product, weight of moisture = $\frac{(4 \times 20)}{100}$ = 0.8 kg

Dry solid weight = 20 – 0.8 = 19.2 kg

Total initial moisture content = $19.2 \times \left[\frac{0.12}{(1 - 0.12)}\right]$ = 2.6182 kg

Water evaporated = 2.6182 – 0.8 = 1.8182 kg/hour

$\lambda_S$ at 85°C = 2296.1 kJ/kg

Heat required = $W \times \lambda_S$ (Assuming the solid mixture enters at 85°C and only moisture removal by evaporation is alone considered)

= 1.8182 × 2296.1 = 4174.73 kJ/hour

$U \times A \times \Delta T = W \times \lambda_S$

$1700 \times A \times 85 = 4174730/3600$

$\therefore \quad A = 8.025 \times 10^{-3}$ m$^2$ or 80.25 cm$^2$. **Ans.**

**17.** A sample of porous sheet material of mineral origin is dried from both sides by cross circulation of air in a laboratory drier. The sample was 0.3 m × 0.3 m and 6 mm thick and edges were sealed. The air velocity is 3 m/s. DBT and WBT of air were 52°C and 21°C respectively. There was no radiation effect. Constant rate drying was $7.5 \times 10^{-5}$ kg/s until critical moisture content of 15% (on wet basis) was obtained. In the falling rate period, rate of drying fell linearly with moisture content until the sample was dry. The dry weight of the sheet was 1.8 kg. Estimate the time needed for drying similar sheets 1.2 m × 1.2 m × 12 mm thick from both sides from 25% to 2% moisture on wet basis using air at a DBT of 66°C but of the same absolute humidity and a linear velocity of 5 m/s. Assume the critical moisture content remains the same.

**Solution.**

Constant drying rate $7.5 \times 10^{-5}$ kg/s

Area of the specimen = 0.3 × 0.3 × 2 = 0.18 m$^2$

Drying rate = $N_{C1} = \frac{7.5 \times 10^{-5}}{0.18} = 4.167 \times 10^{-4}$ kg/m$^2$ s

$$X_C = \frac{0.15}{0.85} = 0.1765$$

$$X_O = \frac{0.25}{0.75} = 0.3333$$

$$X_{\text{final}} = \frac{0.02}{0.98} = 0.02041$$

Area of new solid = 1.2 × 0.6 × 2 = 1.44 $m^2$

Bone dry Weight of new solid = 28.8 kg

(Based on total volume of old and new solid)

Volume of old solid = 0.3 × 0.3 × 0.006 = 54 × $10^{-5}$ $m^3$

Volume of new solid = 0.6 × 1.2 × 0.012 = 864 × $10^{-5}$ $m^3$

Weight of old solid = 1.8 kg

$$\text{Weight of new Bone dry solid} = (864 \times 10^{-5}) \times \frac{1.8}{54} \times 10^{-5} = 28.8 \text{ kg}$$

$N_c \propto (T - T_s)$

$\propto (Y_s - Y)$

$\propto (G)^{0.71}$ where, G is the mass flow rate of air

Old velocity = 3 m/s

Old DBT = 52°C and WBT = 21°C $T_s$ = 21°C, Hence $T_G$–$T_S$ = 31°C

Humidity: 0.002 Saturated humidity: 0.015 kg/kg

New velocity = 5 cm/s

New DBT: 66°C but of same humidity as before, $T_s$ = 24°C

Hence $T_G - T_s$ = 41°C

Humidity: 0.002 Saturated humidity: 0.018 kg/kg

Hence, Drying rate of air under new conditions is =

$$4.167 \times 10^{-4}\left[\left(\frac{5}{3}\right)\times\left(\frac{325}{339}\right)\right]^{0.71} \times \left[\frac{(0.018-0.002)}{(0.015-0.002)}\right]\times\left[\frac{4}{3}\right] = 0.933 \times 10^{-3} \text{ kg/m}^2\text{ s}$$

$$\text{Drying time} = \left[\frac{L_S}{AN_C}\right]\times[X_o - X_{\text{Cr}}] + (X_{\text{Cr}} - X^*)\times \ln\left[\frac{(X_{\text{Cr}} - X^*)}{(X_f - X^*)}\right]$$

$$= \left[\frac{28.8}{1.44 \times 0.933 \times 10^{-3}}\right]$$

$$\times\left[(0.3333 - 0.1765) + (0.176 - 0.0)\times \ln\left\{\frac{0.1765 - 0}{0.0204 - 0.0}\right\}\right]$$

= 11524 s

= 3.20 hrs

## EXERCISES

1. Sheet material, measuring 1 $m^2$ and 5 cm thick, is to be dried from 50% to 2% moisture under constant drying conditions. The dry density of the material is 400 kg/$m^3$ and its equilibrium moisture content is negligible. The available drying surface is 1 $m^2$. Experiments showed that the rate of drying was constant at 4.8 kg/(hr)($m^2$) between moisture contents of 50% and 25% and thereafter the rate decreased linearly. Calculate the total time required to dry the material from 50% to 2%. All moisture contents are on wet basis.

   **(Ans:** 6.653 hrs)

2. Calculate the critical moisture content and the drying rate during the constant rate period for drying a wet slab of size 20 cm × 75 cm × 5 cm, whose dry weight is 16 kg. Both the sides are used for drying. The steam used was at 3 atm. pressure and was consumed at the rate of 0.135 g/s $cm^2$ of the contact surface. The following drying data is available for the sample. Assume equilibrium moisture content is negligible.

| Drying time, hrs | 0 | 0.25 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 4.0 | 6.0 | 8.0 | 10.0 | 12.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample weight, kg | 19.9 | 19.7 | 19.2 | 18.9 | 18.6 | 18.3 | 18.1 | 17.65 | 16.92 | 16.4 | 16.15 | 16.05 |

(**Ans:** 0.14375 kg/kg, 2 kg/hr $m^2$)

3. The following data are available for drying a substance. Estimate the drying time needed to dry a similar sample under similar drying conditions from 40% to 12% moisture content, on wet basis. The drying surface is 1 $m^2$/4 kg of dry weight and the initial weight of the wet sample is 80 kg.

| $X$ (dry basis) | 0.35 | 0.25 | 0.2 | 0.18 | 0.16 | 0.14 | 0.10 | 0.08 | 0.065 |
|---|---|---|---|---|---|---|---|---|---|
| $N$, kg/hr·$m^2$ | 0.3 | 0.3 | 0.3 | 0.266 | 0.24 | 0.21 | 0.15 | 0.07 | 0.05 |

(**Ans:** 7.281 hrs)

4. 175 kg of wet material with 25% moisture is to be dried to 10% moisture. Air enters at 65°C DBT and a WBT of 25°C. The velocity of air is 150 cm/s. Drying area equals 1 $m^2$/40 kg dry weight.

| $X$ (dry basis) | 0.26 | 0.22 | 0.20 | 0.18 | 0.16 | 0.14 | 0.12 | 0.1 | 0.08 |
|---|---|---|---|---|---|---|---|---|---|
| $N$, kg/hr·$m^2$ | 1.5 | 1.5 | 1.5 | 1.3 | 1.2 | 1.04 | 0.9 | 0.75 | 0.6 |

(**Ans:** 6.687 hrs)

5. A wet solid is dried from 35% to 8% moisture in 5 hrs under constant drying condition. The critical moisture content is 15% and equilibrium moisture content is 5%. All the moisture contents are reported as percentage on wet basis. Calculate how much longer it would take place under the similar drying conditions to dry from 8% to 6% moisture on wet basis.

(**Ans:** 1.3115 hrs)

6. A certain material was dried under constant drying conditions and it was found that 2 hours are required to reduce the free moisture from 20% to 10%. How much longer would it require to reduce the free moisture to 4%? Assume that no constant rate period is encountered.

(**Ans:** 4.643 hrs)

7. It is desired to dry sheets of material from 73% to 4% moisture content (wet basis). The sheets are 2 m × 3 m × 5 mm. The drying rate during constant rate period is estimated to be 0.1 kg/hr · $m^2$. The bone-dry density of the material is 30 kg/$m^3$. The material is dried from both the sides. The critical

moisture content is 30% on wet basis and equilibrium moisture content is negligible. The falling rate period is linear. Determine the time needed for drying.

(**Ans:** 2.798 hrs)

**8.** A slab of paper pulp 1 m × 1 m × 5 mm is to be dried under constant drying conditions from 60% to 20% moisture (wet basis) and the critical moisture is 1.5 kg water/kg dry pulp. The drying rate at the critical point has been estimated to be 1.40 kg/($m^2$)(hr). The dry weight of each slab is 2.5 kg. Assuming that drying rate is linear in falling rate period and drying takes place from the two large faces only, calculate the drying time needed.

(**Ans:** 4.8 hrs)

**9.** 50 kg of batch of granular solids containing 25% moisture is to be dried in a tray dryer to 12% moisture by passing a stream of air at 92°C tangentially across its surface at a velocity of 1.8 m/s. If the constant rate of drying under these conditions is 0.0008 kg moisture/$m^2$s and critical moisture content is 10%. Calculate the drying time if the surface available is 1.0 $m^2$ (all moisture contents are on wet basis).

(**Ans:** 2.565 hrs)

**10.** A plant wishes to dry a certain type of fibre board in sheets 1.2 m × 2 m × 12 mm. To determine the drying characteristics a 0.3 m × 0.3 m board with the edge sealed, so that drying takes place only from two large faces only, was suspended from a balance in a laboratory dryer and exposed to a current of hot dry air. Initial moisture content is 75%, critical moisture content is 60% and equilibrium moisture content is 10%. Dry mass of the sample weighs 0.9 kg. Constant drying rate 0.0001 kg/$m^2$·s. Determine the time for drying large sheets from 75% to 20% moisture under the same drying conditions (all moisture contents are on wet basis).

(**Ans:** 65.24 hrs)

**11.** A batch of wet solid was dried on a tray drier using constant drying conditions and the thickness of material on the tray was 25 mm. Only the top surface was exposed for drying. The drying rate was 2.05 kg/$m^2$ • hr during constant rate period. The weight of dry solid was 24 kg/$m^2$ exposed surface. The initial free moisture content was 0.55 and the critical moisture content was 0.22. Calculate the time needed to dry a batch of this material from a moisture content of 0.45 to 0.30 using the same drying condition but the thickness of 50 mm with drying from the top and bottom surfaces.

(**Ans:** 1.756 hrs)

**12.** A pigment material, which has been removed wet from a filter press, is to be dried by extending it into small cylinders and subjecting them to through circulation drying. The extrusions are 6 mm in diameter, 50 mm long and are to be placed in screens to a depth of 65 mm. The surface of the particles is estimated to be 295 $m^2/m^3$ of bed and the apparent density is 1040 kg dry solid/$m^3$. Air at a mass velocity of 0.95 kg dry air/$m^2$ · s will flow through the bed entering at 120°C and a humidity of 0.05 kg water/kg dry air, estimate the constant drying rate to be expected.

(**Ans:** 0.002906 g/cm$^2$ s)

**13.** It is necessary to dry a batch of 160 kg of wet solid from 30% to 5% moisture content under constant rate and falling rate period. The falling rate is assumed to be linear. Calculate the total drying time considering an available drying surface of 1 m$^2$/40 kg of dry solid. The flux during constant rate period is 0.0003 kg/m$^2$ s. The critical and equilibrium moisture contents are 0.2 and 0.02 respectively. If the air flow rate is doubled, what is the drying time needed? The critical and equilibrium moisture contents do not change with velocity of air but $N_C$ varies as $G^{0.71}$, where $G$ is the mass flow rate of air.

(**Ans:** 23.33 hrs and 14.265 hrs)

**14.** A rotary dryer using counter-current flow is to be used to dry 12000 kg/hr of wet salt containing 5% water (wet basis) to 0.1% water (wet basis). Heated air at 147°C with a WBT of 50°C is available. The specific heat of the salt is 0.21 kcal/kg °C. The outlet temperatures of air and salt are 72°C and 93°C respectively. Calculate the diameter of the dryer required.

(**Ans:** 2.55 m)

**15.** During the batch drying test of a wet slab of material 0.35 m$^2$ and 7 mm thick, the falling rate $N$ was expressed as 0.95 ($X$ – 0.01) where $N$, is the drying rate in kg/m$^2$ s and $X$ is the moisture content in kg moisture/kg dry solid. The constant drying rate was 0.38 kg/m$^2$ s and slab was dried from one side only with the edges sealed. Density of the dry material is 1200 kg/m$^3$. It is desired to reduce the moisture content from 35% to 5% on wet basis. What is the time needed for drying?

(**Ans:** 22.63 s)

# 8

# ABSORPTION

## 8.1 INTRODUCTION

Absorption is one of the important gas–liquid contact operations in which a gaseous mixture is contacted with a solvent to dissolve one or more components of the gas preferentially and provide a solution of them in the solvent. Some of the applications of this operation are as follows:

(i) Ammonia is removed from coke-oven gas with water

(ii) Benzene and toluene vapours are removed using hydrocarbon oil from the coke-oven gas.

(iii) Hydrogen sulfide is removed from naturally occurring hydrocarbon gases with alkaline solutions.

(iv) Ammonia and other water soluble harmful gases from air are removed using water.

## 8.2 GAS SOLUBILITY IN LIQUIDS AT EQUILIBRIUM

The equilibrium characteristics of gas solubility in liquids are generally represented as partial pressure of solute in gas ($p^*$) vs mole fraction of solute in liquid ($x$). A typical gas solubility curve drawn at a particular temperature and pressure for different gases is shown in Fig. 8.1. If the gas solubility is low, then the equilibrium pressure for that particular system is very high. The solubility of gas is significantly affected by the temperature. Generally absorption processes are exothermic and if the temperature is increased at equilibrium, the solubility of gases, but not always, will be decreased due to evolution of heat.

## 8.3 IDEAL AND NON-IDEAL LIQUID SOLUTIONS

In an ideal solution, all the components present in the solution approach similarity with regard to their chemical nature. When the gas mixture is in equilibrium with an ideal solution, then it follows Raoult's law.

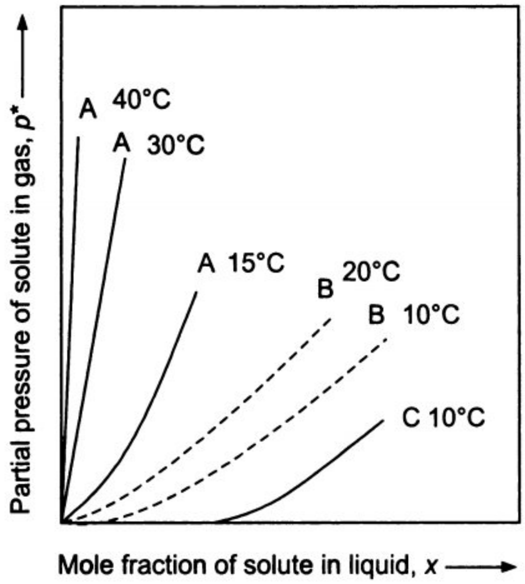


**Fig. 8.1** Solubility of gas in liquid.

$$p^* = Px \tag{8.1}$$

where $p^*$ is the partial pressure of solute, $P$ is the vapour pressure of solute at the same temperature and $x$ is the mole fraction of solute in liquid.

For non-ideal solutions, Henry's law can be applied and is given by,

$$y^* = \frac{p^*}{P_t} = m \cdot x \tag{8.2}$$

where $m$ is Henry's constant, $P_t$ is the total pressure and $y^*$ is the mole fraction of solute in gas.

## 8.4 CHOICE OF SOLVENT FOR ABSORPTION

The following properties are to be considered while choosing a particular solvent in any absorption system.

(i) ***Gas solubility:*** Solubility of the solute to be absorbed in solvent should be relatively high, as it will decrease the quantum of solvent requirement.

(ii) ***Chemical nature:*** Generally solvent should be chemically similar in structure to that of the solute to be absorbed as it will provide good solubility.

(iii) ***Recoverability:*** The solvent should be easily recovered and as it will help in reusing it.

(iv) ***Volatility:*** The solvent should have a low vapour pressure, i.e. less volatile.

(v) ***Corrosiveness:*** The solvent should not be corrosive to the material of construction equipment.

(vi) ***Cost and availability:*** The solvent should be inexpensive and readily available.

(vii) ***Viscosity:*** The solvent should have low viscosity as it will reduce pumping and transportation costs.

(viii) ***Toxic, flammability and stability:*** The solvent should be non-toxic, inflammable, chemically stable and non-reactive.

## 8.5 DESIGN OF ISOTHERMAL ABSORPTION TOWERS

The design of isothermal absorption towers is based on material balance in them. The flow of streams could be either co-current or countercurrent. The operation is either carried out as a single stage operation or as a multistage operation.

### 8.5.1 Single Stage—One Component Transferred—Countercurrent and Isothermal Operation

Consider a single stage isothermal absorber as shown in Fig. 8.2, where (1) and (2) refer to the bottom and top sections of the equipment respectively. Gaseous mixture entering the absorber at the bottom is contacted countercurrently with liquid solvent, entering from the top.

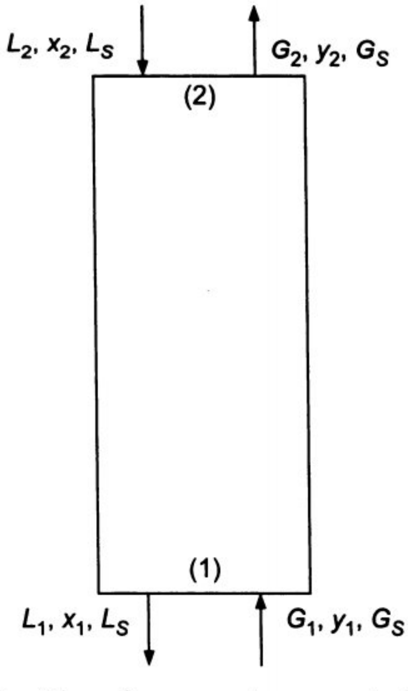


**Fig. 8.2** Flow in a countercurrent absorber.

Let $G_1$ and $L_2$ be the molar flow rates of entering binary gaseous mixture and liquid respectively in moles/(area) (time).

Let $G_2$ and $L_1$ be the molar flow rates of leaving gaseous mixture and liquid respectively in moles/(area) (time).

Let $G_S$ and $L_S$ be the molar flow rates of inert gas and pure liquid respectively in moles/ (area) (time).

Let $x$, $y$ be the mole fractions of solute in liquid and gas phases respectively.

Let $X$, $Y$ be the mole ratios of solute to inert component in liquid and gas phases respectively.

In the gas phase, only one component is transferred and the other component remains as inert. Similarly, in the liquid phase, solvent is the inert component. It is more convenient to represent the concentrations of solute in liquid and gas phases in terms of mole ratios ($X$ and $Y$) of solute to inert component. So

$$X = \frac{x}{(1-x)} \text{ and } Y = \frac{y}{(1-y)} \tag{8.3}$$

Likewise,

$$x = \frac{X}{(1+X)} \text{ and } y = \frac{Y}{(1+Y)} \tag{8.4}$$

$$G_S = G_1\ (1 - y_1) \text{ or } G = \frac{G_S}{(1-y)} \tag{8.5}$$

Writing the material balance on solute basis for the above countercurrent operation, we get

$$G_SY_1 + L_SX_2 = G_SY_2 + L_SX_1 \tag{8.6}$$

$\therefore$

$$Gs(Y_1 - Y_2) = L_S(X_1 - X_2) \tag{8.7}$$

i.e.

$$\left[\frac{L_S}{G_S}\right] = \left[\frac{(Y_1 - Y_2)}{(X_1 - X_2)}\right] \tag{8.8}$$

Equation (8.8) represents the operating line for a single stage countercurrent absorber. The operating line is linear which passes through the coordinates $(X_1, Y_1)$ and $(X_2, Y_2)$ with a slope of $(L_S/G_S)$. Since the solute transfer is taking place from gas to liquid phase, the operating line always lies above the equilibrium curve, which is shown in Fig. 8.3.

Suppose, if the flow rates of gas and liquid streams are not considered on inert basis, i.e. when considered on mole fraction basis, then the operating line would be a non-linear one passing through the coordinates $(x_1, y_1)$ and $(x_2, y_2)$ as shown in Fig. 8.3. It is highly impossible to know the intermediate concentrations which will enable one to draw this operating curve passing through the terminal points $(x_1, y_1)$ and $(x_2, y_2)$. Hence, it is more preferable to obtain the linear operating line with the known terminal concentrations of the system by considering mole ratio basis as shown in Fig. 8.4.

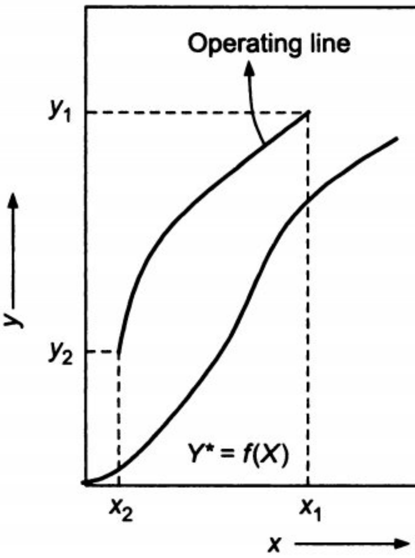


Fig. 8.3 Equilibrium curve and operating line on mole fraction basis.

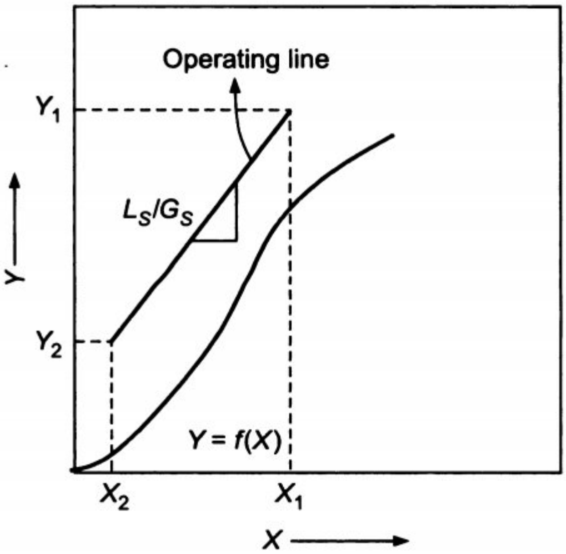


Fig. 8.4 Equilibrium Curve and Operating Line in Mole Ratio Basis.

### 8.5.2 Determination of Minimum $(L_S/G_S)$ Ratio

In absorption, minimum $(L_S/G_S)$ ratio indicates a slope for operating line, at which the maximum amount of solute concentration is obtained in the final liquid. It will be achieved only in the presence of infinite number of stages for a desired level of absorption of solute. When the operating line is tangent to the equilibrium curve, then there is no net driving force and the required time of contact for the concentration change desired is infinite and an infinitely tall tower will result. This is highly uneconomical. So, the tower is operated at the $(L_S/G_S)$ ratio of 1.2 to 2.0 times the minimum $(L_S/G_S)$ ratio.

### 8.5.3 Steps Involved in Determining $(L_S/G_S)_{min}$

1. Plot $X$ and $Y$ data to draw the equilibrium curve.
2. Locate point $A(X_2,Y_2)$.
3. From point A draw a tangent to the equilibrium curve.
4. Determine the slope of this line which will be $(L_S/G_S)_{min}$.
5. Extend the line from $Y_1$ to intersect this operating line which corresponds to the point $B'[(X_1)_{max}, Y_1]$.
6. Determine $(L_S/G_S)_{actual}$ and find the slope.
7. Using the operating line equation, obtain $(X_1)_{actual}$ and draw the actual operating line AB as shown in Fig. 8.5

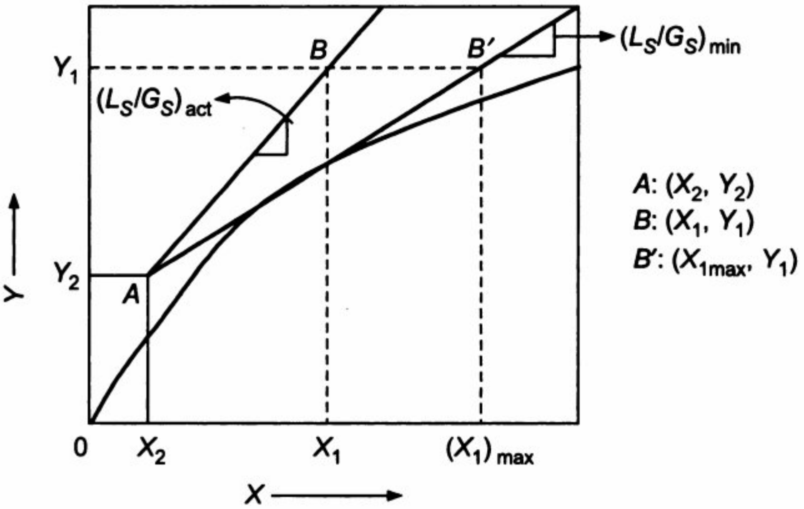
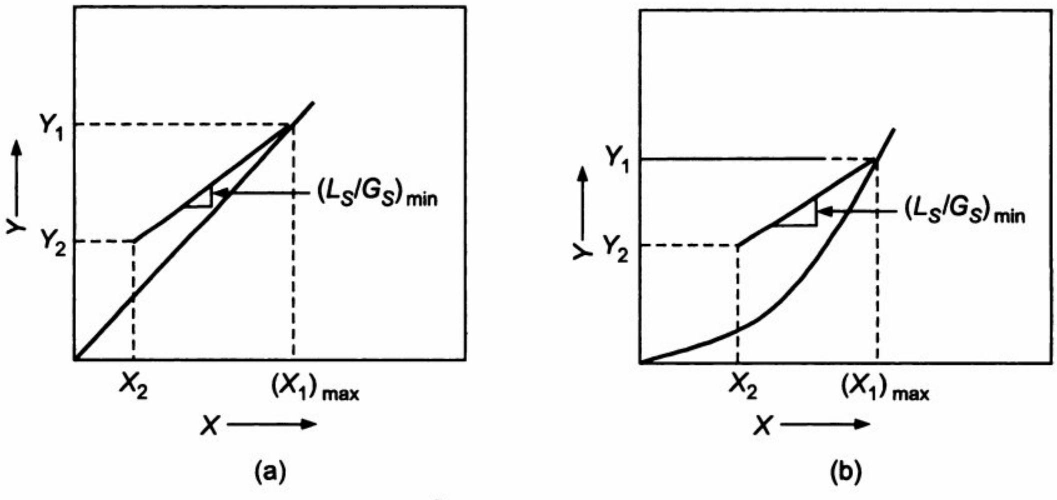


Fig. 8.5 Minimum L/G ratio.

In some cases, the equilibrium curve will be more or less a straight line or concave upward. In such cases the minimum $(L_S/G_S)$ ratio can be determined as shown in Figs. 8.6(a) and (b).



(a) (b)

Fig. 8.6 Equilibrium curve and operating line for special cases.

Therefore,
$$(L_S/G_S)_{\min} = \left[\frac{(Y_1 - Y_2)}{(X_1)_{\max} - X_2}\right] \tag{8.9}$$

Since $(L_S/G_S)_{\min}$ is known, $(X_1)_{\max}$ can be determined as all the other quantities in Eq. (8.9) are known.

### 8.5.4 Multistage Countercurrent Isothermal Absorption

Let us consider a multistage tray tower containing $N_p$ number of stages as shown in Fig. 8.7, where the suffix $p$ represents the tray number. The operation is isothermal and it is assumed that the average composition of gas leaving from a tray is in equilibrium with the average composition of liquid leaving from the same tray. The flow of streams is countercurrent. The liquid flows downwards and the gas upwards and only one component is transferred. The number of theoretical or ideal stages required for the desired operation in the tower is determined as follows:

The material balance on inert basis gives,

$$G_S Y_{Np+1} + L_S X_0 = G_S Y_1 + L_S X_{Np} \tag{8.10}$$

$$G_S(Y_{Np+1} - Y_1) = L_S (X_{Np} - X_0) \tag{8.11}$$

i.e.
$$\left[\frac{L_S}{G_S}\right] = \left[\frac{(Y_{Np+1} - Y_1)}{(X_{Np} - X_0)}\right] \tag{8.12}$$

Equation (8.12) represents a linear operating line for a multistage countercurrent absorber which passes through the coordinates $(X_0, Y_1)$ and $(X_{Np}, Y_{Np+1})$ with a slope $(L_S/G_S)$. Between the equilibrium curve and the operating line, a stepwise construction is made to obtain the number of theoretical trays. The stepwise construction is started from $(X_0, Y_1)$ since it represents operating condition in plate number 1 (as per our convention). This is illustrated in Fig. 8.8.

### 8.5.5 Analytical Method to Determine the Number of Trays

In some special cases such as dilute gaseous mixtures or solutions, the equilibrium curve is a straight line, the number of trays can be determined analytically by using Kremser-Brown–Souders equation given in Eq. (8.13) without going in for a graphical method.

$$N_p = \frac{\log\left[\left(\frac{y_{Np+1} - mx_0}{y_1 - mx_0}\right)(1 - 1/A) + 1/A\right]}{\log A} \tag{8.13}$$

where $A$ is the absorption factor given by $L/mG$ and $m$ is the slope of the equilibrium curve.

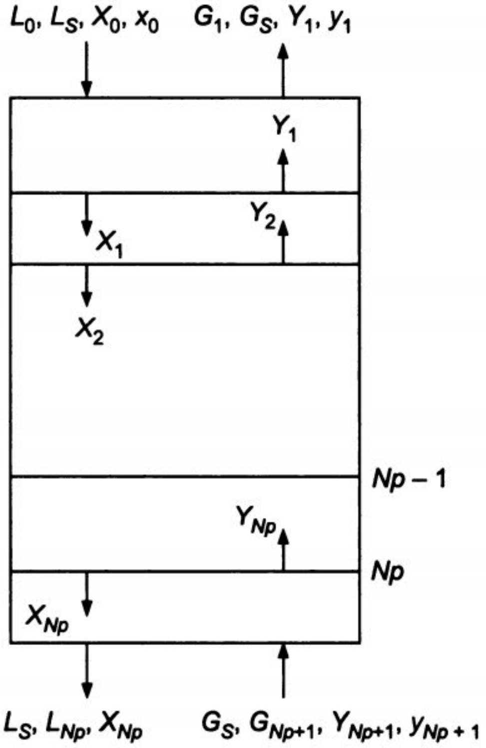


**Fig. 8.7** Various streams in a countercurrent multistage tray tower.

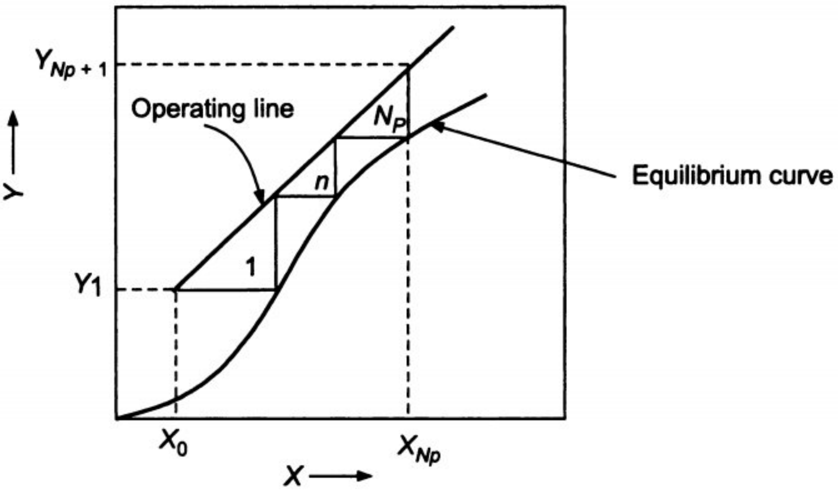


**Fig. 8.8** Stepwise construction for estimating the number of plates/stages.

Absorption factor, *A*, is defined as the ratio of the slope of the operating line to that of the equilibrium curve. If '*A*' varies due to small changes in *L*/*G* from bottom to top of the tower, then the geometric mean value of *A* has to be considered.

Hence, geometric mean value of $A = \sqrt{A_1 A_2}$ (8.14)

where, $$A_1 = \frac{L_1}{mG_1} \approx \frac{L_0}{mG_1}$$

$$A_2 = \frac{L_{Np}}{mG_{Np}} \approx \frac{L_{Np}}{mG_{Np+1}}$$

where $A_1$ is the absorption factor at the top of the tower and $A_2$ is the absorption factor at the bottom of the tower. For larger variations in $A$, graphical computations must be followed.

### 8.5.6 Significance of Absorption Factor

If $A < 1$, the operating line and equilibrium curve converge at the lower end of the tower indicating that the solubility of solute is limited even when large number of trays are provided.

If $A > 1$, any degree of separation is possible with adequate number of trays. However, as $A$ increases beyond 1.0 for a fixed quantity of gas and a given degree of absorption, the absorbed solute is dissolved in a larger quantity of liquid (solvent) and hence becomes less valuable. In addition to that, the number of trays also decreased leading to a lower cost of equipment. This leads to a variation in total cost of operation which will pass through a minimum. Hence, for an economical operation, the value of $A$ has been estimated for various systems and found to be in the range of 1.2 to 2.0.

## 8.6 DESIGN OF MULTISTAGE NON-ISOTHERMAL ABSORBER

Generally the absorption operations are exothermic in nature. Hence, the solubility of gas decreases as temperature of the liquid increases which in turn decreases the capacity of the absorber. When concentrated gaseous mixtures are to be absorbed in solvent then the temperature effects have to be taken into account. If the heat liberated is more, then cooling coils should be provided for an efficient absorption operation. Since the temperature is varying from tray to tray, it influences the concentration changes as well as the flow rate of streams. Hence, energy balance should also be incorporated along with material balance to determine the number of trays. It is very difficult to compute manually the tray to tray calculations. A simple algorithm is developed for one ideal tray involving trial and error calculations and then the programming is extended to other trays for the determination of the number of trays.

Consider a stagewise tray tower operating non-isothermally as shown in Fig. 8.9.

Total mass balance around the entire tower gives

$$G_{Np+1} + L_0 = L_{Np} + G_1 \tag{8.15}$$

Component balance gives,

$$[G_{Np+1}]\, y_{Np+1} + L_0 x_0 = L_{Np} x_{Np} + G_1 y_1 \tag{8.16}$$

Energy balance gives,

$$[G_{Np+1}] H_{G,Np+1} + L_0 H_{L_0} = L_{Np} H_{L_{Np}} + G_1 H_{G_1} \tag{8.17}$$

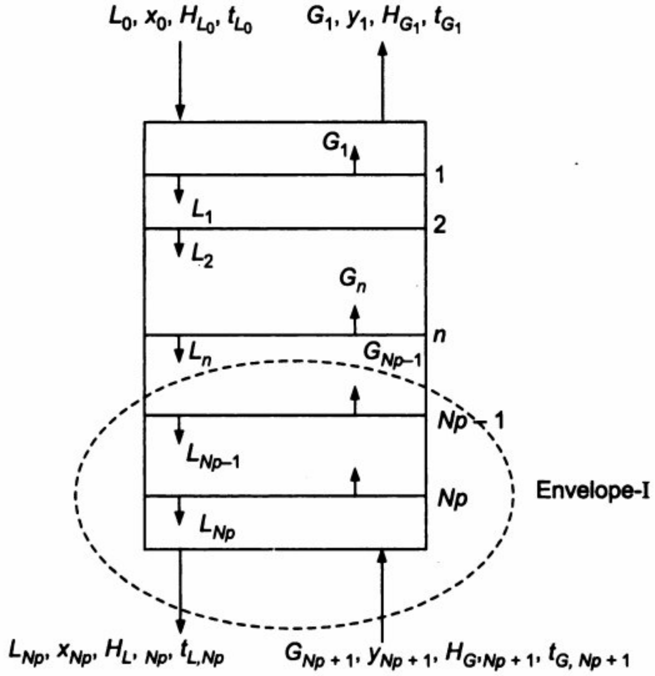


**Fig. 8.9** Streams in a countercurrent multistage tray tower and envelope-I.

where, $H$ is the molal enthalpy of streams. Enthalpies can be determined using the available literature data with reference to some base temperature say, $t_0$, (Pure state).

$$H_G = C_{pG,\text{inert}}\,(t_G - t_0)(1 - y) + y[C_{pG,\text{solute}}\,(t_G - t_0) + \lambda_0] \tag{8.18}$$

$$H_L = C_{pL,\text{inert}}\,(t_L - t_0)(1 - x) + x[C_{pL,\text{solute}}\,(t_L - t_0)] \tag{8.19}$$

where $C_p$ is the specific heat of the component and $\lambda_0$ is the latent heat of vapourization at reference temperature, $t_0$.

Now let us consider the envelope-I.

Mass and energy balance in envelope-I follow,

$$L_n + G_{Np+1} = G_{n+1} + L_{Np} \tag{8.20}$$

$$L_n x_n + [G_{Np+1}]\; y_{Np+1} = G_{n+1} y_{n+1} + L_{Np} x_{Np} \tag{8.21}$$

$$L_n H_{L_n} + [G_{Np+1}]\; H_{G_{Np+1}} = G_{n+1}\; H_{G_{n+1}} + L_{Np} H_{L_{Np}} \tag{8.22}$$

Let $n = N_p - 1$,

$$L_{Np-1} + G_{Np+1} = G_{Np} + L_{Np} \tag{8.23}$$

$$L_{Np-1} x_{Np-1} + [G_{Np+1}] y_{Np+1} = G_{Np} y_{Np} + L_{Np} x_{Np} \tag{8.24}$$

$$L_{Np-1}\; H_{L,\,Np-1} + [G_{Np+1}]\; H_{G,\,Np+1} = G_{Np} H_{G,\,Np} + L_{Np} H_{L,\,Np} \tag{8.25}$$

To solve the above system of equations and determine the number of trays, the following procedure is used.

1. Assume the top tray temperature, $t_{G1}$. The other values like $G_{Np+1}$, $y_1$, $y_{Np+1}$, $L_0$, $x_0$, $t_{L_0}$ and $t_{G\;Np+1}$ are known.
2. Calculate $G_S$ from the relationship, $G_S = G_{Np+1}(1 - y_{Np+1})$
3. Calculate $G_1$ from the relationship, $G_1 = \dfrac{G_S}{1 - y_1}$
4. Using Eq. (8.15), Calculate $L_{Np}$.
5. Find $x_{Np}$ from Eq. (8.16).

6. Calculate $H_{G_{Np+1}}$, $H_{L_0}$ and $H_{G_1}$ using Eqs. (8.18) and (8.19).
7. Find $H_{L_{Np}}$ from Eq. (8.17).
8. Determine $t_{L_{Np}}$ making use of Eq. (8.19).
9. With this knowledge of the temperature of the last tray $N_p$, the compositions can be determined by $y^* = (\text{V.P/T.P})x$ or $y^* = m.x$, where V.P. is vapour pressure, T.P. is total pressure and $m$ is equilibrium constant. Hence $y_{Np} = (m)\ x_{Np}$
10. Now for the last tray, $x_{Np}$, $y_{Np}$, $t_{L_{Np}}$ are known.
11. Find $G_{Np} = \dfrac{G_s}{(1 - y_{Np})}$
12. Now calculate $L_{Np-1}$ using Eq. (8.23).
13. Find $x_{Np-1}$ from Eq. (8.24).
14. Calculate $H_{L_{Np-1}}$ using Eq. (8.25).
15. Find $t_{L_{Np-1}}$ from Eq. (8.19).
16. Now determine the composition, $y_{Np-1}$ and $G_{Np-1}$ as mentioned in step (9) and step (11) respectively.
17. Similarly calculate for the next tray by taking $n = N_{p-2}$ and starting from step (12), by making use of material and enthalpy balances.
18. Finally, the computation is stopped on reaching the value of $y_1$ and also satisfying the assumed $t_{G_1}$. If these two values namely, the assumed composition and computed composition $y_1$ and the assumed temperature and the calculated temperature $t_0$ are not satisfied together, once again the iteration has to be started fresh by assuming a new temperature $t_{G_1}$. However, when both the values of $y_1$ and $t_{G_1}$, are satisfied, the number of trays are known from the computation values.

## 8.7 DESIGN OF COCURRENT ABSORBER

In a cocurrent absorber both gas and solvent streams enter at the same end and flow in the same direction as shown in Fig. 8.10.

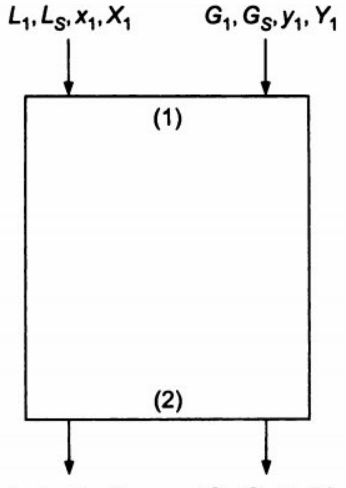


**Fig. 8.10** Cocurrent absorber.

Making material balance,

$$L_SX_1 + G_SY_1 = L_SX_2 + G_SY_2 \tag{8.26}$$

$\therefore$

$$L_S(X_1 - X_2)] = G_S(Y_2 - Y_1) \tag{8.27}$$

i.e.

$$-\left[\frac{L_s}{G_s}\right] = \left[\frac{(Y_1 - Y_2)}{(X_1 - X_2)}\right] \tag{8.28}$$

Equation (8.28) is the equation of operating line for cocurrent absorption operation with the slope – $(L_S/G_S)$ and this is presented in the $X$–$Y$ diagram of Fig. 8.11. If the leaving streams are in equilibrium with each other, then the compositions are represented by $(X'_2, Y'_2)$ and for a typical liquid phase composition of $X_2$, the gas phase composition will be $Y_2$.

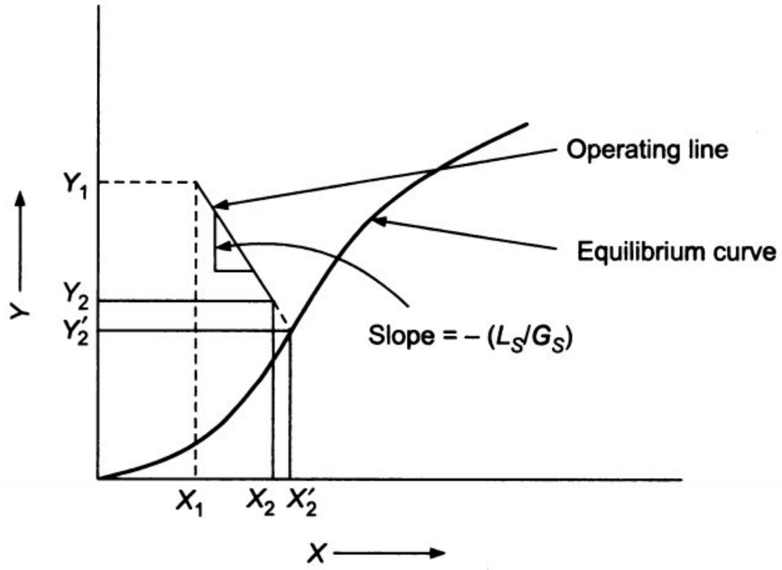


Fig. 8.11 Equilibrium curve and operating line in a cocurrent absorber.

## 8.8 DESIGN OF CONTINUOUS CONTACT EQUIPMENT FOR ABSORPTION

Packed columns and spray towers fall in the category of continuous contact or differential contact towers. They are different from stagewise contactors in the sense that the fluids are in continuous contact throughout the tower. So the liquid and gas compositions change continuously with respect to the height of the tower.

Consider a packed tower of unit cross sectional area as shown in the Fig. 8.12. The characteristics of inlet and outlet streams are also indicated.

Let $Z$ be the total height of the tower and $dZ$ be the differential height which is same as the differential volume. $S$ is the total effective interfacial surface per unit tower cross section. Hence,

$$S = \frac{\text{Interfacial area}}{\text{Area of tower}} = \frac{(a)\cdot[A\cdot Z]}{A} \tag{8.29}$$

$\therefore$

$$dS = a \cdot dZ \tag{8.30}$$

where $dS$ is the differential interfacial surface in the differential volume of packing.

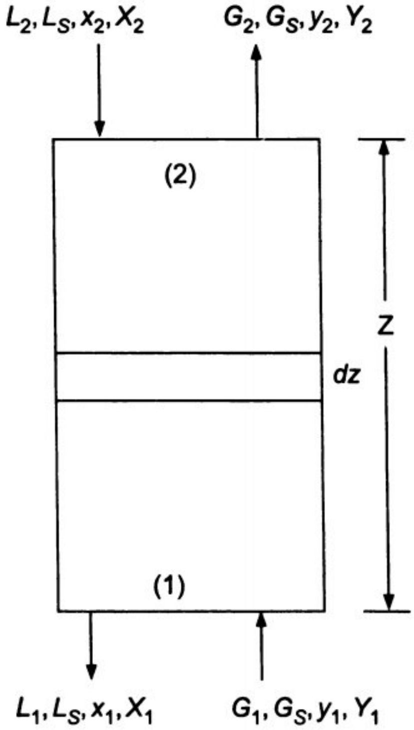


**Fig. 8.12** Continuous countercurrent absorber.

As shown in Fig. 8.12, the quantity of solute A passing through the differential section is $G \cdot y$ moles/(area) (time). The rate of mass transfer is $d(G \cdot y)$ mole $A$/ (differential volume) (time). Since $N_B = 0$, $N_A/(N_A + N_B) = 1.0$. The molar flux of $A$ is obtained by applying the original basic flux equation,

$$N_A = \frac{\text{Rate of absorption of solute } A}{\text{Interfacial area}}$$

$$= \frac{d(Gy)}{adZ} = F_G \ln\left[\frac{(1-y_i)}{(1-y)}\right] \tag{8.31}$$

$d(Gy)$ can be written as

$$d(Gy) = d\left[\frac{G_s y}{(1-y)}\right] \tag{8.32}$$

Since one component is transferred, $G$ and $y$ vary throughout the tower.

i.e.
$$d\left[\frac{G_s y}{(1-y)}\right] = \frac{G_s dy}{(1-y)^2} = \frac{Gdy}{(1-y)} \tag{8.33}$$

Substituting Eq. (8.33) in Eq. (8.31), rearranging and integrating, we get

$$Z = \int_0^Z dZ = \int_{y_2}^{y_1} \frac{Gdy}{F_G a(1-y)\ln[(1-y_i)/(1-y)]} \tag{8.34}$$

It is more convenient to write, $y - y_i = [(1 - y_i) - (1 - y)]$ (8.35)

The numerator and denominator of Eq. (8.34) can be multiplied by the right and left hand sides of Eq. (8.35) respectively to obtain

$$Z = \int_{y_2}^{y_1} \frac{G(1-y)_{iM}\,dy}{F_G a(1-y)(1-y_i)} \tag{8.36}$$

where $(1 - y)_{iM}$ is logarithmic mean of $(1 - y_i)$ and $(1 - y)$.

$$Z = \frac{G}{F_G a}\int_{y_2}^{y_1} \frac{(1-y)_{iM}\,dy}{(1-y)(y-y_i)} \approx H_{tG} \cdot N_{tG} \tag{8.37}$$

where $H_{tG}$ is the height of a gas transfer unit and $N_{tG}$ is the number of gas transfer units.

Thus,

$$H_{tG} = \frac{G}{F_G a} = \frac{G}{k_y a(1-y)_{iM}} = \frac{G}{k_G a P_t (1-y)_{iM}} \tag{8.38}$$

in terms of other individual mass transfer coefficients.
$N_{tG}$ is simplified further by substituting the arithmetic average instead of logarithmic average of $(1 - y)_{iM}$
Hence,

$$(1-y)_{iM} = \frac{(1-y_i)-(1-y)}{\ln\left[\dfrac{(1-y_i)}{(1-y)}\right]} \approx \frac{(1-y_i)+(1-y)}{2} \tag{8.39}$$

$$N_{tG} = \int_{y_2}^{y_1} \frac{(1-y)_{iM}\,dy}{[(1-y)(y-y_i)} = \int_{y_2}^{y_1} \frac{dy}{(y-y_i)} + \frac{1}{2}\ln\frac{(1-y_2)}{(1-y_1)} \tag{8.40}$$

Similarly, when the above mentioned relations have been applied for liquid compositions, we obtain

$$Z = \frac{L}{F_L a}\int_{x_2}^{x_1} \frac{(1-x)_{iM}\,dx}{[(1-x)(x_i-x)]} \approx H_{tL} \cdot N_{tL} \tag{8.41}$$

where $H_{tL}$ is the height of liquid transfer unit, $N_{tL}$ is the number of liquid transfer units and $(1 - x)_{iM}$ is logarithmic mean of $(1 - x)$ and $(1 - x_i)$
On simplification, we get

$$H_{tL} = \frac{L}{F_L a} = \frac{L}{k_x a(1-x)_{iM}} \tag{8.42}$$

and

$$N_{tL} = \int_{x_2}^{x_1} \frac{dx}{(x_i-x)} + \frac{1}{2}\ln\frac{(1-x_1)}{(1-x_2)} \tag{8.43}$$

Equations (8.38), (8.40), (8.42) and (8.43) can be used to determine the height of the tower.

With the known quantities, $H_{tG}$ or $H_{tL}$ can be easily determined. But $N_{tG}$ and $N_{tL}$ can be determined only through the graphical method. For this, plot of $1/(y - y_i)$ against $y$ is drawn and the area under the curve will give $N_{tG}$. The values of $y$ and $y_i$ can be evaluated by drawing a line between equilibrium curve and operating line with the slope $(-k_xa/k_ya)$ where $y$ and $y_i$ are points of intersection of this line on operating line and equilibrium curve respectively.

### 8.8.1 Overall Transfer Units

In some cases where the equilibrium curve is straight and the ratio of mass transfer coefficients is constant, it is more convenient to make use of overall mass transfer coefficients. The height of the tower can be expressed in such cases as

$$Z = N_{toG} \cdot H_{toG} \tag{8.44}$$

$$N_{toG} = \int_{y_2}^{y_1} \frac{(1-y)_{*M}\,dy}{(1-y)(y-y^*)} \approx \int_{y_2}^{y_1} \frac{dy}{(y-y^*)} + \frac{1}{2}\ln\frac{(1-y_2)}{(1-y_1)} \tag{8.45}$$

$$H_{toG} = \frac{G}{F_{OG}a} = \frac{G}{K_ya(1-y)_{*M}} = \frac{G}{K_GaP_t(1-y)_{*M}} \tag{8.46}$$

$$N_{toL} = \int_{x_2}^{x_1} \frac{dx}{(x^*-x)} + \frac{1}{2}\ln\frac{(1-x_1)}{(1-x_2)} \tag{8.47}$$

$$H_{toL} = \frac{L}{F_{oL}a} = \frac{L}{K_xa(1-x)_{*M}} \tag{8.48}$$

### 8.8.2 Dilute Solutions

For dilute solutions or gaseous mixtures, the above equations become much simpler. The second term in Eq. (8.45) and in Eq. (8.47) become negligible. Hence,

$$N_{toG} = \int_{y_2}^{y_1} \frac{dy}{(y-y^*)} \quad \text{or} \quad N_{toL} = \int_{x_2}^{x_1} \frac{dx}{(x^*-x)} \tag{8.49}$$

If the equilibrium curve in terms of mole fractions is also linear over the entire range of $x$, then

$$y^* = m \cdot x + C \tag{8.50}$$

If the solutions are dilute, there won't be variations in $L/G$ ratio throughout, and the operating line can be considered as a straight line so that the driving force $(y - y^*)$ is also linear. In such cases, Eq. (8.43) is simplified to

$$N_{toG} = \frac{(y_1 - y_2)}{(y - y^*)_M} \tag{8.51}$$

where $(y - y^*)_M$ is logarithmic average of the concentration differences at the terminals of the tower.
Therefore,

$$(y - y^*)_M = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln \dfrac{(y_1 - y_1^*)}{(y_2 - y_2^*)}} \tag{8.52}$$

and

$$H_{toG} = \frac{G}{K_y a} \quad \text{or} \quad \frac{G}{K_G a P_t} \tag{8.53}$$

### 8.8.3 Dilute Solutions Using Henry's Law

In dilute solutions, if Henry's law is applied, then

$$y^* = m \cdot x \tag{8.54}$$

The operating line can be written in a linear form as

$$(y - y_2) = \left(\frac{L}{G}\right)(x - x_2) \tag{8.55}$$

Eliminating $x$ between Eqs. (8.54) and (8.55) and substituting $y^*$ in Eq. (8.49), we get

$$N_{toG} = \frac{\ln\left\{\left[\dfrac{y_1 - mx_2}{y_2 - mx_2}\right]\left[1 - \dfrac{1}{A}\right] + \left[\dfrac{1}{A}\right]\right\}}{\left[1 - \dfrac{1}{A}\right]} \tag{8.56}$$

where $A$ is the absorption factor = $L/mG$
The overall height of transfer units can also be expressed in terms of individual phases,

$$H_{toG} = H_{tG} + \left(\frac{mG}{L}\right)H_{tL} \quad \text{or} \quad H_{toL} = H_{tL} + \left(\frac{L}{mG}\right)H_{tG} \tag{8.57}$$

## 8.9 STRIPPING OR DESORPTION

When mass transfer occurs from liquid to gas, i.e. the solute is removed from the liquid solution by contacting with a gas, then the operation is called Desorption or Stripping.

### 8.9.1 Operating Line for Stripper

The schematic representation of operating lines for both countercurrent and co-current operations of a stripper are shown in Fig. 8.13 and Fig. 8.14.

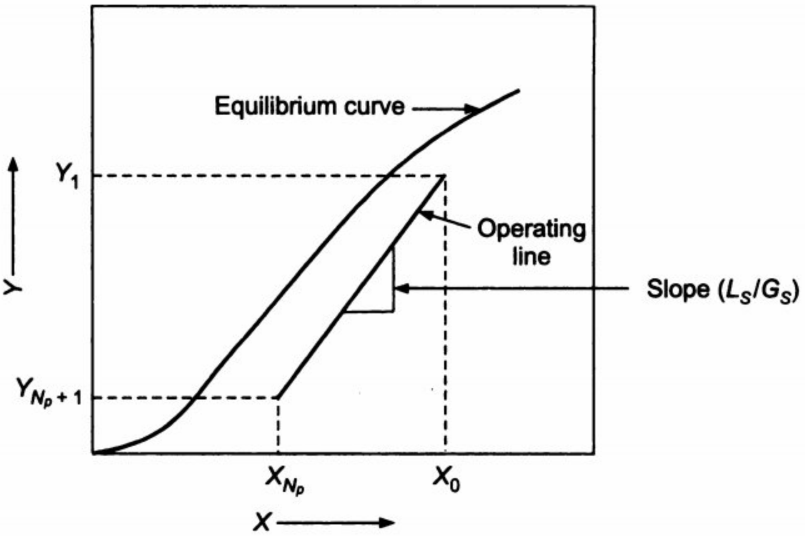


**Fig. 8.13** Equilibrium curve and operating line in a countercurrent stripper.

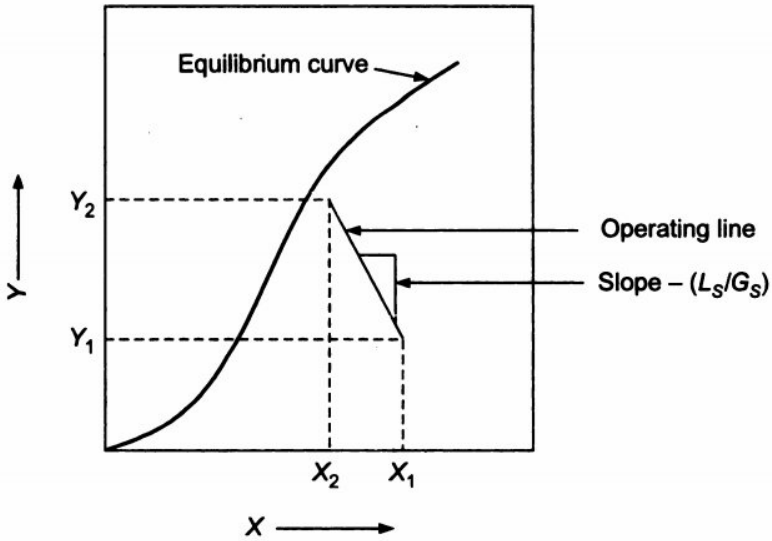


**Fig. 8.14** Equilibrium curve and operating line in a cocurrent stripper.

### 8.9.2 Analytical Relation to Determine Number of Plates

From Kremser–Brown–Souder's Eq. (8.13) on rearranging for desorption we get,

$$N_p = \frac{\log\left\{\left[\frac{x_0 - y_{Np+1}/m}{x_{Np} - y_{Np+1}/m}\right]\left[1-\frac{1}{S}\right]+\left[\frac{1}{S}\right]\right\}}{\log S} \tag{8.58}$$

where $S$ is the stripping factor, $S = \frac{mG}{L}$

For dilute solutions, if Henry's law is applied,

$$N_{toL} = \frac{\ln\left\{\left[\frac{x_2 - y_1/m}{x_1 - y_1/m}\right](1-A) + A\right\}}{1-A} \tag{8.59}$$

## WORKED EXAMPLES

1. An air–$NH_3$ mixture containing 5% $NH_3$ by volume is absorbed in water using a packed tower at 20°C and 1 atm pressure to recover 98% $NH_3$. Gas flow rate is 1200 kg/h m$^2$. Calculate (a) Minimum mass flow rate of liquid; (b) NTU using 1.25 times the minimum liquid flow rate; (c) Height of packed column using $K_Ga = 128$ kg/h m$^2$ atm. The equilibrium relation is $y = 1.154x$ where, $x$, $y$ are expressed in mole fraction units.

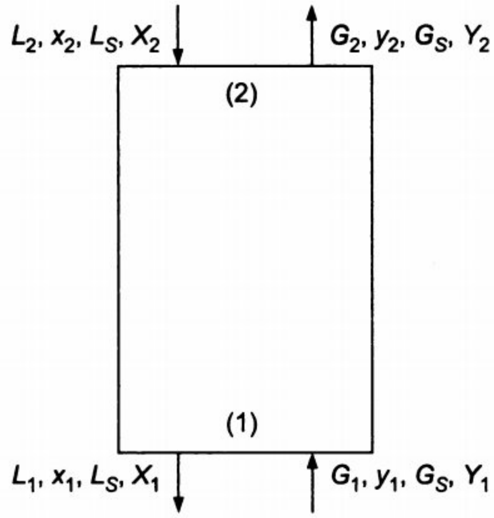


**Fig. 8.15(a)** Example 1.

**Solution.**

(a) Given that

$$y_1 = 0.05,\ P_t = 1 \text{ atm},\ T = 20°\text{C and } X_2 = 0$$

Gas flow rate = 1200 kg/h m$^2$

Average molecular weight of mixture = (0.05 × 17) + (0.95 × 28.84) = 28.25

$$G_1 = \frac{1200}{28.25} = 42.478 \text{ kmol/h m}^2$$

$$G_s = G_1(1 - y_1) = 42.478\ (1 - 0.05) = 40.354 \text{ kmol/h m}^2$$

$$Y_2 = 0.02 \times 0.0526 = 0.001052$$

$$Y_1 = \frac{y_1}{1-y_1} = \frac{0.05}{1-0.05} = 0.0526 \text{ kmol NH}_3\text{/kmol dry air}$$

$$Y_2 = 0.001052$$

$$y_2 = \frac{Y_2}{1+Y_2} = \frac{0.001052}{1.001052} = 0.00105$$

$$G_2 = \frac{G_s}{1-y_2} = \frac{40.354}{1-0.00105} = 40.396 \text{ kmol/h m}^2$$

$$y = 1.154x$$

$$\frac{Y}{1+Y} = \frac{1.154X}{1+X}$$

$$Y = \frac{1.154X}{1-0.154X}$$

| $X$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|
| $Y$ | 0.0116 | 0.0232 | 0.0348 | 0.0464 | 0.058 |

For minimum liquid flow rate

$$y = 1.154x, \text{ then } y_1 = 1.154x_1$$

$$0.05 = 1.154x_1, \text{ so } x_1 = 0.0433$$

$$X_1 = \frac{x_1}{1-x_1} = 0.04526 \text{ kmol NH}_3\text{/kmol water}$$

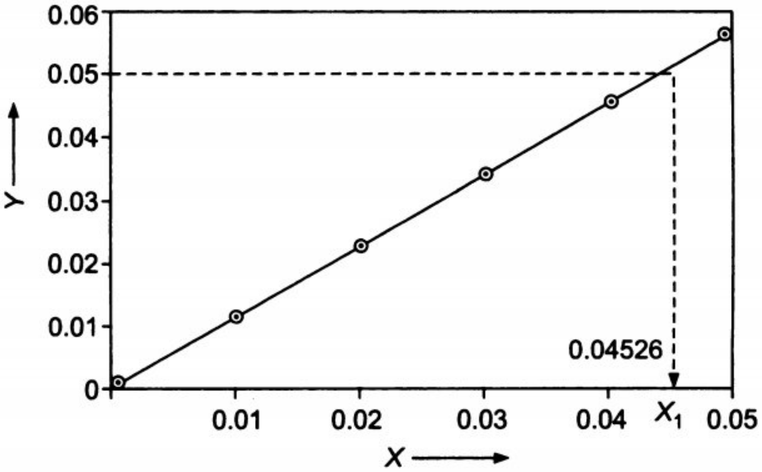


**Fig. 8.15(b)** Example 1.

(This can also be obtained from graph as shown in Fig. 8.15(b))

$$\left(\frac{L_S}{G_S}\right)_{\min} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.0526 - 0.001052)}{(0.04526 - 0)} = 1.139$$

$$(L_S)_{\min} = 40.354 \times 1.139 = 45.969 \text{ kmole/h m}^2$$

Mass flow rate of minimum water required = 45.969 × 18 = 827.44 kg/h $m^2$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = 1.25\left(\frac{L_S}{G_S}\right)_{\min} = 1.25 \times 1.139 = 1.42375$$

(b) Again, $\left(\frac{L_S}{G_S}\right)_{\text{actual}} = \left(\frac{(Y_1 - Y_2)}{(X_{1,\text{actual}} - X_2)}\right) = 1.42375 = \frac{(0.0526 - 0.001052)}{(X_{1,\text{actual}} - 0)}$

$$X_{1,\text{ actual}} = 0.0361.$$

Hence,

$$x_1 = \frac{X_1}{(1 + X_1)} = 0.0349$$

$$y_1{}^* = m\, x_1 = 1.154 \times 0.0349 = 0.0403$$

$$y_2{}^* = m\, x_2 = 0$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}}$$

$$(y - y^*)_{lm} = \frac{[(y_1 - y_1{}^*) - (y_2 - y_2{}^*)]}{\ln\left[\frac{(y_1 - y_1{}^*)}{(y_2 - y_2{}^*)}\right]}$$

$$= \frac{[(0.05 - 0.0403) - (0.001052 - 0)]}{\ln\frac{(0.05 - 0.0403)}{(0.001052)}} = 3.89 \times 10^{-3}$$

$$\text{NTU} = \frac{(0.05 - 0.001)}{3.89 \times 10^{-3}} = 12.581 \approx 13$$

(c) Average gas flow rate

$$= \frac{(G_1 + G_2)}{2} = \frac{(42.478 + 40.396)}{2} = 41.437 \text{ kmol/h/m}^2$$

$$HTU = \frac{G}{K_G a P_t} = \frac{41.437}{128} = 0.3237 \text{ m}$$

Height of the tower, Z = NTU × HTU = 12.581 × 0. 3237 = 4.073 m

**2.** Air containing methanol vapour (5-mole %) is scrubbed with water in a packed tower at 26°C and 760 mm Hg pressure to remove 95% of the methanol. The entering water is free of methanol. The gas-phase flow rate is 1.22 kmol/m$^2$ s and the liquid-phase flow rate is 0.631 kmol/m$^2$ s. If the overall height of a transfer unit based on the liquid phase resistance is 4.12 m, determine NTU and the overall liquid phase mass transfer coefficient. The equilibrium relation is $p = 0.280\ x$, where $p$ is the partial pressure of methanol in atmospheres and $x$ is the mole fraction of methanol in liquid.

**Solution.**

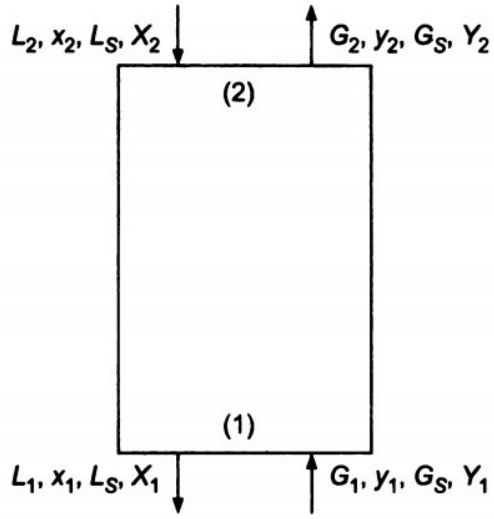


**Fig. 8.16** Example 2.

$y_1 = 0.05$, $T = 26°\text{C}$, pressure = 760 mm Hg

$$Y_1 = \frac{y_1}{(1-y_1)} = \frac{0.05}{0.95} = 0.0526$$

$$Y_2 = 0.05 \times 0.0526 = 0.00263$$

Gas flow rate = 1.22 kmol/m$^2$ s

Liquid flow rate = 0.631 kmol/m$^2$ s

$H_{toL} = 4.12$ m

Equilibrium relationship is: $p = 0.280x$

where, $p$ = partial pressure

$x$ = mole fraction of methanol in liquid.

$$L_2 = L_S = \frac{0.631}{18} = 0.0351 \text{ kmol/m}^2 \text{ s}$$

(Assuming entering water is pure)

$$\text{Average molecular weight of entering gas} = \frac{(0.05 \times 32) + (0.95 \times 28.84)}{1}$$

$$= 2.8.998$$

$$G_1 = \frac{1.22}{28.998} = 0.0421 \text{ kmol/m}^2 \text{ s}$$

$$G_S = G_1(1 - y_1) = 0.0421(1 - 0.05) = 0.04 \text{ kmol/m}^2 \text{ s}$$

Equilibrium relation is: $p = 0.280x$

$$p_t y = 0.280x \quad (p_t = \text{total pressure, } x = \text{mole fraction in liquid phase, } y = \text{mole fraction in gas phase})$$

$$1 \times y = 0.280x$$

$$y = 0.280x$$

$$\left(\frac{L_S}{G_S}\right) = \frac{(Y_1 - Y_2)}{(X_1 - X_2)}$$

$$y_2 = \frac{Y_2}{1 + Y_2} = \frac{0.00263}{(1 + 0.00263)} = 0.00262$$

$X_2 = 0$ (assuming pure water enters the absorber)

$$\left(\frac{L_S}{G_S}\right) = \frac{(Y_1 - Y_2)}{(X_1 - X_2)}$$

i.e. $$\frac{0.0351}{0.04} = \frac{(0.0526 - 0.00263)}{(X_1 - 0)}$$

Therefore, $X_1 = 0.0569$

$$L_1 = L_S(1 + X_1) = 0.0351\ (1.0569) = 0.0371 \text{ kmol/m}^2\text{ s}$$

$$L_{\text{Avg}} = (L_1 \times L_2)^{0.5} = (0.0371 \times 0.0351)^{0.5} = 0.0361 \text{ kmol/m}^2\text{ s}$$

$$x_1 = \frac{X_1}{(1 + X_1)} = \frac{0.0569}{(1 + 0.0569)} = 0.0539$$

We have, $y_1^* = 0.280x_1$

$x_1 = 0.0539$

$x_2 = 0.0$

$y_1 = 0.05$

$y_2 = 0.00262$

$$x_1^* = \frac{y_1}{0.280} = \frac{0.05}{0.280} = 0.1786$$

$$x_2^* = \frac{y_2}{0.280} = \frac{0.00262}{0.280} = 0.00936$$

$$(x^* - x)_{lm} = \frac{[(x_1^* - x_1) - (x_2^* - x_2)]}{\ln\left[\frac{(x_1^* - x_1)}{(x_2^* - x_2)}\right]}$$

$$= \frac{[(0.1786 - 0.0539) - (0.00936 - 0)]}{\ln\left[\frac{(0.1786 - 0.0539)}{(0.00936 - 0)}\right]} = 0.04455$$

$$N_{toL} = \frac{(x_1 - x_2)}{(x^* - x)_{lm}} = \frac{(0.0539 - 0)}{0.04455} = 1.21$$

$$H_{toL} = 4.12 \text{ m}$$

$$H_{toL} = \frac{L_{\text{avg}}}{K_{La}}$$

Therefore, $K_{La} = \dfrac{0.0361}{4.12} = 8.76 \times 10^{-3} \text{ kmol/m}^2 \text{ s } (\Delta x)$

3. An air-$NH_3$ mixture containing 20-mole % $NH_3$ is being treated with water in a packed tower to recover $NH_3$. The incoming gas rate is 1000 kg/h $m^2$. The temperature is 35°C and the total pressure is 1 atm. Using 1.5 times the minimum water flow rate, 95% of $NH_3$ is absorbed. If all the operating conditions remain unchanged, how much taller should the tower be to absorb 99% of $NH_3$? Henry's law is valid and $y_e = 0.746x$. Variations in gas flow rate may be neglected.

**Solution.**

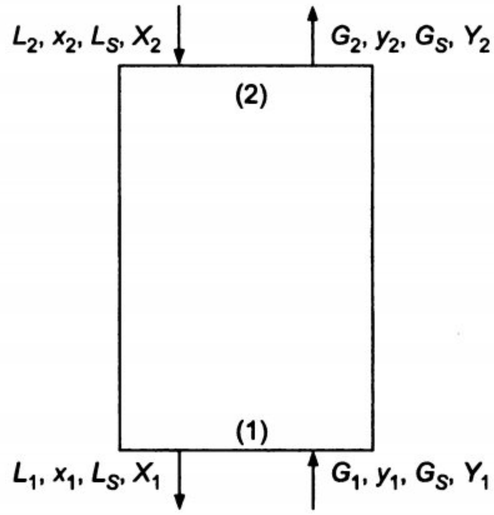


**Fig. 8.17(a)** Example 3.

Given that

$$y_1 = 0.2$$

Gas flow rate (incoming) = 1000 kg/h $m^2$
Temperature = 35°C, pressure = 1 atm

$$\text{HTU} = 1 \text{ m}$$

$$(L_S)_{\text{actual}} = 1.5 \times (L_S)_{\text{min}}$$

Assuming, incoming water to be pure, its flow rate $L_2$ is $L_S$ kmol/h $m^2$
Equilibrium relation = $y_e = 0.746x$

$$Y_1 = \frac{y_1}{(1 - y_1)} = \frac{0.2}{(1 - 0.2)} = 0.25$$

95% Ammonia is absorbed

$$Y_2 = (1 - 0.95) \times 0.25 = 0.0125$$

$$y_2 = \frac{Y_2}{1 + Y_2} = \frac{0.0125}{(1 + 0.0125)} = 0.0123$$

Average molecular weight of incoming gas mixture

$$= [(0.2 \times 17) + (0.8 \times 28.84)] = 26.472$$

$$G_1 = \frac{1000}{26.472} = 37.776 \text{ kmol/h m}^2$$

$$G_S = \mathrm{G}_1\ (1 - y_1) = 37.776(1 - 0.2) = 30.221 \text{ kmol/h m}^2$$

For minimum liquid flow rate.

$$y_1{}^* = 0.746x_1$$

$$x_1 = \frac{0.2}{0.746} = 0.2681$$

$$X_1 = \frac{x_1}{1 - x_1} = 0.3663$$

$$\left(\frac{L_s}{G_s}\right)_{\min} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.25 - 0.0125)}{(0.3663 - 0)} = 0.648$$

(Assuming pure water enters, $X_2 = 0$)

We can also obtain this graphically for which $X$–$Y$ data has to be computed.

$$y_1{}^* = 0.746x_1$$

$$\frac{Y}{(1 + Y)} = 0.746\frac{X}{(1 + X)}$$

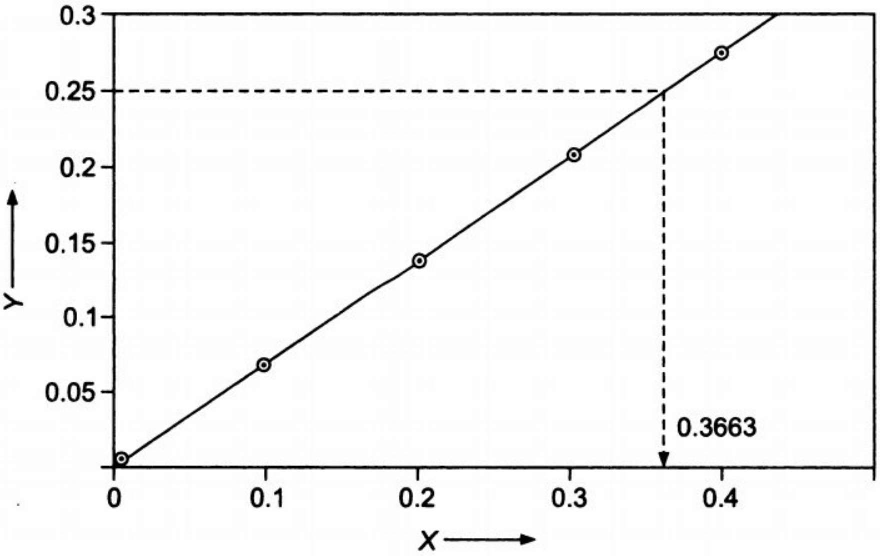


**Fig. 8.17(b)** Example 3.

$$Y = \frac{0.746X}{(1+0.254X)}$$

| X | 0 | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|---|
| Y | 0 | 0.0728 | 0.142 | 0.208 | 0.271 |

From the graph $X_{1,\max}$ = 0.3663 (which is also the same as obtained from calculation)

$$\left(\frac{L_S}{G_S}\right)_{\min} = 0.648$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = 1.5\left(\frac{L_S}{G_S}\right)_{\min} = 1.5 \times 0.648 = 0.972$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.25 - 0.0125)}{(X_1 - 0)}$$

$$X_1 = 0.2443$$

$$X_1 = \frac{X_1}{(1 + X_1)} = 0.1963$$

$$y_1{}^* = 0.746x_1$$

$$y_1{}^* = 0.746 \times 0.1963 = 0.1464$$

$$(y - y^*)_{lm} = \frac{[(y_1 - y_1{}^*) - (y_2 - y_2{}^*)]}{\ln\left[\dfrac{(y_1 - y_1{}^*)}{(y_2 - y_2{}^*)}\right]}$$

$$= \frac{[(0.2 - 0.1464) - (0.0123 - 0)]}{\ln\dfrac{(0.2 - 0.1464)}{(0.0123 - 0)}} = 0.0281$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}} = \frac{(0.2 - 0.0123)}{0.0281} = 6.68$$

$$Z = \text{HTU} \times \text{NTU} = 1 \times 6.68 = 6.68 \text{ m}$$

Now if 99% of $NH_3$ is absorbed,

$$Y_2 = 0.25 \times 0.01 = 0.0025$$

$$y_2 = \frac{Y_2}{1 + Y_2} = 0.0025$$

For, $\left(\dfrac{L_S}{G_S}\right)_{\min}$

$$y_1{}^* = 0.746x_1$$

$$x_1 = \frac{X_1}{(1 + X_1)} = \frac{0.2}{0.746} = 0.2681$$

$$X_1 = \frac{x_1}{1 - x_1} = 0.3663$$

$$\left(\frac{L_S}{G_S}\right)_{\text{min}} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.25 - 0.0025)}{(0.3663 - 0)} = 0.6755$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = 1.5 \times 0.6755 = 1.013$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.25 - 0.0025)}{(X_1 - 0)}$$

$$X_1 = 0.2443$$

$$x_1 = \frac{X_1}{(1 + X_1)} = 0.1963$$

$$y_1^* = 0.746 \times 0.1963 = 0.1464$$

$$(y - y^*)_{lm} = \frac{[(y_1 - y_1^*) - (y_2 - y_2^*)]}{\ln\left[\frac{(y_1 - y_1^*)}{(y_2 - y_2^*)}\right]}$$

$$= \frac{[(0.2 - 0.1464) - (0.0025 - 0)]}{\ln \frac{(0.2 - 0.1464)}{(0.0025 - 0)}} = 0.01667$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}} = \frac{(0.2 - 0.0025)}{0.01667} = 11.847$$

$$Z = \text{NTU} \times \text{HTU} = 11.847 \times 1 = 11.847 \text{ m.}$$

In the first case, when 95% of $NH_3$ was absorbed, $Z = 6.68$ m
Increase in length of tower = 11.847 − 6.68 = 5.168 m
So, when 99% of $NH_3$ is to be absorbed, the tower should be 5.168 m taller than that needed for 95% $NH_3$ absorption, or 77.36% taller.

**4.** An effluent gas containing 12% $C_6H_6$ is to be scrubbed in a packed column, operating at 43°C and 1 atm. pressure. The column is to be designed for treating 15 $m^3$ of entering gas per hour per $m^2$ of column cross-section, such that the exit gas will contain 1% benzene. The solvent for scrubbing is mineral oil which will enter the top of the column at a rate of 28 kg/h $m^2$ and a benzene content of 1%. Determine the height of the column assuming height of transfer unit to be 0.75 m. The equilibrium concentration at the operating conditions is given by $y^* = 0.263x$, where $x$ and $y$ are in mole fraction units.

**Solution.**

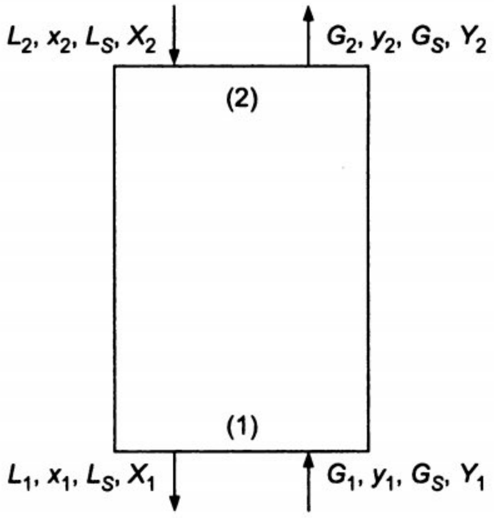


**Fig. 8.18** Example 4.

$y_1 = 0.12$, $T = 43°\text{C}$, pressure = 1 atm

Gas flow rate = 15 $m^3/h\ m^2$

$y_2 = 0.01$

Solvent is mineral oil

$L_2 = 28$ kmol/h $m^2$, $x_2 = 0.01$,

HTU = 0.75 m

The equilibrium relation is $y^* = 0.263x$

Assuming the gas mixture to be ideal,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{(1 \times 15)}{(273+43)} = \frac{(1 \times V_2)}{273}$$

$$V_2 = 12.9589 \text{ m}^3 \text{ (at N.T.P)}$$

or Molar flow rate $= \dfrac{12.9589}{22.414} = 0.5782$ kmol

$$G_1 = 0.5782 \text{ kmol/h m}^2$$

$$G_S = G_1\,(1 - y_1) = 0.5782(1 - 0.12) = 0.5088 \text{ kmol/h m}^2$$

$$L_S = L_2\,(1 - x_2) = 28(1 - 0.01) = 27.72 \text{ kmol/h m}^2$$

$$\left(\frac{L_S}{G_S}\right) = \frac{(Y_1 - Y_2)}{(X_1 - X_2)}$$

$$Y_1 = \frac{y_1}{(1-y_1)} = \frac{0.12}{(1-0.12)} = 0.1364$$

$$Y_2 = \frac{y_2}{(1 - y_2)} = \frac{0.01}{(1 - 0.01)} = 0.0101$$

$$X_2 = \frac{x_2}{(1 - x_1)} = \frac{0.01}{(1 - 0.01)} = 0.0101$$

$$\left(\frac{L_S}{G_S}\right) = \frac{27.72}{0.5088} = \frac{(0.1364 - 0.0101)}{(X_1 - 0.0101)}$$

$$X_1 = 0.01242$$

$$x_1 = \frac{X_1}{(1 + X_1)} = 0.0123$$

$$y_1^* = mx_1$$

$$y_1^* = 0.263 \times 0.0123 = 0.00323$$

$$y_2^* = 0.263 \times 0.01 = 0.00263$$

$$(y - y^*)_{lm} = \frac{[(y_1 - y_1^*) - (y_2 - y_2^*)]}{\ln\left[\dfrac{(y_1 - y_1^*)}{(y_2 - y_2^*)}\right]}$$

$$= \frac{[(0.12 - 0.00323) - (0.01 - 0.00263)]}{\ln\dfrac{(0.12 - 0.00323)}{(0.01 - 0.00263)}} = 0.0395$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}} = \frac{(0.12 - 0.01)}{0.0395} = 2.786 \approx 3$$

Height of tower, $Z = \text{NTU} \times \text{HTU} = 2.786 \times 0.75 = 2.0895$ m.

**5.** An air–$NH_3$ mixture containing 5% $NH_3$ is being scrubbed with water in a packed tower to recover 95% $NH_3$. $G_1 = 3000$ kg/h m$^2$, $L_s = 2500$ kg/h m$^2$. Tower is maintained at 25°C and 1 atm pressure. Find NTU and height of the tower. The equilibrium relation is given by $y^* = 0.98x$, where $x$ and $y$ are mole fraction units. $K_Ga = 65$ kmol/h m$^3$ atm

**Solution.**

$$y_1 = 0.05$$

$$Y_1 = \frac{0.05}{(1 - 0.05)} = 0.0526$$

$$Y_2 = 0.05 \times 0.0526 = 0.00263$$

$$y_2 = \frac{Y_2}{1 + Y_2} = \frac{0.00263}{(1 + 0.00263)} = 0.00262$$

Entering gas flow rate = 3000 kg/h m$^2$

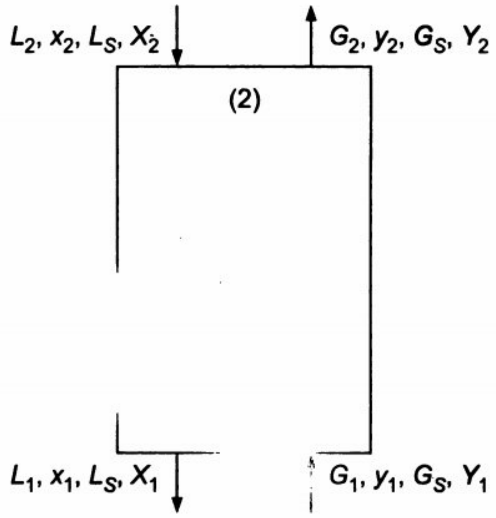


**Fig. 8.19** Example 5.

$L_S$ = 2500 kg/h m$^2$, T = 25°C, Pressure = 1 atm,

$K_G a$ = 65 kmol/h m$^3$ atm

Equilibrium relation = $y^* = 0.98x$

Average molecular weight of incoming gas mixture

$$= \frac{(0.05 \times 17) + (0.95 \times 28.84)}{1} = 28.248$$

$$G_1 = \frac{3000}{28.248} = 106.20 \text{ kmol/h m}^2$$

$$L_S = \frac{2500}{18} = 138.89 \text{ kmol/h m}^2$$

$$G_S = G_1(1 - y_1) = 106.2(1 - 0.05) = 100.89 \text{ kmol/h m}^2$$

$$G_2 = \frac{G_S}{1 - y_2} = \frac{100.89}{(1 - 0.00262)} = 101.16 \text{ kmol/h m}^2$$

$$\left(\frac{L_S}{G_S}\right) = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{138.89}{100.89} = \frac{(0.0526 - 0.00263)}{(X_1 - 0)}$$

Therefore, $X_1 = 0.0363$

$$x_1 = \frac{X_1}{(1 + X_1)} = \frac{0.0363}{(1 + 0.0363)} = 0.035$$

$$y_1^* = 0.98x_1$$

$$y_1^* = 0.98 \times 0.035 = 0.0343$$

$$x_2 = 0;\ y_2^* = 0$$

$$(y - y^*)_{lm} = \frac{[(y_1 - y_1^*) - (y_2 - y_2^*)]}{\ln\left[\dfrac{(y_1 - y_1^*)}{(y_2 - y_2^*)}\right]}$$

$$= \frac{[(0.05 - 0.0343) - (0.00262 - 0)]}{\ln\dfrac{(0.05 - 0.0343)}{(0.00262 - 0)}} = 0.0073$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}} = \frac{(0.05 - 0.00262)}{0.0073} = 6.486$$

$$G_{\text{avg}} = \frac{(G_1 + G_2)}{2} = \frac{(106.2 + 101.16)}{2} = 103.68 \text{ kmol/m}^2 \text{ h}$$

$$\text{HTU} = \frac{G_{\text{avg}}}{K_{Ga}P_t} = \frac{103.68}{(65 \times 1)} = 1.595 \text{ m}$$

$$Z = \text{NTU} \times \text{HTU} = 6.486 \times 1.595 = 10.346 \text{ m.}$$

**6.** An air-$C_6H_6$ mixture containing 5% benzene enters a countercurrent absorption tower where it is absorbed with hydrocarbon oil. $G_s = 600$ kmol/h. The solubility follows Raoult's law. Temperature at 26.7°C and 1 atm pressure are the operating conditions. The average molecular weight of oil is 200. The vapour pressure of benzene at 26.7°C is 103 mm Hg.

Find:

(i) $(L_S)_{\min}$ to recover 90% of entering $C_6H_6$.

(ii) The number of theoretical stages if 1.5 times the minimum liquid rate used.

(iii) The concentration of solute in liquid learning the absorber for condition (ii).

**Solution.**

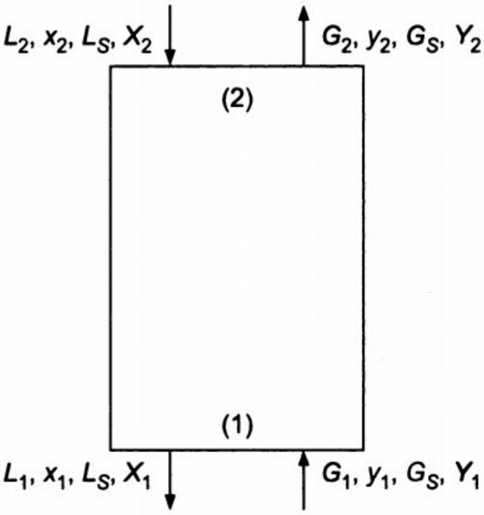


**Fig. 8.20(a)** Example 6.

Given that

$y_1 = 0.05$, $G_S = 600$ kmol/h $T = 26.7°C$, Pressure = 1 atm

Average molecular weight of oil = 200, $p_A = 103$ mm Hg
According to Raoult's law,

$$p^* = p_A x_A$$

$$y^* = \frac{p^*}{P_t} = \frac{(p_A x_A)}{P_t} = \frac{103}{760}x = 0.1355x$$

$$Y_1 = \frac{y_1}{(1-y_1)} = \frac{0.05}{(1-0.05)} = 0.0526$$

$$Y_2 = (0.1 \times 0.0526) = 0.00526$$

$$y_2 = \frac{Y_2}{1+Y_2} = \frac{0.00526}{(1+0.00526)} = 0.00523$$

$$X_2 = 0. \text{ (Assuming pure oil enters)}$$

We have, $y^* = 0.1355x$

$$\frac{Y}{(1+Y)} = 0.1355\frac{X}{(1+X)}$$

Therefore, $$Y = \frac{0.1355x}{(1+0.8645x)}$$

| $X$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|---|
| $Y$ | 0 | 0.0125 | 0.023 | 0.03228 | 0.0403 | 0.0473 | 0.0535 |

From the graph, we can get, $X_{1,\,\max} = 0.54$

$$\left(\frac{L_S}{G_S}\right)_{\min} = \frac{(Y_1 - Y_2)}{(X_{1,\max} - X_2)}$$

For minimum flow rate of oil,

$$\left(\frac{L_S}{G_S}\right)_{\min} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.0526 - 0.00526)}{(0.54 - 0)} = 0.0877$$

$$(L_S)_{\min} = 0.0877 \times 600 = 52.62 \text{ kmol/h}$$

$$(L_S)_{\text{actual}} = 1.5(L_S)_{\min}$$

$$(L_S)_{\text{actual}} = 1.5 \times 52.62 = 78.93 \text{ kmol/h}$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{78.93}{600} = \frac{(0.0526 - 0.00526)}{(X_1 - 0)}$$

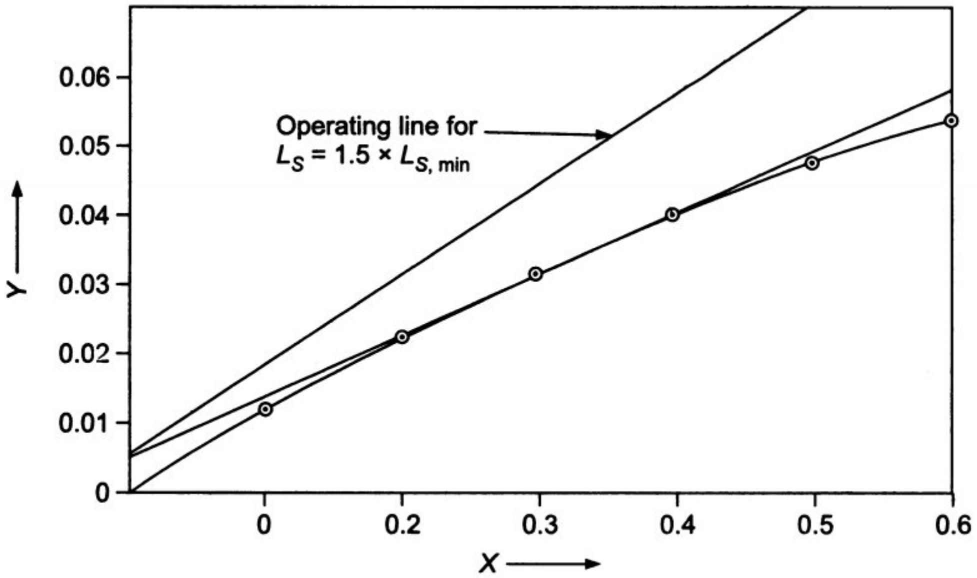


**Fig. 8.20(b)** Example 6.

$X_1$ = 0.36 (which is the same as obtained from graph)

$$x_1 = \frac{X_1}{(1 + X_1)} = \frac{0.36}{(1 + 0.36)} = 0.265$$

The number of stages by stepwise construction is 6.

7. It is desired to absorb 95% of acetone from feed mixture of acetone and air containing 2 (mole) % of acetone using a liquid flow rate of 20 % more than the minimum. Gas flow rate is 450 kg/h. The gas mixture enters at 25°C and 1 atm pressure, which is the operating condition. The equilibrium relation is $y^* = 2.5x$. Find (i) Flow rate of water, and (ii) Number of theoretical plates, when the operation is carried out countercurrently.

**Solution.**

$$y_{Np+1} = 0.02 \qquad Y_{Np+1} = \frac{y_{Np+1}}{(1 - y_{Np+1})} = 0.0204$$

$$Y_1 = 0.0204 \times 0.05 = 0.00102,\ y_1 = 0.00102$$

$$(L_S)_{\text{actual}} = 20\% \text{ more than } (L_S)_{\min}$$

Gas flow rate (entering) = 450 kg/h

$$T = 25°\text{C}, \quad \text{Pressure} = 1 \text{ atm}, \quad y^* = 2.5x$$

Average molecular weight of feed mixture

$$= \frac{(0.02 \times 58) + (0.98 \times 28.84)}{1} = 29.42$$

$$G_{Np+1} = \frac{450}{29.42} = 15.296 \text{ kmol/h}$$

$$G_S = G_{Np+1}(1 - y_{Np+1}) = 15.296\ (1 - 0.02) = 14.99 \text{ kmol/h}$$

Equilibrium relation is $y^* = 2.5x$

$$\text{(i.e.)} \quad \frac{Y}{(1+Y)} = 2.5\frac{X}{(1+X)}$$

$$Y = \frac{2.5X}{(1-1.5X)}$$

| $X$ | 0.0 | 0.002 | 0.004 | 0.006 | 0.008 | 0.01 |
|---|---|---|---|---|---|---|
| $Y$ | 0.0 | 0.005 | 0.01 | 0.0151 | 0.0202 | 0.0254 |

$$X_{Np} = \frac{x_{Np}}{(1-x_{Np})} = \frac{0.008}{(1-0.008)} = 0.00806$$

$$\left(\frac{L_S}{G_S}\right)_{\text{min}} = \frac{(Y_{Np+1} - Y_1)}{(X_{Np} - X_0)} = \frac{(0.0204 - 0.001)}{(0.00806 - 0)}$$

(Assuming pure water enters, $X_0 = 0$) $\left(\frac{L_S}{G_S}\right)_{\text{min}} = 2.4069$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = \left(\frac{L_S}{G_S}\right)_{\text{min}} \times (1 + 0.20) = 2.4069 \times 1.2 = 2.888$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = \frac{(Y_{Np+1} - Y_1)}{(X_{Np} - X_0)} = \frac{(0.0204 - 0.001)}{(X_{Np} - 0)}$$

$$X_{Np} = 0.00672$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = 2.888$$

$$(L_S)_{\text{actual}} = 2.888 \times 14.8205 = 42.802 \text{ kmol/h}$$

(ii) $X_{Np} = 0.00672, \quad x_{Np} = \dfrac{0.00672}{(1+0.00672)} = 0.00667$

$$(L_S) = L_{Np}(1 - x_{Np})$$

$$L_{Np} = \frac{42.802}{(1-0.00667)} = 43.089 \text{ kmol/h}$$

$$G_S = G_1\ (1 - y_1)$$

$$G_1 = \frac{14.8205}{(1-0.01)} = 14.835 \text{ kmol/h}$$

(Assuming pure water enters, $L_0 = L_S$)

$$A_1 = \frac{L_0}{mG_1} = \frac{42.802}{(2.5 \times 14.835)} = 1.154$$

$$A_2 = \frac{L_{Np}}{mG_{Np+1}} = \frac{43.089}{(2.5 \times 15.213)} = 1.133$$

$$A = \sqrt{A_1A_2} = \sqrt{(1.154 \times 1.133)} = 1.143$$

$$N_p = \frac{\log\left[\frac{(y_{Np+1} - m_{x0})}{(y_1 - m_{x1})} \times \left(1 - \frac{1}{A}\right) + \left(\frac{1}{A}\right)\right]}{\log A}$$

$$= \frac{\log\left[\frac{(0.02 - (2.5 \times 0))}{(0.001 - 0)} \times \left(1 - \frac{1}{1.143}\right) + \left(\frac{1}{1.143}\right)\right]}{\log 1.143} = 9.1 \approx 10$$

8. A soluble gas is absorbed in water using a packed tower. The equilibrium relation is $Y_e = 0.06X_e$. $H_x = 0.24$ m, $H_y = 0.36$ m. Find $H_{toG}$.

**Solution.**

Given

$X_2 = 0$, $X_1 = 0.08$, $Y_2 = 0.0005$, $Y_1 = 0.009$, where $X$ and $Y$ are mole ratios.

$Y_e = 0.06X_e$.

| $X$ | 0 | 0.02 | 0.04 | 0.06 | 0.08 |
|---|---|---|---|---|---|
| $x = \frac{X}{(1+X)}$ | 0 | 0.0196 | 0.038 | 0.057 | 0.074 |
| $Y = 0.06X$ | 0 | 0.0012 | 0.0024 | 0.0036 | 0.0048 |
| $y = \frac{Y}{(1+Y)}$ | 0 | 0.0012 | 0.0024 | 0.0036 | 0.0048 |
| $m = \frac{y}{x}$ | — | 0.0612 | 0.0632 | 0.0632 | 0.0649 |

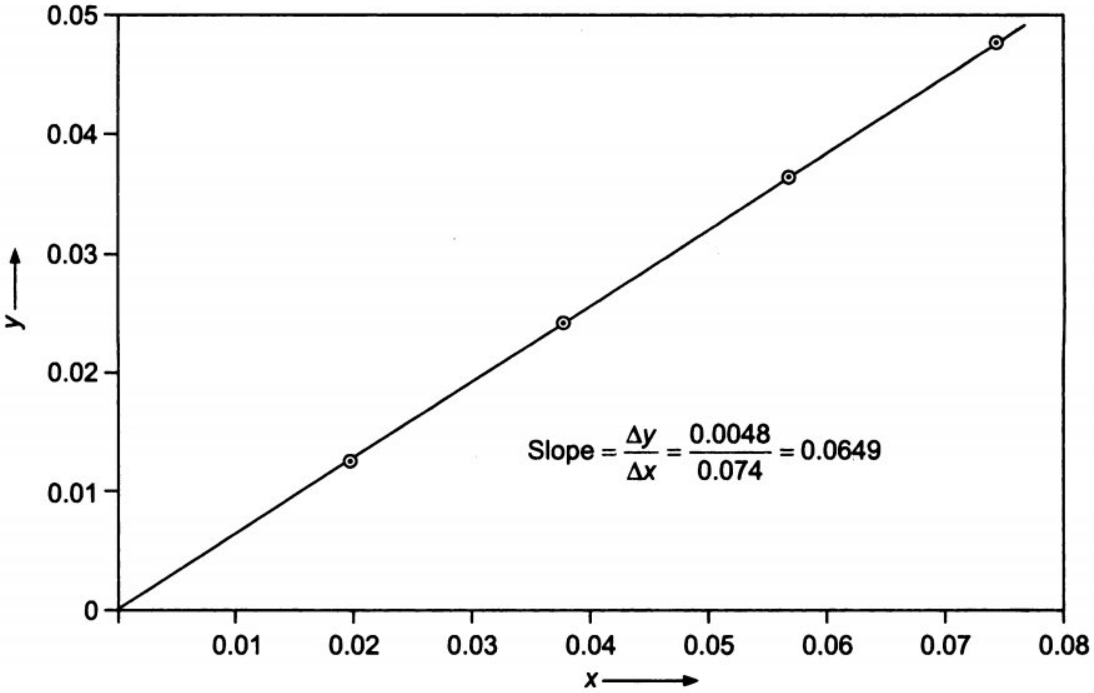


**Fig. 8.21** Example 8.

Average $m$ = 0.063

$$H_{toG} = H_y + \left(\frac{mG}{L}\right)H_x$$

$$H_x = 0.24 \text{ m}, H_y = 0.36 \text{ m}, X_1 = 0.08, Y_1 = 0.10, X_2 = 0, Y_2 = 0.005$$

$$\left(\frac{L_S}{G_S}\right) = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.1 - 0.005)}{(0.08 - 0)} = 1.1875$$

$$H_{toG} = H_y + \left(\frac{mG}{L}\right)H_x = 0.36 + \left(\frac{0.063 \times 1}{1.1875}\right) \times 0.24 = 0.3727 \text{ m}$$

(Since absolute flow rates are not available, we have taken the flow rates on solute free basis.)

9. Acetone is to be recovered from a 5% acetone air mixture by scrubbing with water in a packed tower using countercurrent flow. Both liquid and gas rates are 0.85 kg/m$^2$ s and 0.5 kg/m$^2$ s respectively. $K_Ga$ =1.5 ×10$^{-4}$ kmol/m$^2$ s (kN/m$^2$) partial pressure difference and the gas film resistance controls the process. What should be the height of the tower to remove 98 % acetone? The equilibrium data in mole fractions are as follows:

| | | | | |
|---|---|---|---|---|
| $x$ | 0.0099 | 0.0196 | 0.036 | 0.04 |
| $y$ | 0.0076 | 0.0156 | 0.0306 | 0.0333 |

**Solution.**

$$y_1 = 0.05;$$

$$Y_1 = \frac{y_1}{(1-y_1)} = \frac{0.05}{(1-0.05)} = 0.05263$$

$$L_2 = 0.85 \text{ kg/m}^2 \text{ s}, \quad K_Ga = 1.5 \times 10^{-4} \text{ kmol/m}^2 \text{ s (kN/m}^2)$$

Gas flow rate = 0.5 kg/m² s

$$Y_2 = 0.05263 \times 0.02 = 0.001053$$

$$y_2 = \frac{0.001053}{(1+0.001053)} = 0.00105$$

| $x$ | 0.0099 | 0.0196 | 0.036 | 0.04 |
|---|---|---|---|---|
| $y$ | 0.0076 | 0.0156 | 0.0306 | 0.0333 |
| $m = \frac{y}{x}$ | 0.7677 | 0.7959 | 0.85 | 0.8325 |
| $X = \frac{x}{(1-x)}$ | 0.01 | 0.02 | 0.037 | 0.042 |
| $Y = \frac{y}{(1-y)}$ | 0.0077 | 0.0158 | 0.0316 | 0.0344 |

$$m_{\text{average}} = \frac{0.7677 + 0.7959 + 0.85 + 0.8325}{4} = 0.8115$$

Hence, the equilibrium relation will be $y^* = 0.8115x$

Average molecular weight of gas feed mixture

$$= \frac{(0.05 \times 58) + (0.95 \times 28.84)}{1} = 30.298$$

$$G_1 = \frac{0.5}{30.298} = 0.0165 \text{ kmol/m}^2 \text{ s}$$

$$G_S = G_1\ (1 - y_1) = 0.0165(1 - 0.05) = 0.0157 \text{ kmol/m}^2 \text{ s}$$

$$G_2 = Gs(1 + Y_2) = 0.0157 \times (1 + 0.001053) = 0.01572 \text{ kmol/m}^2 \text{ s}$$

Assuming pure water enters, so $L_2 = L_S = \frac{0.85}{18} = 0.0472 \text{ kmol/m}^2 \text{ s}$

$$\left(\frac{L_S}{G_S}\right) = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{0.0472}{0.0157} = \frac{(0.05263 - 0.001053)}{(X_1 - 0)}; \qquad X_1 = 0.01716$$

$$x_1 = \frac{X_1}{(1+X_1)} = \frac{0.01716}{(1+0.01716)} = 0.01687$$

$$y_1^* = mx_1 = 0.8115 \times 0.01687 = 0.01369,\ y_2^* = 0 \text{ (since } x_2 = 0)$$

$$(y - y^*)_{lm} = \frac{[(y_1 - y_1^*) - (y_2 - y_2^*)]}{\ln\left[\dfrac{(y_1 - y_1^*)}{(y_2 - y_2^*)}\right]}$$

$$= \frac{[(0.05 - 0.01369) - (0.00105 - 0)]}{\ln\left[\dfrac{(0.05 - 0.01369)}{(0.00105 - 0)}\right]} = 0.00995$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}} = \frac{(0.05 - 0.00105)}{0.00995} = 4.92$$

$$G_1 = 0.0165 \text{ kmol/m}^2 \text{ s};\ G_2 = 0.01572 \text{ kmol/m}^2 \text{ s}$$

$$G_{\text{average}} = G = \sqrt{G_1 G_2} = 0.01611 \text{ kmol/m}^2 \text{ s}$$

$$\text{HTU} = \frac{G}{K_{Ga}} = \frac{0.01611}{\left[(1.5 \times 10^{-4}) \times (1.013 \times 10^2)\right]} = 1.06$$

$$Z = \text{NTU} \times \text{HTU} = 4.92 \times 1.06 = 5.216 \text{ m}$$

**(Alternative method)**
We can also draw the equilibrium curve and operating line (on mole ratio basis) and evaluate $\int \frac{dY}{(Y - Y^*)}$ between the limits $Y_1 = 0.001$ and $Y_2 = 0.0525$ graphically. The values of $Y$ and $Y^*$ have been presented below.

| $Y^*$ | $Y$ | $\frac{1}{(Y - Y^*)}$ |
|---|---|---|
| 0.000 | 0.001 | 1000 |
| 0.001 | 0.005 | 250 |
| 0.0025 | 0.01 | 133.3 |
| 0.005 | 0.0175 | 80 |
| 0.0075 | 0.0275 | 50 |
| 0.01 | 0.03625 | 38.1 |
| 0.01125 | 0.04125 | 33.33 |
| 0.0125 | 0.04625 | 29.6 |
| 0.014 | 0.0525 | 25.97 |

The $N_{OG}$ thus calculated is 4.9, which is in close agreement with the value reported above.

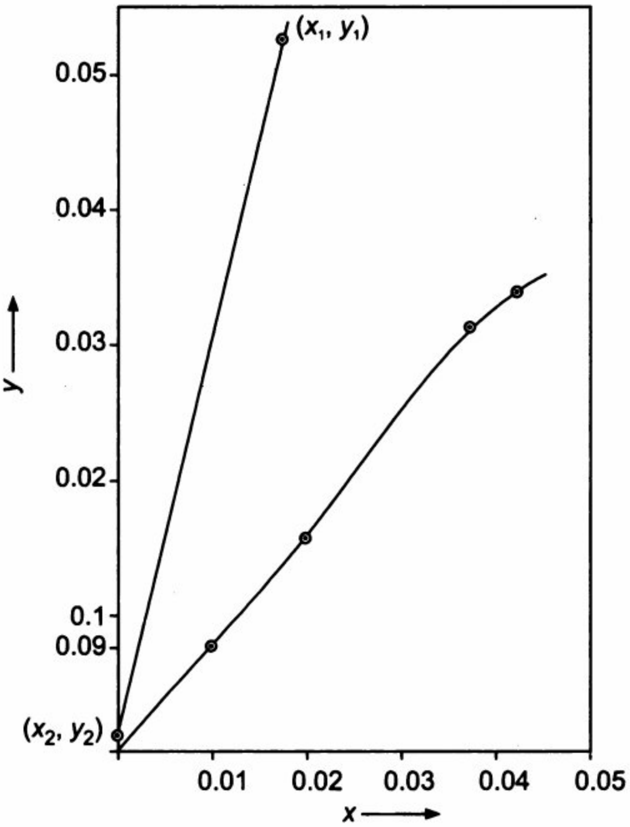


Fig. 8.22(a) Example 9.

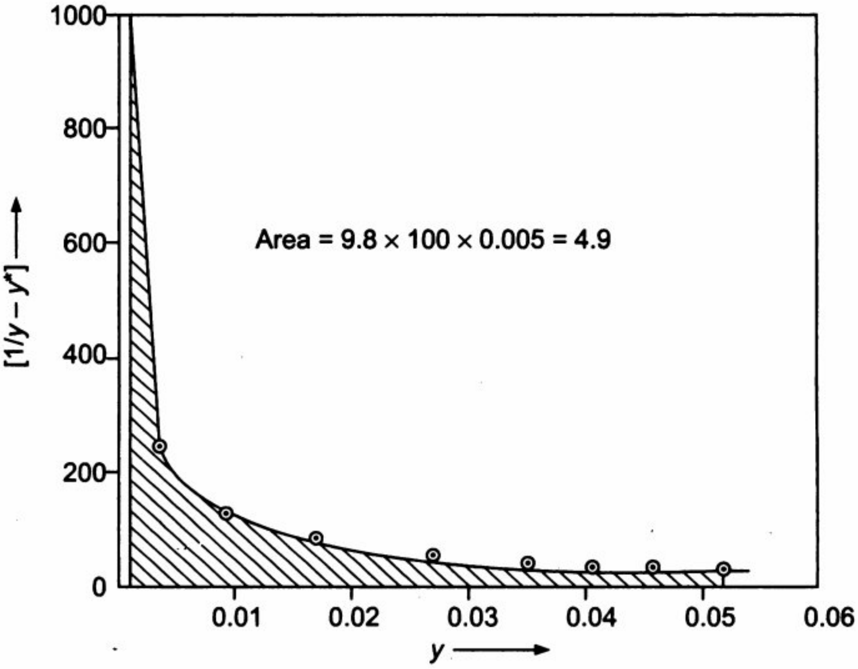


Fig. 8.22(b) Example 9.

10. A countercurrent packed absorption tower is to be designed to handle a gas containing 5% $C_6H_6$, 95% air at 26.5°C and 1 atm. At the top of the tower, a non-volatile oil is to be introduced containing 0.2% $C_6H_6$ by weight. The other data are as follows

$$L_S = 2000 \text{ kg/h}$$

Molecular weight of oil = 230
Vapour pressure of $C_6H_6$ at 26.5°C = 106 mm Hg.
Volumetric flow rate of inlet gas = 1140 $m^3$/h at 26.5°C and 1 atm.

$$K_y a = 34.8 \text{ kmol/h m}^3 \text{ (mole fraction).}$$

Mass velocity of entering gas = 1460 kg/h $m^2$.
Calculate the height and the diameter of packed tower for 90% $C_6H_6$ recovery. Raoult's law is valid.

**Solution.**

Given that

$$y_1 = 0.05, \text{ gas flow rate} = 1140 \text{ m}^3\text{/h at } 26.5°\text{C, 1 atm}$$

Pressure ($p_t$) = 1 atm

Liquid flow rate = 2000 kg/h

Molecular weight of oil = 230

Vapour pressure of $C_6H_6$ = 106 mm Hg

$$K_y a = 34.8 \text{ kmol/h m}^3 \text{ (mole fraction)}$$

Mass velocity of inert gas = 1460 kg/h $m^2$

$$y_2 = 0.05 \times 0.1 = 0.005$$

According to Raoult's law:

$$p = p_A x_A$$

$$P_t \times y = p_A \times x$$

$$y = \frac{p}{P_t} = \frac{(p_A x_A)}{P_t} = \frac{106}{706} x = 0.1395x$$

$$\text{Average moleculor weight of incoming gas} = \frac{(0.05 \times 78) + (0.95 \times 28.84)}{1}$$

$$= 31.3$$

$$\text{Mass velocity of incoming gas in moles} = \frac{1460}{31.3} = 46.645 \text{ kmol/h m}^2$$

$$Y_1 = \frac{y_1}{(1 - y_1)} = \frac{0.05}{(1 - 0.05)} = 0.0526$$

$$Y_2 = (0.1 \times 0.0526) = 0.00526$$

$$y_2 = \frac{Y_2}{1+Y_2} = \frac{0.00526}{(1+0.00526)} = 0.00523$$

$$x_2 = 0.002,\ X_2 = \frac{x_2}{(1-x_1)} = \frac{0.002}{(1-0.002)} = 0.00204$$

Mass velocity of incoming gas $= \dfrac{1460}{31.3} = 46.645$ kmol/h m$^2$

Volumetric flow rate of incoming gas = 1140 m$^3$/h at 26.5°C and 1 atm
Assume that mixture follows ideal gas law,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{\left(1\times 1140\right)}{\left(299.5\right)} = \frac{\left(1\times V_2\right)}{273}$$

$$V_2 = 1039.132 \text{ m}^3\text{/h.}$$

$$\text{Molar flow rate} = \frac{1039.132}{22.414} = 46.361 \text{ kmol/h}$$

$$G_1 = 46.361 \text{ kmol/h.}$$

We know that $y = 0.1395x$

$$\frac{Y}{(1+Y)} = 0.1395\frac{X}{(1+X)}$$

$$Y = \frac{0.1395X}{(1+0.8605X)}$$

$$\text{Area of cross section} = \frac{\text{Volumetric flow rate}}{\text{Mass velocity}}$$

$$\frac{\pi D^2}{4} = \frac{46.361}{46.645} \qquad D = 1.1249 \text{ m}$$

$$L_S = \frac{2000}{230} = 8.696 \text{ kmol/h}$$

$$G_S = G_1(1-y_1) = 46.361(1-0.05) = 44.043 \text{ kmol/h}$$

$$G_2 = \frac{G_S}{1-y_2} = \frac{44.043}{(1-0.00523)} = 44.275 \text{ kmol/h}$$

$$\left(\frac{L_S}{G_S}\right) = \frac{\left(Y_1 - Y_2\right)}{\left(X_1 - X_2\right)} = \frac{8.696}{44.043} = \frac{\left(0.0526 - 0.00526\right)}{\left(X_1 - 0.00204\right)}$$

Therefore, $X_1 = 0.242$

$$x_1 = \frac{X_1}{(1+X_1)} = \frac{0.242}{(1+0.242)} = 0.1948$$

$$y_1{}^* = mx_1 = 0.1395 \times 0.1948 = 0.0272$$

$$y_2{}^* = 0.1395 \times 0.002 = 0.000279$$

$$\left(y - y^*\right)_{lm} = \frac{\left[(y_1 - y_1{}^*) - (y_2 - y_2{}^*)\right]}{\ln\left[\dfrac{(y_1 - y_1{}^*)}{(y_2 - y_2{}^*)}\right]}$$

$$= \frac{\left[(0.05 - 0.0272) - (0.00523 - 0.00279)\right]}{\ln\dfrac{(0.05 - 0.00272)}{(0.00523 - 0.00279)}} = 0.01133$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}} = \frac{(0.05 - 0.005)}{0.01133} = 3.95 \approx 4$$

$$G_{\text{average}} = \sqrt{G_1 G_2} = 45.306 \text{ kmol/h}$$

$$\text{Cross sectional area} = \frac{\pi D^2}{4}$$

Diameter = 1.1249 m (calculated earlier)

$$\text{Cross sectional area} = \frac{\pi D^2}{4} = \frac{\pi \times (1.1249)^2}{4} = 0.9938 \text{ m}^2$$

$$\text{HTU} = \frac{G}{K_y a} = \frac{45.306}{[0.9938 \times 34.8]} = 1.31 \text{ m}$$

$$Z = \text{NTU} \times \text{HTU} = 4 \times 1.31 = 5.14 \text{ m}$$

**11.** It is desired to recover 98 % of $NH_3$ from air – $NH_3$ mixture containing 2% $NH_3$ at 20°C and 1 atm by scrubbing with water in a tower packed with 2.54 cm stoneware Raschig rings. If the gas flow rate is 19.5 kg/min m$^2$ at the inlet and liquid flow rate is 1.8 times the minimum, estimate the height of the tower for a countercurrent operation. Absorption is isothermal. $y^* = 0.746x$, where $x$ and $y$ are mole fractions. $K_G a$ = 1.04 (kmol/min m$^3$ atm.)

**Solution.**

$$y_1 = 0.02,$$

$$Y_1 = \frac{y_1}{(1 - y_1)} = \frac{0.02}{(1 - 0.02)} = 0.02041$$

$$Y_1 = 0.02041 \times 0.02 = 0.00041$$

$$y_2 = \frac{Y_2}{1+Y_2} = \frac{0.00041}{1.00041} = 0.00041$$

Gas flow rate = 19.5 kg/min m$^2$

$$(L_S)_{\text{actual}} = 1.8 \times (L_S)_{\min}$$

Equilibrium relation = $y^* = 0.746x$

$$\frac{Y}{(1+Y)} = 0.746\frac{X}{(1+X)}$$

Therefore,

$$Y = \frac{0.746X}{(1+0.254X)}$$

| $X$ | 0 | 0.010 | 0.020 | 0.025 | 0.03 |
|---|---|---|---|---|---|
| $Y = \dfrac{0.746X}{(1+0.254X)}$ | 0 | 0.00744 | 0.01484 | 0.0185 | 0.0222 |

We can calculate minimum liquid flow rate using the equilibrium relationship or from the graph shown in Fig. 8.23.

$$y_1^* = mx_1, \quad x_1 = \frac{0.0242}{0.746} = 0.0268$$

$$X_1 = \frac{x_1}{1-x_1} = \frac{0.0268}{(1-0.0268)} = 0.0275$$

$$Y_1 = \frac{y_1}{(1-y_1)} = \frac{0.02}{(1-0.02)} = 0.02041$$

$$Y_2 = \frac{y_2}{(1-y_2)} = \frac{0.0004}{\left(1-0.0004\right)} = 0.0004002$$

From Graph also we get, $X_1 = 0.0275$

$$\left(\frac{L_S}{G_S}\right)_{\min} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = \frac{(0.02041 - 0.00041)}{(0.0275 - 0)} = 0.7273$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = 0.7273 \times 1.8 = 1.309$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)} = 1.309 = \frac{(0.02041 - 0.00041)}{(X_1 - 0)} = X_1 = 0.01528$$

$$x_1 = \frac{X_1}{(1+X_1)} = \frac{0.01528}{(1+0.01528)} = 0.01505$$

$$y_1^* = 0.746x_1 = 0.746 \times 0.01505 = 0.01123, \qquad y_2^* = 0$$

$$(y - y^*)_{lm} = \frac{[(y_1 - y_1^*) - (y_2 - y_2^*)]}{\ln\left[\dfrac{(y_1 - y_1^*)}{(y_2 - y_2^*)}\right]}$$

$$= \frac{[(0.02 - 0.01123) - (0.00041 - 0)]}{\ln\left[\dfrac{(0.02 - 0.01123)}{(0.00041 - 0)}\right]} = 0.00273$$

$$\text{NTU} = \frac{(y_1 - y_2)}{(y - y^*)_{lm}} = \frac{(0.02 - 0.00041)}{0.00273} = 7.176$$

Average molecular weight of incoming gas

$$= \frac{(0.02 \times 17) + (0.98 \times 28.84)}{1} = 28.6$$

$$G_1 = 19.5 \text{ kg/min m}^2 = \frac{19.5}{28.6} = 0.682 \text{ kmol/min m}^2$$

$$G_s = G_1(1 - y_1) = 0.682 \times (1 - 0.02) = 0.6684 \text{ kmol/min m}^2$$

$$G_2 = \frac{G_S}{1 - y_2} = \frac{0.6684}{(1 - 0.00041)} = 0.6687 \text{ kmol/min m}^2$$

$$G_{\text{average}} = \sqrt{G_1 G_2} = 0.6753 \text{ kmol/min m}^2$$

$$\text{HTU} = \frac{G}{K_{Ga}P_c} = \frac{0.6753}{1.04 \times 1} = 0.649 \text{ m}$$

$$Z = \text{NTU} \times \text{HTU} = 7.176 \times 0.649 = 4.657 \text{ m}$$

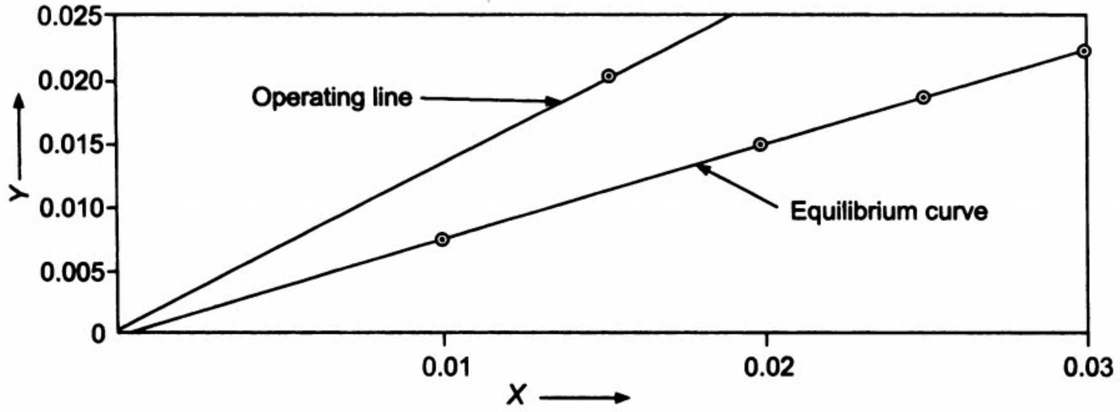


Fig. 8.23 Example 11.

**12.** $CS_2 - N_2$ mixture containing 7% $CS_2$ is to be absorbed by using absorption oil. The gas mixture enters at 24°C and 1 atm at a rate of 0.4 $m^3/s$. The vapour content is to be brought down to 0.5%. The oil enters free from $CS_2$. Raoults law is valid. Determine:

(i) Minimum liquid/gas ratio.

(ii) For a liquid/gas ratio of 1.5 times the minimum, determine the kgs of oil entering the tower and the number of theoretical stages required.

Vapour pressure of $CS_2$ = 346 mm Hg, Molecular weight of oil = 180.

**Solution.**

Average molecular weight of feed gas = (0.07 × 32) + (0.93 × 28) = 28.28

Gas flow rate = 0.4 $m^3$

$$\frac{P_0V_0}{T_0} = \frac{P_1V_1}{T_1}$$

$$V_0 = \frac{V_1}{T_1} \times T_0 = \frac{0.4}{0.297} \times 273 = 0.3677 \text{ m}^3/\text{s (At NTP condition)}$$

$$G_1 = \frac{0.3677}{22.414} \text{ kmol/s}$$

$$G_1 = 59.06 \text{ kmol/h}$$

$$G_S = 59.06(1 - 0.07) = 54.93 \text{ kmol/h}$$

Now,

$$y = mx$$

$$y = \frac{346}{760} x = 0.455x$$

$$\frac{Y}{(1+Y)} = 0.455 \frac{X}{(1+X)}$$

$$\frac{(1+Y)}{Y} = \frac{(1+X)}{0.455X}$$

$$\frac{1}{Y} = \frac{1+X-0.455X}{0.455X} = \frac{(1+0.545X)}{0.455X}$$

∴

$$Y = \frac{0.455X}{(1+0.545X)}$$

| X | 0 | 0.05 | 0.1 | 0.15 | 0.2 |
|---|---|---|---|---|---|
| Y | 0 | 0.022 | 0.043 | 0.0631 | 0.082 |

$$X_{1,\,\max} = 0.1775$$

$$\left(\frac{L_S}{G_S}\right)_{\min} = \frac{(Y_1 - Y_2)}{(X_{1,\max} - X_2)} = \frac{(0.0753 - 0.005)}{(0.1775 - 0)} = 0.396$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = (1.5 \times 0.395) = 0.594$$

$$\therefore \qquad L_S = (0.594 \times 54.93) = 32.63 \text{ kmol/h}$$

$$= 32.63 \times 180 = 5873.4 \text{ kg/h}$$

$$0.594 = \frac{(Y_1 - Y_2)}{(X_{1,\,\text{Act}} - X_2)} = \frac{(0.0753 - 0.005)}{(X_{1,\text{Act}} - 0)}$$

$$\therefore \qquad X_{1,\,\text{act}} = \frac{(0.0753 - 0.005)}{0.594} = 0.1184$$

Number of theoretical stages: 5 as shown in Fig. 8.24.

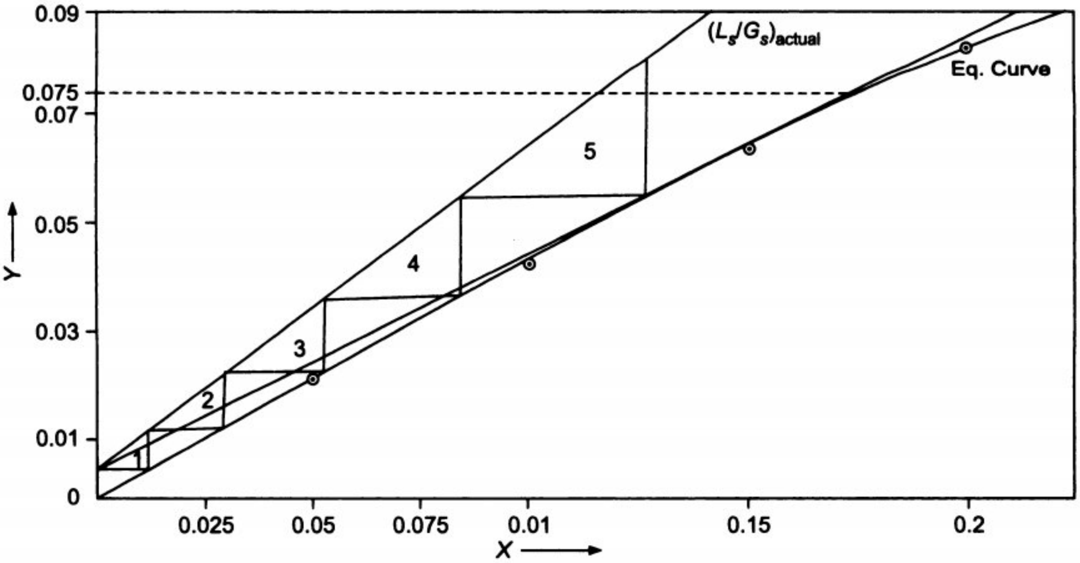


**Fig. 8.24** Example 12.

13. NH$_3$ is absorbed from a gas by using water in a scrubber under atmospheric pressure. The initial NH$_3$ content in the gas is 0.04(kmol/kmol of inert gas). The recovery of NH$_3$ by absorption is 90 %. The water enters the tower free from NH$_3$. Estimate (i) the concentration of NH$_3$ in the exiting liquid if the actual water used is 1.5 times of the minimum. (ii) the number of theoretical stages required.

| X | 0.005 | 0.01 | 0.0125 | 0.015 | 0.02 | 0.023 |
|---|---|---|---|---|---|---|
| Y | 0.0045 | 0.0102 | 0.0138 | 0.0183 | 0.0273 | 0.0327 |

where *x* and *y* are in mole ratios.

**Solution.**

$$\left(\frac{L_S}{G_S}\right) = \frac{(Y_1 - Y_2)}{(X_{1,\max} - X_2)}$$

$$\left(\frac{L_S}{G_S}\right)_{\min} = \frac{(Y_1 - Y_2)}{(X_{1,\max} - X_2)} = \frac{(0.04 - 0.004)}{(X_{1,\max} - 0)}$$

$$X_{1,\max} = 0.027 \text{ (from graph in Fig. 8.25)}$$

$$\left(\frac{L_S}{G_S}\right)_{\min} = \frac{0.036}{0.027} = 1.333$$

$$\left(\frac{L_S}{G_S}\right)_{\text{actual}} = 1.5 \times \left(\frac{L_S}{G_S}\right)_{\min} = 2$$

$$2 = \frac{(0.04 - 0.004)}{(X_{1,\text{act}} - 0)}$$

$$X_{1,\text{act}} = \frac{(0.04 - 0.004)}{2} = \frac{0.036}{2} = 0.018$$

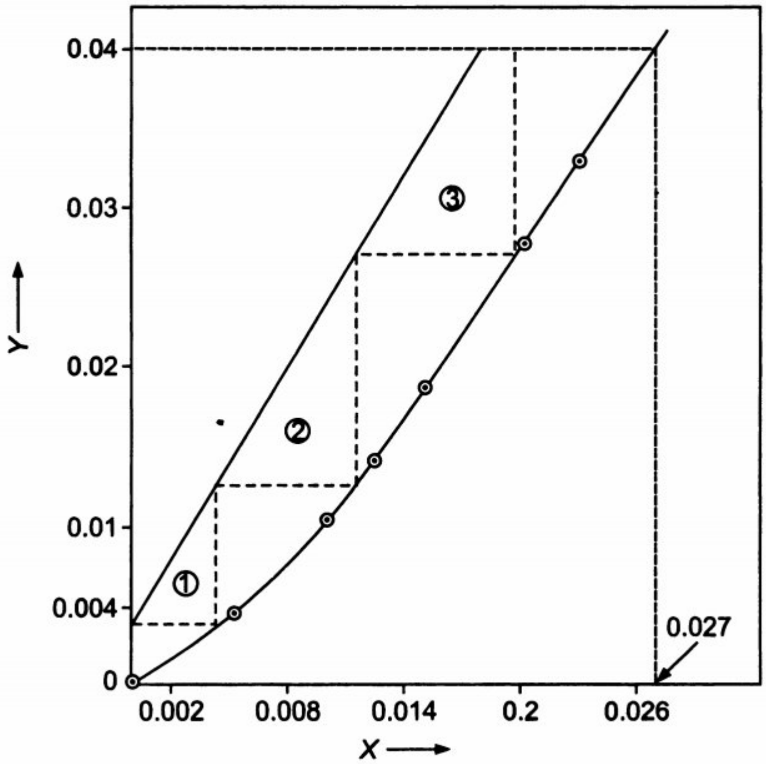


**Fig. 8.25** Example 13.

The concentration of ammonia in the exiting liquid: 0.018 kmol/kmol of water.
The number of theoretical stages required: 3 (from Fig. 8.25).

**14.** Gas from petroleum refinery has its concentration of $H_2S$ reduced from 0.03 (kmol $H_2S$/kmol inert gas) to 1% of these value by scrubbing with a solvent in a countercurrent tower at 27°C and 1 atm. The equilibrium relation is $Y^* = 2X$, where $X$ and $Y^*$ are in mole ratios. Solvent enters free of $H_2S$ and leaves at a concentration of 0.013 kmol $H_2S$/kmol of solvent. If the flow rate of incoming gas is 55.6 kmol/hr m$^2$, calculate the height of absorber used if the entire resistance to mass transfer lies in gas phase. Assume $K_ya = 0.04$ kmol/(m$^3$ of tower volume × $s$ × $\Delta y$.)

**Solution.**

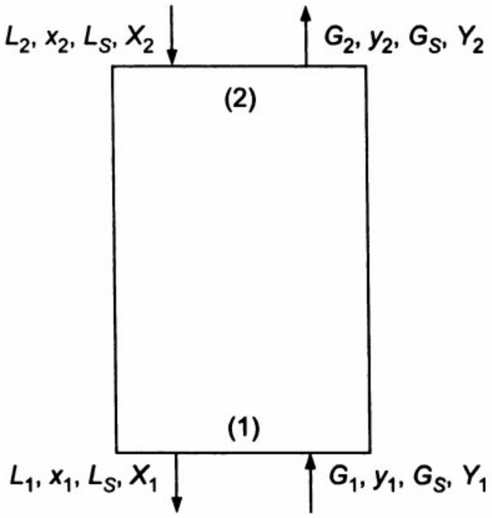


**Fig. 8.26** Example 14.

$$X_1 = 0.013; \quad X_2 = 0;$$
$$Y_1 = 0.03; \quad Y_2 = 0.0003$$
$$Y_1^* = 2 \times 0.013 = 0.026; \quad Y_2^* = 0$$
$$y_1 = \frac{Y_1}{1+Y_1} = \frac{0.03}{1.03} = 0.029$$
$$y_2 = \frac{Y_2}{1+Y_2} = \frac{0.0003}{1.0003} = 0.0003$$

Inert gas flow rate = $G_s = G_1(1 - y_1) = 55.6 \times 0.971 = 54$ kmol/h m$^2$

$$G_2 = Gs(1 + Y_2) = 54 \times 1.0003 = 54.016 \text{ kmol/h m}^2$$
$$G = (54.016\ 55.6)^{0.5} = 54.6 \text{ kmol/h m}^2$$

$$\text{NTU} = \frac{(Y_1 - Y_2)}{\left[\dfrac{(Y_1 - Y_1^*) - (Y_2 - Y_2^*)}{\ln\dfrac{(Y_1 - Y_1^*)}{(Y_2 - Y_2^*)}}\right]} = \frac{(Y_1 - Y_2)}{(\Delta Y)_{lm}}$$

$$(\Delta Y)_{lm} = \frac{(0.03 - 0.026) - (0.0003 - 0.0)}{\ln\dfrac{(0.03 - 0.026)}{(0.0003 - 0.0)}} = 1.428 \times 10^{-3}$$

$$\text{NTU} = \frac{(0.03 - 0.0003)}{(1.428 \times 10^{-3})} = 20.79$$

$$\text{HTU} = \frac{G}{K_y a} = \frac{(54.6)}{(0.04 \times 3600)} = 0.379 \text{ m}$$

Height of tower = HTU × NTU = 0.379 × 20.79 = 7.879 m

## EXERCISES

1. An air–$NH_3$ mixture containing 20% (mole) $NH_3$ is being treated with water in a packed tower to recover $NH_3$. Incoming gas rate = 1000 kg/h m$^2$. Water used is 1.5 times the minimum. The temperature is 35°C and the pressure is 1 atm. The equilibrium relation is $y^* = 0.746x$, where $x$ and $y$ are mole fraction units. Find the NTU for removing 95% $NH_3$ in the feed.
2. An air–$SO_2$ mixture containing 5% $SO_2$ is scrubbed with water to remove $SO_2$ in a packed tower. 20 kmol/s of gas mixture is to be processed, to reduce $SO_2$ concentration at exit to 0.15%. If $(L_s)$ actual is twice $(L_s)$ min, and the equilibrium relationship is $y = 30x$, HTU = 30 cms, find the height of packing to be used.
3. It is desired to absorb 95% $NH_3$ from a feed mixture containing 10% $NH_3$ and rest air. The gas enters the tower at a rate of 500 kmol/h. If water is used as solvent at a rate of 1.5 times of the minimum, estimate (i) NTU, (ii) $(L_s)$ actual.
4. An air–$SO_2$ mixture containing 5.5% $SO_2$ is scrubbed with water to remove $SO_2$. 500 kg/hr of gas mixture is to be processed and the $SO_2$ content in the exit should be brought to 0.15%. Calculate the height of packing required if the liquid used is 2.5 times the minimum liquid rate. Dilute solutions are involved in operation. The equilibrium lines are given by $y = 30x$, where $x$ and $y$ are mole fractions. The HTU is 30 cm.
5. An air–$NH_3$ mixture containing 5% $NH_3$ enters a packed tower at the rate of 500 kmol/h m$^2$. It is desired to recover 95% $NH_3$ using a liquid flow rate of 1.5 times the minimum. Estimate the height of the tower. HTU is 0.25 m. Fresh solvent enters the absorber. The equilibrium relation is $y^* = 1.08x$ where $x$ and $y$ are mole fractions.
6. A packed tower is to be designed to absorb $SO_2$ from air by scrubbing the gas with water. The entering gas contains 20% of $SO_2$ by volume and the leaving gas contains 0.5% of $SO_2$ by volume. The entering water is $SO_2$ free. The water flow to be used is twice the minimum. The airflow rate on $SO_2$ free basis is 975 kg/h m$^2$. The temperature is 30°C and pressure is 1 atm. $y^* = 21.8x$, where $x$ and $y$ are mole fractions. Find the NTU.

7. $NH_3$ is to be absorbed from air at 20°C and 1 atm pressure in a packed tower using water as absorbent. $G_S$ = 1500 kmol/h m$^3$, $L_S$ = 2000 kmol/h m$^2$. $y_1 = 0.0825$; $y_2 = 0.003$. $K_y a$ = (0.3 kmol/h m$^2$ ($\Delta y$)). Determine the height of the tower by $N_{toG}$ method.

| $X$ | 0.0164 | 0.0252 | 0.0359 | 0.0455 | 0.072 |
|---|---|---|---|---|---|
| $Y$ | 0.021 | 0.032 | 0.042 | 0.053 | 0.08 |

$X$ and $Y$ are mole ratios.

8. An air–$NH_3$ mixture containing 20 mole % of $NH_3$ is being treated with water in a packed tower to recover $NH_3$. The incoming gas rate is 700 kg/h m$^2$. The water used is 1.5 times the minimum and enters the tower free of $NH_3$. Under these conditions, 95% of $NH_3$ is absorbed from the incoming feed. If all the operating conditions remain unchanged, how much taller the tower should be to absorb 99% of $NH_3$, under the given conditions $y^* = 0.75x$ where $x$ and $y$ are mole fractions of $NH_3$ in liquid and gas phase respectively.

9. A packed tower is to be designed to recover 98% carbon dioxide from a gas mixture containing 10% carbon dioxide and 90% air using water. The equilibrium relationship is $Y = 14X$, where $Y$ = (kg $CO_2$/kg dry air) and $X$ = (kg $CO_2$/kg dry water). The water to gas rate is kept 30% more than the minimum value. Calculate the height of the tower if $(HTU)_{OG}$ is 1 metre. **(Ans:** 11.42 m**)**

10. An air–$NH_3$ mixture containing 6% of $NH_3$ is being scrubbed with water to recover 90% of $NH_3$. The mass velocities of gas and water are 3200 kg/h m$^2$ and 2700 kg/h m$^2$ respectively. The operating conditions are 25°C and 1 atm. Find NTU and height of the tower. Given that, $K_G a$ = 65 kmol/h m$^3$ atm, $y^* = 0.987x$, where $x$ and $y$ are mole fractions.

11. 500 m$^3$/h of a gas at 760 mm Hg and 35°C containing 3% by volume of toluene is absorbed using a wash oil as an absorbent to remove 95% of toluene. The wash oil enters at 35°C contains 0.5% toluene and has an average molecular weight of oil 250. The oil rate used is 1.5 times the minimum. Wash oil is assumed to be ideal. Vapour pressure of toluene is 110 mm Hg. Find the amount of wash oil used and the number of theoretical stages.

12. Ammonia is recovered from a 10% $NH_3$–air mixture by scrubbing with water in a packed tower at 20°C and 1 atm. pressure such that 99% of the $NH_3$ is removed. What is the required height of the tower? Gas and water enter at the rate of 1.2 kg/m$^2$ s and 0.94 kg/m$^2$ s respectively. Assume $K_G a$ = 0.0008 kmol/m$^3$ s. atm. The equilibrium data is as follows:

| $x$ | 0.021 | 0.031 | 0.042 | 0.053 | 0.079 | 0.106 | 0.159 |
|---|---|---|---|---|---|---|---|
| $p$ (mm Hg) | 12 | 18.2 | 24.9 | 31.7 | 50 | 69.6 | 114 |

where $x$ is the mole fraction of $NH_3$ in liquid, $p$ is the partial pressure of $NH_3$ in mm Hg.

13. An air–acetone mixture, containing 5% acetone by volume, is to be scrubbed with water in a packed tower to recover 95% of the acetone. Airflow rate is 1400 $m^3/h$ at 20°C and 1 atmosphere. The water rate is 3000 kg/h. The equilibrium relation is $Y_e = 1.68\ X$, where $Y_e$ and $X$ are mole fractions of acetone in vapour and liquid respectively. The flooding velocity is 1.56 metre per second and the operating velocity is 25% of the flooding velocity. The interfacial area of the packing is 204 $m^2/m^3$ of packing and the overall mass transfer coefficient $K_y$ is 0.40 kmol/h $m^2$ mole fraction. Estimate the diameter and packed height of the tower operating at 1 atmosphere.

14. $CO_2$ evolved during the production of ethanol by fermentation contains 1 mole ratio of alcohol. It is proposed to remove alcohol by absorption in water at 40°C. The water contains 0.0001-mole ratio of alcohol. 500 moles/hr of gas is to be processed. Equilibrium data is given by $y = 1.05x$, where $x$ and $y$ are mole fractions. Calculate the water rate for 98% absorption of alcohol by using 1.5 times the minimum liquid rate and determine the number of plates.

15. A gas stream containing a valuable hydrocarbon (molecular weight = 44) and air is to be scrubbed with a non-volatile oil (molecular weight = 300) in a tower placed with 2.54 cm Raschig rings. The entering gas analyses 10 mole % hydrocarbon and 95% of this hydrocarbon is to be recovered. The gas stream enters the bottom of the column at 2270 kg/h and the hydrocarbon free oil used is 1.5 times the minimum. Find $N_{toG}$ for this operation. The equilibrium data is as follows:

| $X$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.458 |
|---|---|---|---|---|---|---|
| $Y$ | 0 | 0.01 | 0.02 | 0.06 | 0.118 | 0.2 |

where $X$ and $Y$ are mole ratios. (ii) If the flow rate of liquid is 4600 kg/h, estimate the number of transfer units needed and the solute concentration in mole fraction in leaving liquid?

**(Ans:** (ii) 4, 0.322)

16. A soluble gas is absorbed in water using a packed tower. The equilibrium relationship may be taken as $y = 0.06x$.

| | *Terminal conditions* | |
|---|---|---|
| | Top | *Bottom* |
| $x$ | 0 | 0.08 |
| $y$ | 0.001 | 0.009 |

($x$, $y$: Mole fraction of solute in liquid and vapour phase respectively)
If the individual height of transfer units based on liquid and gas phase respectively are $H_x = 0.24$ m and $H_y = 0.36$ m, (i) what is the value of $(HTU)_{OG}$ and (ii) what is the height of packed section?

**(Ans:** (i) 0.511 m and (ii) 1.833 m)

17. An air–$NH_3$ mixture containing 20-mole % $NH_3$ is being treated with water in a packed tower to recover $NH_3$. The incoming gas rate is 1000 kg/h $m^2$. The temperature is 35°C and the total pressure is 1 atm. The water flow rate is 3000 kg/h $m^2$. 95% of incoming $NH_3$ is to be absorbed. If all the operating conditions remain unchanged, how much taller should the tower be to absorb 99% of $NH_3$? Henry's law is valid and Henry's constant is 0.746. Variations in gas flow rates may be neglected.

(**Ans:** 58.15%)